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An Extension of the Multi-machine Multi-level Proportional Lot Sizing and Scheduling Model for Product-Life-Cycle Demand

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Abstract**—**Often lot sizing and scheduling models have neglected the consideration of influence of product life cycles (PLCs). But, accordingly as desirable in a firm's production strategy, decisions of lot sizing scheduling may have to respond to the stage changes in a product's life cycle. In this paper, we propose an extension of the lot sizing scheduling, the multi-item, multi-level product-structure and multi-machine proportional lot sizing scheduling model by Kimms (1999), by the concept of PLC. The advantages have been realized and include both the setup and holding cost improvements and an improved lot sizing scheduling that better matches the products' demands in the PLCs. In addition, the combined effects of the setup (cost) learning effect and cash flow are examined in the model and the genetic algorithm is adopted as the solution aid tool. A numerical example is provided and demonstrates the advantages of this model and the effects of the factors considered.

Keywords**—**Proportional lot sizing and scheduling problem; Product life cycle; Cash flow, Learning curve; Genetic algorithms

1. INTRODUCTION

Lot sizing and scheduling or determining which quantities of what items have to be produced at what time to meet the demands of the products is two of the most important problems in production planning. Many researchers have developed models and approaches to these problems (e.g., see Drexl and Kimms, 1997 for a good review). Four types of problems have been reported and investigated and include the capacitated lot sizing problems (e.g. Barany et al. , 1984; Chen and Thizy, 1990; Haase, 1996), discrete lot sizing and scheduling problems (e.g. Fleschman, 1994), continuous setup lot sizing problems (e.g. Bitran and Matsuo, 1986;Karnarkar and Schrage, 1985), and proportional lot sizing and scheduling problems (PLSP) (e.g. Drexl and Haase, 1995; Kimms, 1996). In this paper, we intend to propose an extension of the PLSP with multi-item multi-level-product-structure and multi-machines (PLSP-MM) (Kimms, 1999) for the concern of product-life-cycle (PLC) demands. As the PLC concern can be an important factor, but it has been neglected in the existing model.

In this PLSP-MM problem, we make the same assumptions as those done by Kimms (1999) besides the PLC demands. We assume a finite horizon and which is divided into discrete periods with dynamic and deterministic external demands for the items. No shortage or backlog is allowed. Items may consist of other items that are to be produced before the former items can be produced. The intermediate items may in turn cause internal demands for other items and so on. Also, we assume that the capacity per period of each machine is constrained. The production of one item consumes an item-specific amount of machine capacity. To produce an item the machine has to be set up for the item. Every setup causes item-specific setup costs. Setup time and sequence dependent setup cost are ignored. If a remaining capacity occurs in a period it can be used to schedule another item. And idle periods between lots of a same item do not cause additional setup costs. Items that are produced within a period without a demand in that period are stored in inventory to meet future demands, while inventory is assumed to be uncapacitated but have holding costs. Therefore, the objective is to find the production plan feasible and cheap, i.e., the sum of setup and holding costs must be as low as possible.

Furthermore, under product life cycles (Polli and Cook, 1969; Rink and Swan, 1979), a firm often has to change its market strategies several times (Lambkin and Day, 1989), (Levitt, 1965), which is not only due to the competitors' challenges but also that the chance of the product's market economic state and its role as acting in the market may have changed (Aaker, 1996). Therefore, a firm often has to prepare a series of strategies for different stages of PLCs (Kurawarwala and Matsou, 1998) and the strategies for production to meet the different stages of PLCs. Existing lot sizing and scheduling models often have focused on the problems without the consideration of PLC stages and changes of demands in the PLCs. Though Hill (1996) has proposed a model with the PLC concern, but his assumption was made on a single item, single-product-level model and cannot solve the multi-item multi-level product structure problems. Especially it is not considered that the lot sizes of the products therefore may have to respond to

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the stage changes of the PLC. It is only a pure inventory model instead of multi-item scheduling and inventory model. Therefore, in this paper the PLSP model will be considered with the PLC concept. Besides that, the effects of the cash flow and learning curve (Argote and Epple, 1990) will also be examined. The technique of genetic algorithms (GAs) will be adopted as the solution tool for the current model.

The remainder of this paper is organized as: Section 2 provides the model development. Section 3 provides the solution procedure by incorporating the genetic algorithm. Section 4 provides a numerical example for demonstration. In section 5 we draw conclusions.

2. MODEL DEVELOPMENT

2.1 The multi-level PLSP with multiple machines (PLSP-MM)

In this subsection the PLSP-MM model by Kimms (1999) will be reviewed. Additional assumptions in addition to those given in the last section will be provided as follows. In a multi-level product structure, the precedence relationships among the items can be defined as an acyclic gozinto-structure of a general type. Several resources (or machines) can be available for manufacturing the items and each item is produced on item-specific machines (or resources). Also, positive lead-times of the items can be defined due to the technological restrictions such as cooling or transportation for instance. Moreover, items may share common machines (or resources), some of them may be scarce, and capacities of these resources may vary over time.

