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Integration of ART2 Neural Network and Fuzzy Sets Theory for Market Segmentation

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Abstract — In order to simulate human beings' thinking, this study is dedicated to proposing a novel fuzzy neural network (FNN) to cluster the fuzzy data being collected from the fuzzy questionnaires. The proposed FNN is the integration of adaptive resonance theory 2 (ART2) neural network and fuzzy sets theory. It can handle the fuzzy inputs as well as the fuzzy weights. A case study for mobile phone market segmentation revealed that the proposed fuzzy ART2 neural network is able to cluster the fuzzy data precisely.

Keywords —ART2, Fuzzy sets theory, Fuzzy neural networks, Market segmentation.

1. INTRODUCTION

The establishment of market segmentation for analysis and decision-making using clustering analysis method can be regarded as the core issue in marketing research. Conventional market segmentation approaches usually involve designing a questionnaire to capture the opinions from the customers, and employing a multivariate statistical analysis to process the collected data from the questionnaire in order to segment appropriately the market. Nevertheless, human thinking and recognition process are quite complicated and involve a lot of uncertainty. The traditional discrete questionnaire used by these approaches can hardly obtain complete opinions from the customers. Almost all conventional clustering analysis methods for market segmentation are always performed through crisp partition, that is, one sample can be assigned to one cluster only (Punj & Stewart, 1983). This type of crisp partition method cannot describe effectively the shape and inner structure of the origin market, because the boundaries between the clusters might be fuzzy, and it is not always true that one customer belongs to one cluster only.

Artificial neural networks (ANNs) with functions such as learning and fault tolerance have been employed successfully in pattern recognition, clustering analysis, fault diagnosis, prediction, and automatic control (Masson and Wang, 1990; Kohonen, 1988b). Although their performances in these areas have been confirmed, more research is needed to determine how useful they are for the market analysis and decision-making process. In contrast to the traditional statistical methods, ANNs usually fail to make a definite and reasonable explanation for the meaning of the inner network. This feature might be advantageous or disadvantageous depending on the purpose of the study. However, the meaning of the inner network does not have much relevance to most of the applications. Fuzzy sets theory, another popular technique, has the potential advantage of handling the non-quantifiable problem with semantic judgments. It has also been successfully applied to control and decision-making. In the utilization of the fuzzy models for decision-making in a specified problem domain, however, there should be a method developed for learning the previous experiences for solving the problem. This kind of method is the so-called pre-processed data collection method. Fuzzy neural network, which integrates neural network and fuzzy sets theory, may have the potential to be developed as a pre-processed data collection method for market segmentation.

In this research, a fuzzy questionnaire is designed to get the fuzzy data sets, which represent the characteristics of the credit card holders, as the input of the proposed model for generating fuzzy market segments. This model aims to integrate fuzzy sets theory with unsupervised neural network to create a fuzzy neural network model. It takes advantages of the learning function and the capability of handling uncertainty in human recognition process. Furthermore, it employs a fuzzy clustering algorithm to carry out automatic clustering analysis.

The proposed model for market segmentation of mobile phone market is an integration of adaptive resonance theory 2 (ART2) neural network and fuzzy sets theory. The proposed fuzzy ART2 neural network not only is able to process the fuzzy inputs, but also has the fuzzy weights. This is an pilot study for combine the fuzzy set theory with the unsupervised neural network, though the main author and his colleagues have presented a fuzzy self-organizing feature map neural network for grouping the parts into several families (Kuo et al. 2001). However, it needs further visual examination for determining the number of clusters. As a result, the intelligent clustering analysis model with fuzzy ART2 neural network for this application is created to solve the general problems in clustering, such as number of groups, and assignment of marginal samples between two groups.

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Same as other clustering analysis models previously developed, the proposed model can explain completely the results from experiments. Moreover, this model seems to be more useful and practical. The advantage of this model is that fuzzy ART2, which integrates the superior characteristics of fuzzy sets theory and artificial neural network, can cluster database automatically without any visual examination. In addition, this intelligent model can manipulate directly fuzzy data for analysis and has learning capability.

The remainder of this paper is organized as follows. Section 2 overviews related background information, while the proposed approach is presented in Section 3. Section 4 summarizes the evaluation results and discussion. Finally, the concluding remarks are made in Section 5

2. BACKGROUND

This section briefly presents the necessary background information including artificial neural networks in market segmentation and fuzzy neural networks.

2.1 Artificial neural networks in market segmentation

The learning algorithms of ANNs can be divided into two categories: supervised and unsupervised. In supervised learning, the network has its output compared with a known answer, and receives feedback about any errors. The most widely applied unsupervised learning scheme is Kohonen's feature maps (Kohonen, 1982).

Venugopal and his colleague (Venugopal and Baets, 1994) presented the possible applications of ANNs in marketing management. Three examples, retail sales forecasting, direct marketing and target marketing, were employed to demonstrate the capability of ANNs. Bigus (Bigus, 1996) suggested that ANNs can be employed as a tool for data mining and presented a network with three different dimensions of data, population (sex, age, and marriage), economic information (salary and family income), and geographic information (states, cities, and level of civilization). Balakrishnan (Balakrishnan et al., 1994) compared SOM with the K-means method. The results reveal that the K-means method has a higher rate of classification through the Monte Carlo algorithm. Two years later, Balakrishnan (Balakrishnan et al., 1996) employed the frequency-sensitive competitive learning algorithm (FSCL) and the K-means method for clustering the simulated data and real-world problem data. Also, the combination of these two methods was presented. Neither the simulated nor real-world problem data can determine which method alone is better. However, the combination of the two methods seems to provide better managerial explanation for the brand choice data.

A modified two-stage method, which first uses the SOM to determine the number of clusters and the starting points and then employs the K-means method to find the final solution, is proposed by Kuo (Kuo et al., 2002) for market segmentation. The simulation results show that the modified two-stage method is slightly more accurate than the conventional two-stage method (Ward's minimum variance method followed by the K-means method) with respect to the rate of misclassification.

2.2 Fuzzy neural networks

The ANNs (Lippmann, 1987) and fuzzy model (Zadeh 1973 and Lee, 1990) have been applied in many application areas, each pairing its own merits and disadvantages. Therefore, how to successfully combine these two approaches, ANNs and fuzzy modeling, has become a very potential research area.

Generally, the traditional fuzzy system mentioned above is based on experts' knowledge. However, it is not very objective. Besides, it is very difficult to acquire robust knowledge and find available human experts (Jang, 1992). Recently, the ANN's learning algorithm has been applied to improve the performance of fuzzy system and has been shown to be a new and promising approach. Takagi and Hayashi (1991) have introduced a feed-forward ANN into the fuzzy inference, wherein a ANN represents a rule, while all membership functions are represented by only one ANN. The algorithm is divided into three major parts: (1) the partition of inference rules; (2) the identification of IF parts; and (3) the identification of THEN parts. Since each rule and all the membership functions are represented by different ANNs, they are trained separately. In other words, the parameters can't be updated concurrently.

Jang(1991, 1992) and Jang and Sun (1993) have proposed a method which transforms the fuzzy inference system into a functional equivalent adaptive network, and then employs the EBP-type algorithm to update the premise parameters and the least square method to identify the consequence parameters. Meanwhile, Fukuda and Shibata (1992), Shibata, Fukuda, Kosuge and Arai (1992), and Wang and Mendel (1992) have also presented similar methods. Moreover, Nakayama, Horikawa, Furuhashi and Uchikawa (1992) have propose a so-called FNN, which has a special structure for realizing a fuzzy inference system, wherein each membership function consists of one or two sigmoid functions for each inference rules. Owing to lack of membership function setup procedure, the rule determination and the membership function setup are decided by the so-called experts where the decision is very subjective. Lin and Lee (1991) have proposed a so-called neural-network-based fuzzy logic control system (NN-FLCS), wherein they introduced the low-level learning power of neural networks in fuzzy logic system and provided a high-level human-understandable meaning to normal connectionist architecture. In addition, Kuo and Cohen (1998a,b) have also introduced a feed-forward ANN into the fuzzy inference represented by Takagi's fuzzy modeling and applied it to a multi-senior integration, whereas Buckley and Hayashi (1994) have surveyed recent findings on learning algorithm and applications of FNNs.

Furthermore, Buckley and Hayashi have also introduced several methods in the BEP learning algorithms.

The above-mentioned FNNs are only appropriate for digital data. However, the expert's knowledge is always of fuzzy type. Thus, some researchers have attempted to address the problem. Ishibushi, Okada, Fujioka and Tanaka (1993) and Ishibuchi, Kwon and Tanaka (1995) have proposed the learning methods of neural networks to utilize not only the digital data but also the expert knowledge represented by fuzzy if-then rules. Kuo and Xue (1998c, 1999) have proposed a novel FNN whose inputs, outputs and weights are all asymmetric Gaussian functions. The learning algorithm is an EBP-type learning procedure. Kuo and his colleagues continued to improve the proposed FNN by combining the genetic algorithm (Kuo et al., 2001a and Kuo et al., 2002). Kuo and his colleagues also presented a fuzzy unsupervised neural network, fuzzy SOM (Kuo, 2001b, Kohonen, 1988a, b, and Yang and Killon, 1992) for grouping the parts. In addition, Lin (1995b) and Lin and Lu (1995a) have also presented an FNN, capable of handling both fuzzy inputs and outputs.

3. METHODOLOLGY

Section 2 has presented the background information for fuzzy neural networks as well as market segmentation. The proposed clustering scheme is presented in more detailed in this section.

3.1 Data collection and transformation

To collect data, a questionnaire is designed to solicit fuzzy judgments from the respondents. In the fuzzy questionnaire, seven semantic terms are designed to scale the different importance levels of human judgment. They are "very unimportant," "unimportant," "a little unimportant," "as important as," "a little important," "important," and "very important." In the beginning of the questionnaire, the respondent is requested to fill out a scale interval within the range from 0 to 100 for each semantic term. After that, the respondent can continue to make their judgment for each question about the motivation and preference of applying for a credit card.

Using the interval data, we can obtain the fuzzy data set $\tilde{A}=(l,\mu,u)$ in which *l* stands for the pessimistic index, *u* stands for the optimistic index, and μ stands for the average index. According to the definition of asymmetric bell-shaped fuzzy set (Kuo and Xue, 1998), the interval data can be used to generate an asymmetric bell-shaped membership function. The asymmetric fuzzy sets have been proven to have good performance in reducing the convergence time of training a neural network. The detailed processes of data collection and manipulation can be summarized in the following steps.

Step 1: Design a fuzzy semantic questionnaire

In the questionnaire, the respondent is requested to express her/his semantic opinion of each importance level using a continuous interval within the range from 0 to 100. The upper and lower bounds of the interval represent the pessimistic index (*l*) and the optimistic index (*u*), respectively. For instance, a respondent fills in the interval [34, 67] for a semantic term. That means, 34 is the pessimistic index and 67 is the optimistic index.

Step 2: Compute the average index

The average index μ in the fuzzy set is usually defined to be the geometric mean of the pessimistic index and the optimistic index. However, if the pessimistic index is equal to zero, the average index becomes the arithmetic mean of these two indices. The formulas can be written as

$$\mu=\begin{cases}(l\times u)^{1/2} & , \quad if \ \ l\neq 0\\ (l+u)/2 & , \quad if \ \ l=0\end{cases} \tag{1}$$

In the research, the data set $\tilde{A}=(l,\mu,u)$ is regarded as the semantic judgment of the respondent for a specified importance level.

Step 3: Normalize the fuzzy data sets

The normalization of $\tilde{A}=(l,\mu,u)$ can be completed by dividing the three indices *l*, μ and *u* by 100. The normalized fuzzy set is then represented as $\tilde{A}^*=(l^*,\mu^*,u^*)$.