For this problem, Table 1 defines the decision variables. Table 2 provides the problem parameters needed. Using these symbols, the PLSP-MM may be modelled as a mixed integer program (1999) as follows:

Table 1. Decision variables

Symbol	Definition
I_{jt}	Inventory for item j at the end of period t.
q_{jt}	Production quantity of item j in period t.
x_{jt}	Binary variable for indicating whether a setup for item j occurs in period t x_{jt} = 1) or not (x_{jt}= 0).
y_{jt}	Binary variable which indicates whether machine m_j is set up for item j at the end of period t (y_{jt}= 1) or not (y_{jt}= 0).

$$\text{Min} \ \sum_{j=1}^{J}\sum_{i=1}^{T}(s_j x_{j_t} + h_j I_{j_t}), \tag{1}$$

subject to

$$I_{j_t} = I_{j(t-1)} + q_{j_t} - d_{j_t} - \sum_{i\in\beta_j} a_{ji}q_{it}, \quad j=1,2,\ldots,J,\ t=1,2,\ldots,T, \tag{2}$$

Table 2. Parameters for the PLSP-MM

Symbol	Definition
J	Number of items.
M	Number of machines.
T	Number of periods.
a_{ji}	"Gozinto"-factor. Its value is zero if item i is not an immediate successor of item j. Otherwise, it is the quantity of item j to produce one item i.
C_{mt}	Available capacity of machine m in period t.
d_{jt}	External demand for item j in period t.
h_j	Non-negative holding cost for having one unit of item j one period in inventory.
m_j	Machines on which items j is produced.
p_j	Capacity needs for producing one unit of item j.
s_j	Non-negative setup cost for item j.
v_j	Positive and integral lead time of item j.
I_{j0}	Initial inventory for item j.
y_{j0}	Unique initial setup state.
α_m	Set of all items that share the machine m, i.e., $\alpha_m = \left\{ j\in\{1,2,...,J\} \middle| m_j = m \right\}$.
β_j	Set of immediate successors of item j, i.e., $\beta_j = \left\{ i\in\{1,2,...,J\} \middle| a_{ji} > 0 \right\}$.

$$I_{jt} \geq \sum_{i\in\beta_j} \sum_{\tau=t+1}^{\min\{t+v_j,T\}} a_{ji}q_{i\tau}, 0 \quad j=1,2,\ldots,J,\ t=1,2,\ldots,T-1, \tag{3}$$

$$\sum_{j\in\alpha_m}^{J} y_{j_t} \leq 1 \quad m=1,2,\ldots,M,\ t=1,2,\ldots,T, \tag{4}$$

$$x_{j_t} \geq y_{j_t} - y_{j(t-1)} \quad j=1,2,\ldots,J,\ t=1,2,\ldots,T, \tag{5}$$

$$p_j q_{j_t} \leq C_{m_j t}\left(y_{j(t-1)} - y_{j_t}\right) \quad j=1,2,\ldots,J,\ t=1,2,\ldots,T, \tag{6}$$

$$\sum_{j\in\alpha_m} p_j q_{jt} \leq C_{mt}, \quad m=1,2,\ldots,M,\ t=1,2,\ldots,T \tag{7}$$

$$y_{j_t} \in \{0,1\}, \quad i=1,2,\ldots,I,\ j=1,2,\ldots,J,\ t=1,2,\ldots,T, \tag{8}$$

$$I_{j_t}, q_{j_t}, x_{j_t} \geq 0, \quad j=1,2,\ldots,J,\ t=1,2,\ldots,T, \quad i=1,2,\ldots,I,\ j=1,2,\ldots,J, \tag{9}$$

The objective (1) is to minimize the sum of setup and holding costs. Eqs. (2) are the inventory balances. At the end of a period t there should be in inventory what was in there at the end of period t−1 plus what is produced minus external and internal demands. Constraints (3) guarantee a lower bound of inventory of item j at the end of a period for fulfillment of the internal demands for the amount of items required to produce other items but within the next v_j periods of its positive lead time. Constraints (4) make sure that the setup state of each machine is uniquely defined at the end of each period. Those periods in which setups of a machine m_j for item j happen are spotted by Eqs. (5) that if the machine is not set up for the particular item j at the beginning of a period, $y_{j(t-1)} = 0$, but it is at the end of that period ($y_{jt} = 1$), then a setup must have occurred in period t, i.e., $x_{jt} \geq 1 - 0 = 1$. Idle periods may occur in order to save setup costs. Due to Eqs. (6), production can take place only if there is a proper setup state either at the beginning or at the end of a particular period. Note that if the machine is not set up for an item at the beginning of a period but it is at the end, the setup takes place in that period, which means that production may take place in that period. As the consequence of Eqs. (6), at most two items can be manufactured on each machine per period. Capacity constraints are formulated in Eqs. (7). Eqs. (8) define the binary-valued setup state variables, while Eqs. (9) are simply non-negativity conditions. One should easily convince himself that due to Eqs. (5) in combination with Eq. (1) setup variables x_{jt} are indeed zero-one valued. Hence non-negativity conditions are sufficient for these variables. By letting inventory variables I_{jt} be nonnegative, backlogs cannot occur.