Step 4: Summarize all the judgments

After each respondent *i* gives his/her judgment $\tilde{A}_{ik}^*=(l_{ik}^*,\mu_{ik}^*,u_{ik}^*)$ to the importance level *k*, the summation formulas for integrating all the judgments from *n* respondents can be expressed as

$$l_k^*=Min(l_{ik}^*)\,,\ \ i=1,\ldots,n \tag{2}$$

$$\mu_k^*=\left[\prod_{i=1}^{n}\mu_{ik}^*\right]^{1/n},i=1,\ldots,n \tag{3}$$

$$u_k^*=Max(u_{ik}^*)\,,i=1,\ldots,n \tag{4}$$

If l_{ik}^* is zero, μ_k^* is defined to be the arithmetic mean. That is,

$$\mu_k^*=\sum\mu_{ik}/n\ ,\quad i=1,\ldots,n$$

Step 5: Find the membership function

From Step 4, the summation data set $\tilde{A}_k^*=(l_k^*,\mu_k^*,u_k^*)$ for the importance level k can be transformed into the asymmetric bell-shaped fuzzy data set $\tilde{B}_k=(\sigma_k^L,\mu_k,\sigma_k^R)$, where

$$\mu_k = \text{the average index with the membership of 1} \tag{5}$$

$$\sigma_k^L=\frac{\mu_k^*-l_k^*}{3} \tag{6}$$

$$\sigma_k^R=\frac{u_k^*-\mu_k^*}{3} \tag{7}$$

The membership function of the fuzzy data set can be written as

$$\tilde{B}_k(x)=\begin{cases}\exp(-\frac{1}{2}(\frac{x-\mu_k}{\sigma_k^{\ L}})^2) \quad , & x<\mu_k \\ 1 \quad , & x=\mu_k \\ \exp(-\frac{1}{2}(\frac{x-\mu_k}{\sigma_k^{\ R}})^2) \quad , & otherwise\end{cases} \tag{8}$$

where μ_k , $\sigma_k^{\ L}$ and $\sigma_k^{\ R}$ are the gravity, left-side standard deviation and right-side standard deviation of the fuzzy set $\tilde{B}_k$, respectively. The purpose of this step is to prepare the fuzzy training and testing data for the FSOM neural network.

3.2 Fuzzy ART2 neural network

After the fuzzy inputs have been extracted, the proposed FNN (Kuo et al., 2004) called the fuzzy ART2 neural network is employed to automatically cluster the questionnaire data. Most of the FNNs proposed in the literatures are supervised and they only handle the actual real inputs and outputs. Though Lin (1995b), Ishibuchi, Kwon and Tanaka (1995), Kuo and Xue (1998c), and Kuo, Chen and Hwang (2001a) have presented the FNNs with fuzzy inputs, weight, and outputs, yet they are all supervised networks. However, for the purpose of clustering, supervised networks are not feasible. Though the unsupervised neural network proposed in (Kuo et al., 2001b) is fuzzy SOM, yet it needs visual examination to determine the number of clusters. The proposed fuzzy ART2 does not have the above mentioned shortcomings. Like Kuo and his colleagues' previous works, both the fuzzy inputs and weights are all asymmetric fuzzy numbers defined as $\tilde{A}=(\mu,\sigma^L,\sigma^R)_{L-R}$ and

$$\tilde{A}(x)=\begin{cases}\exp(-\frac{1}{2}(\frac{x-\mu}{\sigma^L})^2) & , x<\mu \\ 1 & , x=\mu \\ \exp(-\frac{1}{2}(\frac{x-\mu}{\sigma^R})^2) & , otherwise\end{cases} \tag{9}$$

where μ , σ^L, and σ^R represent the mean, left width and right width, respectively. Since the input vectors and connection weight vectors of the fuzzy ART2 neural network are fuzzified, the addition, multiplication and nonlinear mapping of fuzzy number numbers are necessary for defining the proposed network.

3.2.1 Operations of fuzzy numbers

The fuzzy operations are defined as follows:

$$\mu_{\tilde{X}+\tilde{Y}}(z)=\max\{\mu_{\tilde{X}}(x)\wedge\mu_{\tilde{Y}}(y)\,|\,z=x+y\} \tag{10}$$

$$\mu_{\tilde{X}\cdot\tilde{Y}}(z)=\max\{\mu_{\tilde{X}}(x)\wedge\mu_{\tilde{Y}}(y)\,|\,z=x\cdot y\} \tag{11}$$

$$\mu_{f(\overline{Net})}(z)=\max\{\mu_{\overline{Net}}(x)\,|\,z=f(x)\} \tag{12}$$

where $\tilde{X},\tilde{Y}$ and $\tilde{Z}$ are all fuzzy numbers, μ(・) denotes the membership function of each fuzzy number, and $\wedge$ is the minimum operator. The α-cut of the fuzzy numbers is $\tilde{X}$ which is defend as:

$$\tilde{X}[\alpha]=\{\ x\ \big|\ \mu_{\tilde{X}}\geq\alpha\ ,\ x\in\Re\ \}\ \text{for}\ 0<\alpha\leq 1 \tag{13}$$

Afterα-cutting the fuzzy number, the above equation can be rewritten as：

$$\tilde{X}[\alpha]=[\bar{X}[\alpha]^L,\ \bar{X}[\alpha]^U] \tag{14}$$

where $\bar{X}[\alpha]^L$ and $\bar{X}[\alpha]^U$ are the upper and the lower bounds of the α-level set. In addition, the corresponding operators are summarized in the following equations.

$$\begin{aligned}\tilde{X}[\alpha]+\tilde{Y}[\alpha]&=[\bar{X}[\alpha]^L,\bar{X}[\alpha]^U]+[\bar{Y}[\alpha]^L,\bar{Y}[\alpha]^U]\\&=[\bar{X}[\alpha]^L+\bar{Y}[\alpha]^L,\bar{X}[\alpha]^U+\bar{Y}[\alpha]^U]\end{aligned} \tag{15}$$

$$\begin{aligned}\tilde{X}[\alpha]-\tilde{Y}[\alpha]&=[\bar{X}[\alpha]^L,\bar{X}[\alpha]^U]-[\bar{Y}[\alpha]^L,\bar{Y}[\alpha]^U]\\&=[\bar{X}[\alpha]^L,\bar{X}[\alpha]^U]+[-\bar{Y}[\alpha]^U,-\bar{Y}[\alpha]^L]\\&=[\bar{X}[\alpha]^L-\bar{Y}[\alpha]^U,\bar{X}[\alpha]^U-\bar{Y}[\alpha]^L]\end{aligned} \tag{16}$$

$$\begin{aligned}&\tilde{X}[\alpha]\cdot\tilde{Y}[\alpha]\\&=[\bar{X}[\alpha]^L,\bar{X}[\alpha]^U]\cdot[\bar{Y}[\alpha]^L,\bar{Y}[\alpha]^U]\\&=[\min\{\bar{X}[\alpha]^L\cdot\bar{Y}[\alpha]^L,\bar{X}[\alpha]^L\cdot\bar{Y}[\alpha]^U\\&\quad,\bar{X}[\alpha]^U\cdot\bar{Y}[\alpha]^L,\bar{X}[\alpha]^U\cdot\bar{Y}[\alpha]^U\}\\&\quad,\max\{\bar{X}[\alpha]^L\cdot\bar{Y}[\alpha]^L,\bar{X}[\alpha]^L\cdot\bar{Y}[\alpha]^U\\&\quad,\bar{X}[\alpha]^U\cdot\bar{Y}[\alpha]^L,\bar{X}[\alpha]^U\cdot\bar{Y}[\alpha]^U\}]\end{aligned} \tag{17}$$

$$\begin{aligned}&\tilde{X}[\alpha]/\tilde{Y}[\alpha]\\&=[\bar{X}[\alpha]^L,\bar{X}[\alpha]^U]/[\bar{Y}[\alpha]^L,\bar{Y}[\alpha]^U]\\&=[\min\{\bar{X}[\alpha]^L/\bar{Y}[\alpha]^L,\bar{X}[\alpha]^L/\bar{Y}[\alpha]^U\\&\quad,\bar{X}[\alpha]^U/\bar{Y}[\alpha]^L/\bar{X}[\alpha]^U/\bar{Y}[\alpha]^U\}\\&\quad,\max\{\bar{X}[\alpha]^L/\bar{Y}[\alpha]^L/\bar{X}[\alpha]^L/\bar{Y}[\alpha]^U\\&\quad,\bar{X}[\alpha]^U/\bar{Y}[\alpha]^L,\bar{X}[\alpha]^U/\bar{Y}[\alpha]^U\}]\end{aligned} \tag{18}$$

$$\begin{aligned}f(\overline{Net}[\alpha])&=f([\overline{Net}[\alpha]^L,\overline{Net}[\alpha]^U])\\&=[f(\overline{Net}[\alpha]^L),f(\overline{Net}[\alpha]^U)]\end{aligned} \tag{19}$$

Thus, under the assumption of $0\leq\bar{Y}[\alpha]^L\leq\bar{Y}[\alpha]^U$, $\tilde{X}[\alpha]\cdot\tilde{Y}[\alpha]$ can be rewritten as:

$$\begin{aligned}&[\min\{\bar{X}[\alpha]^L\cdot\bar{Y}[\alpha]^L,\bar{X}[\alpha]^L\cdot\bar{Y}[\alpha]^U\}\\&\ ,\max\{\bar{X}[\alpha]^U\cdot\bar{Y}[\alpha]^L,\bar{X}[\alpha]^U\cdot\bar{Y}[\alpha]^U\}]\end{aligned} \tag{20}$$

3.2.2 Network structure

Figure 1 presents the framework of fuzzy ART2 neural

network. The input and output relation of the proposed fuzzy ART2 neural network is defined by extension principle and can be written as follows:
Input layer (F1 layer)：

$$\tilde{O}_{pi} = \tilde{X}_{pi} \ , \quad i = 1, 2, ..., n \tag{21}$$

The F1 layer consists of six types of units (the *w*, *x*, *u*, *v*, *p* and *q* units.).

$$\tilde{w}_{kpi} \ , \quad i = 1, 2, ..., n \tag{22}$$

$$\tilde{x}_{kpi} \ , \quad i = 1, 2, ..., n \tag{23}$$

$$\tilde{u}_{kpi} \ , \quad i = 1, 2, ..., n \tag{24}$$

$$\tilde{v}_{kpi} \ , \quad i = 1, 2, ..., n \tag{25}$$

$$\tilde{p}_{kpi} \ , \quad i = 1, 2, ..., n \tag{26}$$

$$\tilde{q}_{kpi} \ , \quad i = 1, 2, ..., n \tag{27}$$

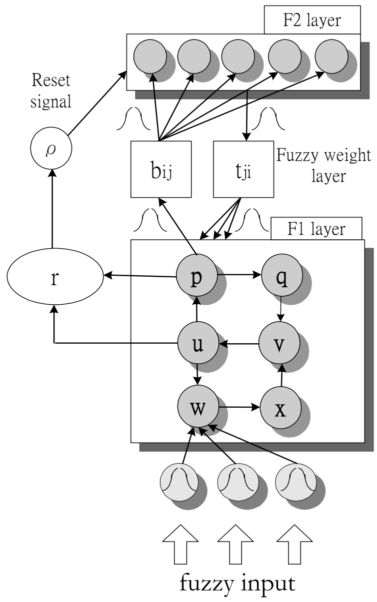

Figure 1. The structure of fuzzy ART2 neural network.

Weight layer:
The weight layer consists of two types of weights, down-top and top-down weights.

Down-top weight: $\tilde{b}_{ij} \ , \ j = 1, 2, ..., m$ 。 (28)

Top-down weight: $\tilde{t}_{ji} \ , \ j = 1, 2, ..., m$ 。 (29)

Output layer (F2 layer):
1. Calculate the fuzzy vector between the fuzzy weight, down-top, and p unit fuzzy vector of F1 layer for each input node. The fuzzy vector is defined as

$$\tilde{T}_j = \vec{\tilde{p}}_{ij} \otimes \vec{\tilde{b}}_{ij} \tag{30}$$

where $\otimes$ is the fuzzy delete operation.
2. Apply the transformation method propose by Chen and Hwang (1992) to defuzzify the fuzzy vector and compute the defuzzified values g_{ij}.
3. Choose the winner with the maximum T_j.

$$T_j = \sum_i g_{ij} \quad , \tag{31}$$

$$T_j^* = \max_j \{T_j\} \tag{32}$$

Reset or resonance layer
The layer will decide the inputting fuzzy vector is "reset" or "resonance" through vigilance parameter testing. The check for a reset gives $\| r \|$. However, $(\| r \| + e) < \rho$ for valid value of ρ (ρ is the vigilance parameter), so the winning cluster unit will be allowed to learn current pattern.