The following will introduce the present extended model.

2.2 The proposed PLC-based PLSP model

This subsection introduces the proposed PLC-based PLSP-MM model. The additional parameters due to this consideration will be first introduced. Then, the extension of the PLSP-MM model will be given.

2.2.1 Notations for the PLC concerns

In applications of PLCs, important steps relevant to this research include determining the transitional points between the consecutive stages, such as the introduction, growth, maturity, and decline stages and forecasting the demands of the PLCs. Useful technique has been found in (Chang and Chang, 2000), (Chang and Chang, 2003) and the reviews therein. Table 3 provides the additional parameters to the PLC-concerned modelling due to the concern of demands of the PLCs. Here we should extend the original PLSP-MM model by extending it to allow for different additional capacity and lot sizing constraints for the various stages of PLCs. The modelling resultant may have a better scheduling for the PLC demands.

2.2.2 Additional capacity-allocation constraints proposed

In lot sizing scheduling models, often, the capacity constraints have been treated equally for all items competing for it. No considerations have been placed on the products' demand-stage changes. As a result when the PLC stages of the products exist strongly, the allocation of capacities for different stages of products may occur inadequately for fulfillment of the demands. Higher inventories have to be used in some periods. However, a reality can be observed as well that in practice a company usually can have different strategies for capacity allocation and different products in different stages for any predictable market changes. Therefore, if PLC is not considered, the solutions of the conventional lot sizing scheduling models may become unsuitable or suboptimal. To overcome this problem, here a termed "stage capacity-allocation weight" for each stage of the products based on their PLCs may be introduced. It may be defined by

Stage capacity-allocation weight

$$= \frac{Stage\ total\ demand}{PLC\ total\ demand} \times \frac{PLC\ total\ time\ length}{Stage\ total\ time\ length}, \tag{10}$$

Table 3. Additional parameters for the PLC based PLSP modelling

Symbol	Definition
T_{B_j}	Starting period of the PLC of item j.
T_{In_j}	Transitional period between the introduction and growth stages of item j.
T_{Gr_j}	Transitional period between the growth and maturity stages of item j.
T_{Ma_j}	Transitional period between the maturity and decline stages of item j.
T_{E_j}	End period of PLC of item j.
In_j	Number of periods in the introduction stage of item j.
Gr_j	Number of periods in the growth stage of item j.
Ma_j	Number of periods in the maturity stage of item j.
De_j	Number of periods in the decline stage of item j.
T_j	Number of periods of the PLC of item j.
n_j	Number of immediate successors of item j.

and taking into consideration the PLC stage demands and lengths of the products in capacity allocation of the lot-sizing scheduling. Therefore, by the stage capacity-allocation weights, the additional constraints of capacity allocations for each item in each period can be defined (will be denoted as $C'_{m_j t}$) and also replace their original $C_{m_j t}$ in Eqs. (6). Also, for consideration of the original constraints C_{mt}, $C'_{m_j t}$ can be defined by multiplying the original C_{mt} by the respective stage capacity-allocation weight of the items at each stage. Hence, for introduction stage,

$$C'_{m_j t} = \min\left\{ C_{mt}, C_{mt} \times \frac{\sum_{t=T_{B_j}}^{T_{In_j}} d_{jt}}{\sum_{t=T_{B_j}}^{T_{E_j}} d_{jt}} \times \frac{T_j}{In_j} \right\}, \tag{11}$$
for $j = 1,2,\ldots,J,\ t = T_{B_j}, T_{B_j}+1,\ldots,T_{In_j}$

For growth stage,

$$C'_{m_j t} = \min\left\{ C_{mt}, C_{mt} \times \frac{\sum_{t=T_{In_j}+1}^{T_{Gr_j}} d_{jt}}{\sum_{t=T_{B_j}}^{T_{E_j}} d_{jt}} \times \frac{T_j}{Gr_j} \right\}, \tag{12}$$
for $j = 1,2,\ldots,J,\ t = T_{In_j}+1,\ldots,T_{Gr_j}$

For maturity stage,

$$C'_{m_j t} = \min\left\{ C_{mt}, C_{mt} \times \frac{\sum_{t=T_{Gr_j}+1}^{T_{Ma_j}} d_{jt}}{\sum_{t=T_{B_j}}^{T_{E_j}} d_{jt}} \times \frac{T_j}{Ma_j} \right\}, \tag{13}$$
for $j = 1,2,\ldots,J,\ t = T_{Gr_j}+1,\ldots,T_{Ma_j}$

For decline stage,

$$C'_{m_j t} = \min\left\{ C_{mt}, C_{mt} \times \frac{\sum_{t=T_{Ma_j}+1}^{T_{E_j}} d_{jt}}{\sum_{t=T_{B_j}}^{T_{E_j}} d_{jt}} \times \frac{T_j}{De_j} \right\}, \tag{14}$$
for $j = 1,2,\ldots,J,\ t = T_{Ma_j}+1,\ldots,T_{E_j}$

In Eqs. (11)-(14), the minimum is taken to the purpose of maintaining the original capacity constraints un-exceeded. And the results fulfill the strategic reassignment of capacity for each item in each period at each stage. Therefore, the stage capacity-allocation weight will reflect the capacity assignment requirement (constraints) of the items in different stages.