3.2.3 Learning algorithm

For the above equations, the winner unit is calculated for fuzzy inputs and the fuzzy weights. The fuzzy relation of network structure and parameters definition can be found as follows.
Parameters definition:
n : number of input units.
m : number of cluster units.
a, b : fixed weights in the F1 layer.
c : fixed weight used in testing for reset.
d : activation of winning of F2 units.
e: a small parameter introduced to prevent division by zero where the norm of a vector is zero.
θ: noise suppression parameter.
ρ: vigilance parameter.
Input layer (F1 layer):

$$\tilde{X}_i[\alpha] = [\bar{X}_i[\alpha]^L, \bar{X}_i[\alpha]^U] \ , \quad i = 1, 2, ..., n \tag{33}$$

$$\begin{aligned} &\vec{\tilde{X}}[\alpha] \\ &= (\ \tilde{X}_1[\alpha], ..., \tilde{X}_i[\alpha], ..., \tilde{X}_n[\alpha]\ ) \\ &= (\ [\bar{X}_1[\alpha]^L, \bar{X}_1[\alpha]^U], ..., [\bar{X}_i[\alpha]^L, \bar{X}_i[\alpha]^U] \\ &\quad , ...[\bar{X}_n[\alpha]^L, \bar{X}_n[\alpha]^U]\ ) \end{aligned} \tag{34}$$

Six types of units:
The w unit:

$$\begin{aligned} &\vec{\tilde{w}}_i[\alpha] \\ &= (\tilde{X}_1[\alpha], ..., \tilde{X}_i[\alpha], ..., \tilde{X}_n[\alpha]\ ) \\ &\quad + a\ (\ \tilde{u}_1[\alpha], ..., \tilde{u}_i[\alpha], ..., \tilde{u}_n[\alpha]\ ) \\ &= ([\bar{X}_1[\alpha]^L, \bar{X}_1[\alpha]^U], ..., [\bar{X}_i[\alpha]^L, \bar{X}_i[\alpha]^U] \end{aligned}$$

$$
\begin{aligned}
&,...[\overline{X}_n[\alpha]^L,\overline{X}_n[\alpha]^U])+([a\,\overline{u}_1[\alpha]^L,a\,\overline{u}_1[\alpha]^U]\\
&,...,[a\,\overline{u}_i[\alpha]^L,a\,\overline{u}_i[\alpha]^U],...[a\,\overline{u}_n[\alpha]^L,a\,\overline{u}_n[\alpha]^U])\\
&=([\overline{X}_1[\alpha]^L+a\,\overline{u}_1[\alpha]^L,\overline{X}_1[\alpha]^U+a\,\overline{u}_1[\alpha]^U],...,\\
&[\overline{X}_i[\alpha]^L+a\,\overline{u}_i[\alpha]^L,\overline{X}_i[\alpha]^U+a\,\overline{u}_i[\alpha]^U],...,\\
&[\overline{X}_n[\alpha]^L+a\,\overline{u}_n[\alpha]^L,\overline{X}_n[\alpha]^U+a\,\overline{u}_n[\alpha]^U])\\
&=([\overline{w}_1[\alpha]^L,\overline{w}_1[\alpha]^U],...,[\overline{w}_i[\alpha]^L,\overline{w}_i[\alpha]^U]\\
&,...[\overline{w}_n[\alpha]^L,\overline{w}_n[\alpha]^U])\\
&=(\tilde{w}_1[\alpha],...,\tilde{w}_i[\alpha],...,\tilde{w}_n[\alpha])
\end{aligned}
\tag{35}
$$

The *x* unit:

$$
\begin{aligned}
\vec{\tilde{x}}_i[\alpha]&=\frac{\vec{\tilde{w}}}{e+\left\|\vec{\tilde{w}}\right\|}\\
&=\frac{[\overline{w}_i[\alpha]^L,\overline{w}_i[\alpha]^U]}{e+(\sum_i\{\overline{w}_i[\alpha]^L,\overline{w}_i[\alpha]^U\}^2)^{1/2}}\\
&=\frac{[\overline{w}_1[\alpha]^L,\overline{w}_1[\alpha]^U]}{e+[\overline{W}_i[\alpha]^L,\overline{W}_i[\alpha]^U]}\\
&=\frac{[\overline{w}_i[\alpha]^L,\overline{w}_i[\alpha]^U]}{[e+\overline{W}_i[\alpha]^L,e+\overline{W}_i[\alpha]^U]}\\
&=[\frac{\overline{w}_i[\alpha]^L}{e+\overline{W}_i[\alpha]^U},\frac{\overline{w}_i[\alpha]^U}{e+\overline{W}_i[\alpha]^L}]\\
&=(\tilde{x}_1[\alpha],...,\tilde{x}_i[\alpha],...,\tilde{x}_n[\alpha])
\end{aligned}
\tag{36}
$$

$,\overline{W}[\alpha]=\sum\overline{w}[\alpha]$.

The *u* unit:

$$
\begin{aligned}
&\vec{\tilde{u}}_i[\alpha]\\
&=\frac{\vec{\tilde{v}}}{e+\left\|\vec{\tilde{v}}\right\|}\\
&=\frac{[\overline{v}_i[\alpha]^L,\overline{v}_i[\alpha]^U]}{e+(\sum_i\{\overline{v}_i[\alpha]^L,\overline{v}_i[\alpha]^U\}^2)^{1/2}}\\
&=\frac{[\overline{v}_i[\alpha]^L,\overline{v}_i[\alpha]^U]}{e+[\overline{V}_i[\alpha]^L,\overline{V}_i[\alpha]^U]}\\
&=\frac{[\overline{v}_i[\alpha]^L,\overline{vi}[\alpha]^U]}{[e+\overline{V}_i[\alpha]^L,e+\overline{V}_i[\alpha]^U]}\\
&=[\frac{\overline{v}_i[\alpha]^L}{e+\overline{V}_i[\alpha]^U},\frac{\overline{v}_i[\alpha]^U}{e+\overline{V}_i[\alpha]^L}]\\
&=(\tilde{u}_1[\alpha],...,\tilde{u}_i[\alpha],...,\tilde{u}_n[\alpha])
\end{aligned}
\tag{37}
$$

$,\overline{V}[\alpha]=\sum\overline{v}[\alpha]$.

The *q* unit:

$$
\begin{aligned}
\vec{\tilde{q}}_i[\alpha]&=\frac{\vec{\tilde{p}}}{e+\left\|\vec{\tilde{p}}\right\|}\\
&=\frac{[\overline{p}_i[\alpha]^L,\overline{p}_i[\alpha]^U]}{[e,e]+(\sum_i\{\overline{p}_i[\alpha]^L,\overline{p}_i[\alpha]^U\}^2)^{1/2}}\\
&=\frac{[\overline{p}_i[\alpha]^L,\overline{p}_i[\alpha]^U]}{e+[\overline{P}_i[\alpha]^L,\overline{P}_i[\alpha]^U]}\\
&=\frac{[\overline{p}_i[\alpha]^L,\overline{p}_i[\alpha]^U]}{[e+\overline{P}_i[\alpha]^L,e+\overline{P}_i[\alpha]^U]}\\
&=[\frac{\overline{p}_i[\alpha]^L}{e+\overline{P}_i[\alpha]^U},\frac{\overline{p}_i[\alpha]^U}{e+\overline{P}_i[\alpha]^L}]\\
&=(\tilde{q}_1[\alpha],...,\tilde{q}_i[\alpha],...,\tilde{q}_n[\alpha])
\end{aligned}
\tag{38}
$$

$,\ \overline{P}[\alpha]=\sum\overline{p}[\alpha]$.