2.2.3 Lot sizing constraints proposed

Analogous to the arguments in capacity allocation constraints, PLC demands vary with stages. The lot-sizing decisions of the products should also be given with the consideration of the stage variations too not only the capacity availability. To supply this idea into the lot sizing decision of different stage production requirements of the items, additional lot-sizing constraints may be defined. Note that an item may be required for manufacturing other item(s). For these lot-sizing constraints to be feasible, the lot sizing and thus resultant inventories should take the number of immediate successors of an item into consideration too, to avoid shortages for the demands. The additional required lot-sizing constraints may be defined by

Additional lot sizing constraint
$$= \textit{No. of immediate successors} \times \frac{\textit{Stage total demand}}{\textit{Stage total time length}} \tag{15}$$

Hence For introduction stage:

$$0 \le q_{jt} \le n_j \times \frac{\sum_{t=T_{B_j}}^{T_{In_j}} d_{jt}}{In_j}, \tag{16}$$
for $j = 1,2,\ldots,J,\ t = T_{B_j}, T_{B_j}+1,\ldots,T_{In_j}$

For growth stage:

$$0 \le q_{jt} \le n_j \times \frac{\sum_{t=T_{In_j}+1}^{T_{Gr_j}} d_{jt}}{Gr_j}, \tag{17}$$
for $j = 1,2,...,J,\ t = T_{In_j}+1,...,T_{Gr_j}$,

For maturity stage:

$$0 \le q_{jt} \le n_j \times \frac{\sum_{t=T_{Gr_j}+1}^{T_{Ma_j}} d_{jt}}{Ma_j}, \tag{18}$$
for $j = 1,2,...,J,\ t = T_{Gr_j}+1,...,T_{Ma_j}$,

For decline stage:

$$0 \le q_{jt} \le n_j \times \frac{\sum_{t=T_{Ma_j}+1}^{T_{E_j}} d_{jt}}{De_j}, \tag{19}$$
for $j = 1,2,...,J,\ t = T_{Ma_j}+1,...,T_{E_j}$.

2.2.4 Cash flow and learning curve considerations

Practically time value of money is considered necessitated in any investment, revenue and expenses. Some lot-sizing models have investigated a cash-flow-oriented model to replace the non-cash-flow one (e.g., see Hofmann, 1998; Ram and Thomas, 1995). The model proposed here will also examine the cash flow effects for a further investigation. Moreover, as an item's production usually exhibits the learning curve, which is important in PLCs, the effects of learning curve cost will also be examined in the current model.

The setup costs may be considered with the learning curve effect. Following these results (Argote and Epple, 1990; Ram and Thomas, 1995; Yelle, 1979), the setup cost for an item may be modelled in a power function:

(Setup cost)

$$S_{jt} = S_{j1} \times t^{-b}, \quad \text{for } j = 1,2,...,J, \forall \{t | S_{jt} \geq S_{\min}\} \tag{20}$$

where the learning rate b = 0.321928 is used in the numerical example.

For the effects of cash flows, discounting of the costs at the various periods with a rate of return i per period to the equivalent present worth or projecting them into the future worth or equivalent annual worth performs the task (William et al. , 1997) The equivalent present worth is adopted here. They include:

(Setup costs at each period)

$$\sum_{j=1}^{J} \left(S_{jt} x_{jt} \Big/ (1+i)^t \right), \text{ for } t = 1,2,....,T \tag{21}$$

(Holding costs at each period)

$$\sum_{j=1}^{J} \left(h_j I_{jt} \Big/ (1+i)^t \right), \text{ for } t = 1,2,....,T \tag{22}$$

The entire model proposed, PLC-based PLSP-MM, can be defined as, by collecting the above formulations:

$$\text{Min} \sum_{j=1}^{J} \sum_{i=1}^{T} \left[\left(s_{jt} x_{j_t} \Big/ (1+i)^t \right) + \left(h_j I_{j_t} \Big/ (1+i)^t \right) \right] \tag{23}$$

subject to

$$I_{jt} = I_{j(t-1)} + q_{jt} - d_{jt} - \sum_{i \in \beta_j} a_{ji} q_{it},$$
$$j = 1,2,\ldots,J, \; t = 1,2,\ldots,T,$$

$$I_{jt} \geq \sum_{i \in \beta_j} \sum_{\tau = t+1}^{\min\{t+v_j, T\}} a_{ji} q_{i\tau},$$
$$j = 1,2,\ldots,J, \; t = 1,2,\ldots,T-1,$$