The *p* unit:

$$
\begin{aligned}
&\vec{\tilde{p}}_i[\alpha]\\
&=(\tilde{u}_1[\alpha],...,\tilde{u}_i[\alpha],...,\tilde{u}_n[\alpha])\\
&+d\,(\tilde{t}_i[\alpha],...,\tilde{t}_i[\alpha],...,\tilde{t}_n[\alpha])\\
&=([\overline{u}_1[\alpha]^L,\overline{u}_1[\alpha]^U],...,[\overline{u}_i[\alpha]^L,\overline{u}_i[\alpha]^U]\\
&,...,[\overline{u}_n[\alpha]^L,\overline{u}_n[\alpha]^U])+([d\,\overline{t}_1[\alpha]^L,d\,\overline{t}_1[\alpha]^U]\\
&,...,[d\,\overline{t}_i[\alpha]^L,d\,\overline{t}_i[\alpha]^U],...[d\,\overline{t}_n[\alpha]^L,d\,\overline{t}_n[\alpha]^U])\\
&=([\overline{u}_1[\alpha]^L+d\,\overline{t}_1[\alpha]^L,\overline{u}_1[\alpha]^U+d\,\overline{t}_1[\alpha]^U]\\
&,...,[\overline{u}_i[\alpha]^L+d\,\overline{t}_i[\alpha]^L,\overline{u}_i[\alpha]^U+d\,\overline{t}_i[\alpha]^U]\\
&,...,[\overline{u}_n[\alpha]^L+d\,\overline{t}_n[\alpha]^L,\overline{u}_n[\alpha]^U+d\,\overline{t}_n[\alpha]^U])\\
&=([\overline{p}_1[\alpha]^L,\overline{p}_1[\alpha]^U],...,[\overline{p}_i[\alpha]^L,\overline{p}_i[\alpha]^U]\\
&,...[\overline{p}_n[\alpha]^L,\overline{p}_n[\alpha]^U])=(\tilde{p}_1[\alpha],...,\tilde{p}_i[\alpha],...,\tilde{p}_n[\alpha])
\end{aligned}
\tag{39}
$$

The *v* unit:

For this unit, apply the transformation method proposed by Chen and Hwang (1992) to defuzzify $\tilde{x}$ and $\tilde{q}$, and then test the defuzzified numbers through the activation functions shown in the below:

$$
f(X)=\begin{cases}X & \text{if} \quad X\geq\theta\\ 0 & \text{if} \quad X<\theta\end{cases}
\tag{40}
$$

$$
f(Q)=\begin{cases}Q & \text{if} \quad Q\geq\theta\\ 0 & \text{if} \quad Q<\theta\end{cases}
\tag{41}
$$

This functions treats any signal, which is less than θ as noise and suppresses it (set it to zero and fuzzy numbers to [0,0]).

Weight layer:

Down-top weight:

$$
\tilde{b}_{ij}[\alpha]=[\overline{b}_{ij}[\alpha]^L,\overline{b}_{ij}[\alpha]^U],\ j=1,2,...,m
\tag{42}
$$

Top-down weight:

$$\tilde{t}_{ji}[\alpha]=[\overline{t}_{ji}[\alpha]^{L},\overline{t}_{ji}[\alpha]^{U}]\ ,\ j=1,2,...,m \tag{43}$$

Transform the asymmetric fuzzy numbers:

$$\begin{cases}\overline{b}_{ij}[\alpha]^{L}=\mu_{ij}-\sigma_{ij}^{L}\cdot(-2\ln\alpha)^{1/2}\ ,\ \ if\ x\le\mu\\ \overline{b}_{ij}[\alpha]^{U}=\mu_{ij}+\sigma_{ij}^{R}\cdot(-2\ln\alpha)^{1/2}\ ,\ \ if\ x>\mu\end{cases} \tag{44}$$

$$\begin{cases}\overline{t}_{ji}[\alpha]^{L}=\mu_{ji}-\sigma_{ji}^{L}\cdot(-2\ln\alpha)^{1/2}\ ,\ \ if\ x\le\mu\\ \overline{t}_{ji}[\alpha]^{U}=\mu_{ji}+\sigma_{ji}^{R}\cdot(-2\ln\alpha)^{1/2}\ ,\ \ if\ x>\mu\end{cases} \tag{45}$$

Output layer (F2 layer):

1. Calculate the fuzzy vector between the fuzzy weight, down-top, and p unit fuzzy vector of F1 layer for each input node. The fuzzy vector is defined as

$$\begin{aligned}\tilde{T}_{j}&=\vec{\tilde{p}}_{ij}\otimes\vec{\tilde{b}}_{ij}\\ &=[\ \overline{p}_{ij}[\alpha]^{L},\overline{p}_{ij}[\alpha]^{U}\ ]\otimes[\ \overline{b}_{ij}[\alpha]^{L},\overline{b}_{ij}[\alpha]^{U}\ ]\\ &=[[\ \overline{p}_{ji}[\alpha]^{L}\cdot\overline{b}_{ij}[\alpha]^{L}\ ],[\ \overline{p}_{i}[\alpha]^{U},\overline{b}_{ij}[\alpha]^{U}\ ]]\\ &=[\overline{T}_{j}[\alpha]^{L},\overline{T}_{j}[\alpha]^{U}\ ]\end{aligned} \tag{46}$$

2. Apply the transformation method proposed by Chen and Hwang (1992) to defuzzify the fuzzy vector and compute the defuzzified values g_{ij}.
3. Choose the winner with the maximum Tj.