$$\sum_{j \in \alpha_m}^{J} y_{j_t} \leq 1,$$
$$m = 1,2,\ldots,M, \; t = 1,2,\ldots,T,$$
$$x_{j_t} \geq y_{j_t} - y_{j(t-1)},$$
$$j = 1,2,\ldots,J, \; t = 1,2,\ldots,T,$$
$$p_j q_{j_t} \leq C'_{m_j t} \left(y_{j(t-1)} - y_{j_t} \right),$$
$$j = 1,2,\ldots,J, \; t = 1,2,\ldots,T,$$

$$\sum_{j \in \alpha_m} p_j q_{jt} \leq C_{mt},$$
$$m = 1,2,\ldots,M, \; t = 1,2,\ldots,T,$$
$$y_{j_t} \in \{0,1\},$$
$$i = 1,2,\ldots,I, \; j = 1,2,\ldots,J, \; t = 1,2,\ldots,T,$$
$$S_{jt} = S_{j1} \times t^{-0.321928},$$
$$\text{for } j = 1,2,...,J,$$

$$C'_{m_j t} = \min \left\{ C_{mt}, C_{mt} \times \frac{\sum_{t=T_{\mathrm{B}_j}}^{T_{\mathrm{In}_j}} d_{jt}}{\sum_{t=T_{\mathrm{B}_j}}^{T_{\mathrm{E}_j}} d_{jt}} \times \frac{T_j}{In_j} \right\},$$
$$\text{for } j = 1,2,\ldots,J, t = T_{\mathrm{B}_j}, T_{\mathrm{B}_j}+1,\ldots,T_{\mathrm{In}_j},$$

$$C'_{m_j t} = \min \left\{ C_{mt}, C_{mt} \times \frac{\sum_{t=T_{\mathrm{In}_j}+1}^{T_{\mathrm{Gr}_j}} d_{jt}}{\sum_{t=T_{\mathrm{B}_j}}^{T_{\mathrm{E}_j}} d_{jt}} \times \frac{T_j}{Gr_j} \right\},$$
$$\text{for } j = 1,2,\ldots,J, t = T_{\mathrm{In}_j}+1,\ldots,T_{\mathrm{Gr}_j},$$

$$C'_{m_j t} = \min \left\{ C_{mt}, C_{mt} \times \frac{\sum_{t=T_{\mathrm{Gr}_j}+1}^{T_{\mathrm{Ma}_j}} d_{jt}}{\sum_{t=T_{\mathrm{B}_j}}^{T_{\mathrm{E}_j}} d_{jt}} \times \frac{T_j}{Ma_j} \right\},$$
$$\text{for } j = 1,2,\ldots,J, t = T_{\mathrm{Gr}_j}+1,\ldots,T_{\mathrm{Ma}_j},$$

$$C'_{m_j t} = \min \left\{ C_{mt}, C_{mt} \times \frac{\sum_{t=T_{\mathrm{Ma}_j}+1}^{T_{\mathrm{E}_j}} d_{jt}}{\sum_{t=T_{\mathrm{B}_j}}^{T_{\mathrm{E}_j}} d_{jt}} \times \frac{T_j}{De_j} \right\},$$
$$\text{for } j = 1,2,\ldots,J, t = T_{\mathrm{Ma}_j}+1,\ldots,T_{\mathrm{E}_j},$$

$$0 \leq q_{jt} \leq n_j \times \frac{\sum_{t=T_{\mathrm{In}_j}+1}^{T_{\mathrm{Gr}_j}} d_{jt}}{Gr_j},$$
$$\text{for } j = 1,2,...,J, t = T_{\mathrm{In}_j}+1,...,T_{\mathrm{Gr}_j},$$

$$0 \leq q_{jt} \leq n_j \times \frac{\sum_{t=T_{\mathrm{Ma}_j}+1}^{T_{\mathrm{E}_j}} d_{jt}}{De_j},$$
$$\text{for } j = 1,2,...,J, t = T_{\mathrm{Ma}_j}+1,...,T_{\mathrm{E}_j},$$
$$I_{j_t}, q_{j_t}, x_{j_t} \geq 0, \quad j = 1,2,\ldots,J, \; t = 1,2,\ldots,T.$$

This model will also be denoted as PLC-PLSP-MM-CL, where 'CL' stands for the cash flow ('C') and learning curve ('L') effect. Also a similar denotation will be used for the original model, PLSP-MM, and therefore, PLSP-MM-CL denotes the original PLSP-MM model with the cash flow and learning curve effects. Also, for the comparison purpose, PLC-PLSP-MM will denote the current model without the cash flow and learning curve effects.

The following introduces the solution procedure of the model.

3. THE SOLUTION PROCEDURE

For optimization problems, genetic algorithms have been proved powerful and general for any type of problem and particularly for those of complex nature and modelling. With the vast literature and application cases, here the reader is referred to the references (Disney et al., 2000), (Hyun et al., 1998), (Ip et al., 2000), (Khouja et al., 1998), (Kim and Kim, 1996), (Kimms, 1999), (Chang and Lo, 2001), (S_{ik})provided for the basic idea. The design of the GA for aiding in the solution of the model above is provided as the solution procedure as follows.