Reset or resonance layer:

This layer decides whether the inputted fuzzy vector is "reset" or "resonance" through vigilance parameter testing. The check for a reset gives $\|r\|$.

$$\begin{aligned}&\|r\|\\ &=\frac{\|\vec{\tilde{u}}+c\,\vec{\tilde{p}}\|}{e+\|\vec{\tilde{u}}\|+c\|\vec{\tilde{p}}\|}\\ &=\frac{\|(\ \tilde{u}_{i}[\alpha],...,\tilde{u}_{i}[\alpha],...,\tilde{u}_{n}[\alpha]\ )+c\,(\ \tilde{p}_{i}[\alpha],...,\tilde{p}_{i}[\alpha],...,\tilde{p}_{n}[\alpha]\,)\|}{e+(\sum_{i}\{\ \overline{u}_{i}[\alpha]^{L},\overline{u}_{i}[\alpha]^{U}\ \}^{2})^{1/2}+c(\sum_{i}\{\ \overline{p}_{i}[\alpha]^{L},\overline{p}_{i}[\alpha]^{U}\ \}^{2})^{1/2}}\\ &=\frac{(\sum_{i}\{\,(\overline{u}_{i}[\alpha]^{L}+c\overline{p}[\alpha]^{L},\overline{u}_{i}[\alpha]^{U}+c\overline{p}[\alpha]^{U}\,)\,\}^{2})^{1/2}}{([\ e+\overline{U}_{i}[\alpha]^{L}+c\overline{P}_{i}[\alpha]^{L}\ ],[\ e+\overline{U}_{i}[\alpha]^{U}+c\overline{P}_{i}[\alpha]^{U}\ ])}\\ &=[\frac{\overline{\ _{i}+cp[\alpha]^{L}}}{e+\overline{U}_{i}[\alpha]^{U}+c\overline{P}_{i}[\alpha]^{U}},\frac{\overline{u_{i}+cp[\alpha]^{U}}}{e+\overline{U}_{i}[\alpha]^{L}+c\overline{P}_{i}[\alpha]^{L}}]\end{aligned} \tag{47}$$

If the layer is resonant, modify the weights as below:

$$\begin{aligned}\tilde{b}_{ij}&=\frac{\tilde{u}_{ij}}{1-d}\\ &=[\ \frac{\overline{u}_{ij}[\alpha]^{L}}{1-d},\frac{\overline{u}_{ij}[\alpha]^{U}}{1-d}\ \ ]\end{aligned} \tag{48}$$

$$\begin{aligned}\tilde{t}_{ji}&=\frac{\tilde{u}_{ji}}{1-d}\\ &=[\ \frac{\overline{u}_{ij}[\alpha]^{L}}{1-d},\frac{\overline{u}_{ij}[\alpha]^{U}}{1-d}\ ]\end{aligned} \tag{49}$$

3.2.4 Learning procedures

The learning procedures (Figure 2) of the fuzzy ART2 neural network are summarized as follows.

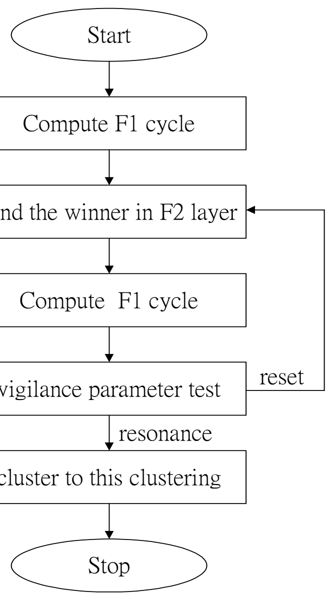

Figure 2. The fuzzy ART2 neural network learning algorithm.

Step 1. Input the fuzzy vector into F1 layer and compute the six units until the u or p unit value is convergent.

Step 2. Calculate the fuzzy vector between the fuzzy weight —down-top and p unit fuzzy vector of F1 layer for each input node. Find the maximum value and decide which is the winner .

Step 3. Input the fuzzy vector into F1 layer and compute the six units until the u unit value is convergent, again.

Step 4. Test the vigilance parameter and then decide whether the layer state is "reset" or "resonance." If the state is "reset," set the winner's Tj to be zero and repeat Step 2 to find the other winner.

If all winners don't pass the vigilance parameter test, it is necessary to create a new cluster and add the corresponding weights. If the state is "resonance", just make the current fuzzy input belong to this cluster and modify the corresponding weights.

4. MODEL EVALUATION RESULTS

This section intends to apply the questionnaire data being collected from the mobile phone users in Taipei City for verifying the proposed fuzzy ART2 neural network's feasibility. The reason for choosing mobile phone market is that usage percentage has reached 86.32% in May 2001 in Taiwan. Most of the mobile companies have tried to remain their own market shares and competitive advantage.

It is believed that one user with more than one mobile phones has become the trend. Thus, there are still a lot of potentials for the mobile phone in the near future.

Therefore, in here, the proposed fuzzy ART2 neural network is employed to cluster the customers into several segmentations. Based on these segmentations, the mobile phone company is able to make the strategy in order to create her own niche and competence. The main steps are as shown in Figure 3 and they are questionnaire design, data statistics, clustering, and statistical test.

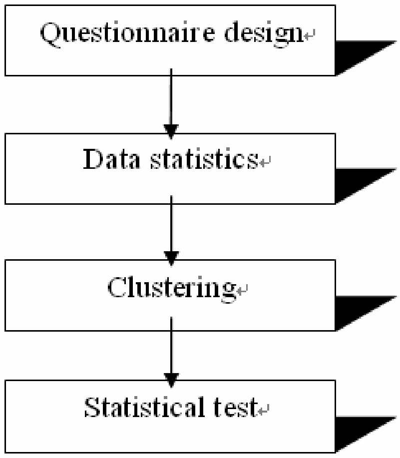

Figure 3. The case study flowchart.

4.1 Questionnaire design

The questionnaire mainly consists of five parts: (1) previous expenditure factors, (2) respondent's scale statistics, (3) buying motivation factors, (4) living style factors, and (5) geographic factors.

(1) Previous expenditure factors

From surveying customers' previous expenditure experiences, the business is able to figure out customers' expenditure characteristics by transforming these survey data into statistic data. Further, the business can make more accurate decision for the marketing strategy. Totally, there are six questions in this part and they are as follows:

1. Length for using the mobile phone,
2. Way to obtain the mobile phone information,
3. Telecommunication company used,
4. Mobile phone used,
5. Expense for mobile phone accessory used, and
6. Monthly fee.

(2) Linguistic Scales

Considering different cognition for each linguistic term, seven scales, Very Important, Important, Slightly Important, Equally Important, Slightly Not Important, Not Important, Not Very Important are determined by the respondents. The respondents can fill in the ranges in [0, 100] for seven scales themselves. By using the summarized function mentioned in Section 3, fuzzy number for each linguistic term can be defined. They will become the input of the fuzzy ART2 neural network.

(3) Motivation for buying the products factors

The main objective of this part is to understand the importance degree of each mobile phone function, which may influence customers' selection. Totally, there are twelve factors as follows:

1. Price,
2. Style,
3. Branding,
4. Length of stand by,
5. Function and convenience,
6. Monthly renting fee,
7. Communication rate,
8. Convenience for applying,
9. Communication quality,
10. Bill accuracy,
11. Communication scope,
12. Special functions.

(4) Life style variables

Based on Berkman & Gilson(1974), they stated that life style is the consistant behavior. It can influence consumer behavior. Thus, this study select 20 items for life style according to Lesser and Hughes. These twenty variables are applied for clustering.

(5) Demographic variables

This part is mainly to collect respondents' fundamental data in order to understand their society, home and economical condition. Kotler (1998) presented ten possible items. This study selects six items, sex, age, education, life cycle, job, and income, as the variables.

4.2 Data statistics

4.2.1 Questionnaire survey

(1) Population

The survey time is ranged from September 1, 2001 to April 15, 2002. The place is in Taipei main station and its neighborhood areas. Thus, the population is set as the mobile phone users in Taipei City.

(2) Sampling

Basically, the respondent is selected every ten persons. The respondent is asked whether he/she has used mobile phone or not. If yes, then he/she is our candidate. Since the current study's questionnaire is fuzzy questionnaire, thus, only one respondent is asked to fill the questionnaire each time. The survey person has to explain how to fill the questionnaire. This can dramatically increase the efficiency and affectivity of survey.

(3) Sampling size

Based on Roscore (1975), there at lease should be 200 samples. Thus, totally 304 respondents are surveyed. Fourteen questionnaires are invalid. So, there are 290 valid questionnaires.

4.2.2 Samples structure

Table 1 lists the samples structure.

4.3 Clustering

4.3.1 Extraction of the fuzzy features

In order to obtain the fuzzy inputs for fuzzy ART2 neural network, summarize the second part of the questionnaire by using procedures mentioned in Subsection 3.1. Though these steps, seven different asymmetric bell-shaped fuzzy numbers can be found. Then, according to different α-cut levels, transform the fuzzy number into fuzzy ranges like presented in Table 2.

Table 1. Sampling structure

Variables		Number of Persons	Percentage (%)
Sex	Male	153	52.76
	Female	137	47.24
Age	Under 20	65	22.41
	21~30	98	33.79
	31~40	73	25.17
	41~50	43	14.83
	50~60	8	2.76
	Above60	3	1.04
Marriage	Single	147	50.69
	Married	143	49.31
Education	Junior high school	6	2.07
	High school	43	14.83
	College	209	72.07
	Graduate school	32	11.03
Occupation	Government employees	38	7.62
	Students	105	36.21
	Service industry	40	13.78
	House wives	9	3.10
	Manufacturing industry	43	14.82
	Professional industry	18	6.21
	Business	28	9.66
	Others	9	3.10
Income	Under 10,000	71	24.48
	10,001~20,000	49	16.89
	20,001~30,000	58	20.00
	30,001~40,000	45	15.52
	40,001~50,000	46	15.86
	50,001~60,000	12	4.14
	60,001~70,000	5	1.72
	Above 70,001	4	1.38

Table 2. The α-cut levels under different linguistic terms

α -cut levels	0		0.2		0.4		0.6		0.8	
Strongly not agree (1)	0.03	0.27	0.078235	0.221765	0.095851	0.204149	0.109569	0.190431	0.123278	0.176722
Not agree (2)	0.15	0.38	0.196225	0.333775	0.213107	0.316893	0.226254	0.303746	0.239392	0.290608
Slightly not agree(3)	0.27	0.49	0.314215	0.445785	0.330363	0.429637	0.342939	0.417061	0.355505	0.404495
No comments(4)	0.41	0.66	0.460245	0.609755	0.478595	0.591405	0.492885	0.577115	0.507165	0.562835
Slightly agree(5)	0.57	0.78	0.612206	0.737794	0.627619	0.722381	0.639623	0.710377	0.651618	0.698382
Agree (6)	0.65	0.86	0.692206	0.817794	0.707619	0.802381	0.719623	0.790377	0.731618	0.778382
Strongly agree (7)	0.73	0.97	0.778235	0.921765	0.795851	0.904149	0.809569	0.890431	0.823278	0.876722

The survey data in the forth part of the questionnaire, life style variables, is applied as the variables for clustering. Thus, use Table 2 to transform the respondent's data to different α-cut levels fuzzy ranges as the fuzzy ART2 neural network's input. Then, fuzzy ART2 can automatically cluster the respondents into several segmentations.

4.3.2 Determination of the number of clusters

The number of cluster for fuzzy ART2 neural network is determined by vigilance parameter. Once the parameter is well set up, the fuzzy ART2 neural network can cluster the data automatically. Also, examining the within cluster variances under different vigilance parameters with different numbers of clusters can determine the final number of clusters. The current study tries to examine the Wilk's Lambda value for determining the number of the clusters. Table 3 reveals that as number of clusters increases from 4 to 5, the Wilk's Lambda value has the largest variance, or increasing. Thus, it is reasonable to select 4 as the number of the clusters.

Table 3. Each cluster's Wilk's Lambda value.

Vigilance parameter	Number of clusters	Wilk's Lambda
0.97	6	0.0995
0.95	5	0.1240
0.91	***4***	***0.1532***
0.84	3	0.2666

5. CONCLUSIONS

This study has presented the application of fuzzy ART2 neural network for market segmentation. Most of the conventional applications of ART2 neural network are on engineering issues, like image processing or cell manufacturing. This study extends its applications to a new but feasible direction. Especially, fuzzy sets theory is integrated. This is because that fuzzy questionnaire is more like the human beings' thinking. It can represent really what the respondents' considerations. The model evaluation results have showed that fuzzy ART2 neural network is really able to conform respondents' thinking. The industries can apply these results to make their own marketing strategies. In the future, one important issue is to look for a better way for determining the parameters of ART2 neural network. In the current study, we only can use Wilk's Lambda value as the criteria. Also, other applications of fuzzy ART2 are desirable, though it has been applied for group technology (Kuo et al., 2004).

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