Step 1. GA representation/coding: Let S_{MT} denote a chromosome consisting of $M \times T$ binary genes s_{mt}, m = 1, 2, …, M, t = 1, …, T. These genes are arranged in the order of machines and time periods as Figure 1. Each gene represents whether a set up of the corresponding machine occurs or not in the particular period t.

Step 2. Initial population of chromosomes: An initial population of chromosomes S_{mt} of a desirable size (PSIZE) is generated randomly and each S_{MT} represents a feasible solution. PSIZE = 50 has been used in the numerical example.

Step 3. Selection rules for setting up for items: For the above model, we have used a backward procedure for deciding the lot size scheduling, i.e., starting from the period T and then working backwards towards period 1. Meanwhile, for each s_{mt} = 1 (t = T, T–1, …,1, m = 1, 2, …, M) of each SMT, selecting the proper item that requires the machine to be set up has been the central of this step. Here five rules are used and applied in order of rule 1 (δ_1) to rule 5 (δ_5). If with a selection rule there are more-than-one items that can be selected, the next rule is applied. Table 4 gives the parameters to be used in these selection rules.

Rule 1 (δ_1)—Based on minimizing the holding cost:

Step (δ_1-*1*): Find the set α_{mt} of items, which require the machine m to produce and have unfulfilled demand at time t.

$$\alpha_{mt} = \left\{ j \in \alpha_m \middle| CD_{j(t+1)} > 0 \right\} \cap \left\{ j \in \alpha_m \middle| Nr_j - \sum_{\tau=t+1}^{T} q_{j\tau} > 0 \right\}. \tag{24}$$

Table 4. Parameters for the selection rules

Symbol	Definition
CD_{jt}	Cumulative demand for item j that has not been met yet in period t, i.e., $CD_{jt} = \sum_{\tau=t}^{T} d_{j\tau} - \sum_{\tau=t+1}^{T} q_{j\tau}$ and $CD_{j(T+1)} = 0$.
Nr_j	Net requirement of item j, i.e., $Nr_j = \sum_{\tau=1}^{T} d_{j\tau}$.
pat_j	Path of an item j (with the meaning to be explained in rule 4).
dep_j	Depth of an item j (with the meaning to be explained in rule 3).
IS_k	Set of items selected by selection rule k.

Step (δ_1-*2*)*:* Evaluate the holding cost that can result from each of the items in α_{mt}. The idea is that in order to let all immediate successors i of an item j can be produced in the next period (t = t + 1), the production of item j at machine m must be completed in period t, or otherwise it will cause holding cost of the item. Therefore, the holding cost that an item $j \in \alpha_{mt}$ could cause may be evaluated as $h_j \times CD_{j(t+1)}$. This rule is to select the item that can result in the possible maximal holding cost and to schedule it first in order to avoid high holding costs. Namely,

$$IS_{\delta_1} = \left\{ l \in \alpha_{mt} \middle| \begin{aligned} & h_l\left(CD_{l(t+1)}\right) \\ & = \max\left\{ j \in \alpha_{mt} \middle| h_j\left(CD_{j(t+1)}\right) \right\} \end{aligned} \right\}, \tag{25a}$$

$$\text{and } S_{jmt} = \begin{cases} 1, & \text{if } j \in IS_{\delta_1}, \\ 0, & \text{otherwise,} \end{cases} \tag{25b}$$

where S_{jmt} indicates the production and setup status of item j in period t whether it is scheduled on machine m or not. If $IS_{\delta_1} \geq 2$, apply the next rule δ_2.

Rule 2 (δ_2)—Based on minimizing the setup cost:

Step (δ_2-*1*)*:* The idea for this rule is that for the same machine if an item can be produced in consecutive periods additional setup can be avoided. For this rule, one would like to schedule an item first if it has already been scheduled for production on machine m in the next period (t + 1). Namely, this rule will select the item(s) as: resetting α_{mt} for δ_2,

Machine m	1				2			…	M		
Period t	1	2	…	T	1	…	T	…	1	…	T
Gene s_{mt}	0	1	…	1	1	…	0	…	0	…	1

Figure 1. Chromosome representation.

$$\alpha_{mt} = \left\{ j \in IS_{\delta_1} \middle| S_{jm(t+1)} = 1 \right\} \cap \alpha'_{mt} , \tag{26a}$$

and

$$\begin{aligned} \alpha'_{mt} &= \left\{ j \in \alpha_m \middle| CD_{j(t+1)} + d_{jt} > 0 \right\} \cap \\ &\left\{ j \in \alpha_m \middle| Nr_j - \sum_{\tau=t+1}^{T} q_{j\tau} > 0 \right\}. \end{aligned} \tag{26b}$$

The purpose of (26*b*) is to ensure that the item selected has an unfulfilled demand including that for period t. If in (26*a*) $\left\{ j \in IS_{\delta_1} \middle| S_{jm(t+1)} = 1 \right\} = \varnothing$, let $IS_{\delta_2} = IS_{\delta_1}$, omit the next step and go to the next rule. Otherwise, continue to the next step.
Step (δ_2*-2*)*:* Select the item from α_{mt} with the minimal setup cost.

$$S_{jmt} = \begin{cases} & \text{if } j \in IS_{\delta_2} = \\ 1, & \\ & \left\{ l \in \alpha_{mt} \middle| s_l = \min\left\{ j \in \alpha_{mt} \middle| s_j \right\} \right\}, \\ 0, & \text{otherwise.} \end{cases} \tag{27}$$

Again if $IS_{\delta_2} \ge 2$, apply the next rule δ_3.
Rule 3 (δ_3)—Based on the maximal depth of the items in the product structure:
Assume a product manufacturing structure is as shown in Figure 2. The depths of the items can be determined as $dep_1 = 1$, $dep_2 = 2$, $dep_3 = 3$, $dep_4 = 4$. In order to reduce the generating of inferior or even infeasible solutions, we would look for the deeper-depth or deeper-associated item to schedule first. This rule realizes that

$$S_{jmt} = \begin{cases} & \text{if } j \in IS_{\delta_3} = \\ 1, & \\ & \left\{ l \in IS_{\delta_2} \middle| dep_l = \max\left\{ j \in IS_{\delta_2} \middle| dep_j \right\} \right\}, \\ 0, & \text{otherwise.} \end{cases} \tag{28}$$

Again if $IS_{\delta_3} \ge 2$, apply the next rule δ_4.

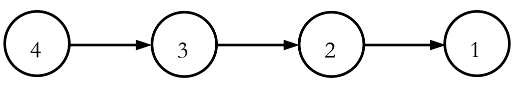

Figure 2. An example of product structure.

Rule 4 (δ_4)—Based on the longest path of the items:
By a similar idea to rule 3, rule 4 focuses on the entire path of the items in the product structure for avoiding inferior solutions due to long influence of an item to the inventory and setup costs if it is not scheduled first. Again by referring to Figure 2, the path lengths of the items as shown for instance can be determined as $pat_1 = pat_2 = pat_3 = pat_4 = 4$ as they are all in a path with the four items. Therefore, this rule realizes that

$$S_{jmt} = \begin{cases} & \text{if } j \in IS_{\delta_4} = \\ 1, & \\ & \left\{ l \in IS_{\delta_3} \middle| pat_l = \max\left\{ j \in IS_{\delta_3} \middle| pat_j \right\} \right\} \\ 0, & \text{otherwise.} \end{cases} \tag{29}$$

If again $IS_{\delta_4} \ge 2$, apply the final rule δ_5.
Rule 5 (δ_5)—Random selection:
If applying the above rules still cannot settle the choice, a random selection may be applied.
Step 4. Carry out the program (23) and determine all other decision variables' values and the objective function value.
Step 5. GA *reproduction/selection*: The GA reproduction is designed and performed here with the roulette wheel selection based on the fitness function values. In GA operation, this requires the evaluation of the fitness (objective) function value of each chromosome and which is for maximization. The fitness function is defined as the reciprocal of the objective function above for each chromosome.
Step 6. GA *crossover*: The crossover operation is designed with a crossover probability P_c for actually performing the crossover operation or otherwise the two randomly selected chromosomes enter directly the next generation. It is performed in two randomly selected chromosomes with two randomly selected gene-points. Then exchange of the genes between the crossover points of the two chromosomes generates two offspring chromosomes.
Step 7. GA *mutation*: Mutation operation is carried out with a mutation probability P_m on a randomly selected gene of a chromosome randomly selected here and mutates it from 0 to 1 or vice versa. After completing this operation, a generation of the GA is reached. Then, if the termination condition is met, the GA process terminates. Or otherwise it returns to step 3.

In addition, in the above procedure we also designed it with a "floating" crossover and mutation probability mechanism for escaping from entrapment of local optimum possible. This is with that, when the process continues to find the same value for a number of generations n, the GA increases gradually the probabilities P_c and P_m with $(P_{cU} - P_{cD})/GEN$ and $(P_{mU} - P_{mD})/GEN$, respectively, where P_{cU} and P_{cD} denote the upper limit and default, respectively, of P_c and similarly for P_m, and GEN denotes the number of generations. Till the solution changes, they are then reset to the default value. If the number of generations GEN is reached and still the same solution is found, the process terminates. After a number of empirical trial runs with all models, these probability parameters, $(P_{cU} - P_{cD})/GEN = (0.9 - 0.6)/100$, $(P_{mU} - P_{mD})/GEN = (0.3 - 0.2)/100$, $GEN = 100$, and $n = 10$ were used.

4. NUMERICAL EXAMPLE

Assume a firm is planning for production of two products X or 1 and Y or 2 for the next ten periods for demands as shown in Table 5. Figure 3. shows the manufacturing structure of the two products according to the bills of materials, where the numbers along the arrows indicate the quantities of the items required to make a unit of another item. Table 6 shows the manufacturing parameters of all these items. There are three machines available and each machine has 80 units of capacity per period of time. Table 7 shows the manufacturing relationships between these machines and items. It shows that which item can be produced on what machine(s). For example, item 3 can be produced on machine 2 or 3.

Table 8 shows the optimal chromosomes of machine setups S_{MT} with each model introduced after 20 runs of the solution procedures. Other results obtained by these models are provided and analyzed in the next sections.

Table 5. Demands for products X and Y

	Demand									
Product	Period1	2	3	4	5	6	7	8	9	10
X	5	10	15	25	35	40	30	20	0	0
Y	0	0	0	10	20	30	35	50	30	20

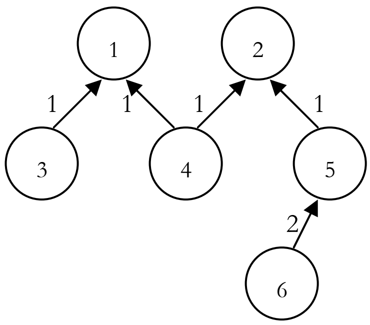

Figure 3. The gozinto-structure for manufacturing.

Table 6. Manufacturing parameters

Parameter	Item 1	2	3	4	5	6
S_j	10	10	10	10	10	10
h_j	3	3	2	2	2	2
P_j	1	1	1	1	1	1
V_j	1	1	1	1	1	1
I_{j0}	0	0	0	0	0	0
y_{j0}	0	0	0	0	0	0

Table 7. Relationships between machines and items

	Relationship between the machine and item					
Machine	Item 1	2	3	4	5	6
1	1	1	0	1[a]	1	1
2	1	1	1	1	1	0
3	1	1	1	0[a]	1	1

[a]1 indicates that an item can be produced on a machine and 0 indicates that it cannot.

4.1 The optimal lot sizing and scheduling

The outputs are from the PLC-PLSP-MM-CL model and the original PLSP-MM, PLC-PLSP-MM, and PLSP-MM-CL models too. Table 9 shows the optimal setup statuses of the machines for the items at the ends of periods, y_{jt}, with the applied selection rule by the PLC-PLSP-MM-CL model.

Table 8. Optimal chromosomes of machine setups S_{mt} with each model

	S_{mt}										
	Machin	t=1	2	3	4	5	6	7	8	9	10
Original PLSP-MM	1	1	1	1	1	1	1	1	1	1	1
	2	1	1	1	1	1	1	1	1	1	0
	3	0	1	1	1	1	1	1	1	1	0
PLC-PLSP-MM	1	1	1	1	1	1	1	1	0	1	1
	2	0	1	0	1	1	1	1	1	1	0
	3	1	0	1	1	1	1	1	1	1	0
PLSP-MM-CL	1	0	0	1	1	1	1	1	1	1	0
	2	1	1	1	1	1	1	1	0	0	1
	3	1	1	1	1	1	1	1	1	1	1
PLC-PLSP-MM-CL	1	1	1	1	1	1	1	1	1	0	1
	2	0	1	0	1	1	1	1	1	1	0
	3	1	0	1	1	1	1	1	1	1	1

Table 9. Optimal setup statuses y_{jt} by the PLC-PLSP-MM-CL model

		y_{jt}									
Machine	Item	t =1	2	3	4	5	6	7	8	9	10
1	1	1(δ_2)	1(δ_2)	1(δ_2)	1(δ_2)	1(δ_2)	1(δ_2)	1(δ_5)	0	0	0
	2	0	0	0	0	0	0	0	0	0	1(δ_2)
	3	0	0	0	0	0	0	0	0	0	0
	4	1(δ_1)	1(δ_1)	0	0	0	0	1(δ_1)	0	0	0
	5	0	0	0	0	0	0	0	1(δ_1)	0	1(δ_3)
	6	0	0	1 (δ_1)	1 (δ_1)	1 (δ_1)	1 (δ_1)	0	1 (δ_1)	0	0
2	1	0	0	0	0	0	0	0	0	0	0
	2	0	0	0	1(δ_2)	1(δ_2)	1(δ_2)	1(δ_2)	1(δ_2)	1(δ_2)	0
	3	0	1(δ_1)	0	1(δ_1)	1(δ_1)	1(δ_1)	1(δ_1)	0	0	0
	4	0	0	0	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0	1(δ_1)	1(δ_1)	0
	6	0	0	0	0	0	0	0	0	0	0
3	1	0	0	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0	0	0
	3	1(δ_5)	0	1(δ_1)	0	0	0	0	0	0	0
	4	1(δ_1)	0	1(δ_1)	1(δ_1)	1(δ_1)	1(δ_1)	1(δ_1)	1(δ_1)	1(δ_1)	1(δ_5)
	5	0	0	0	1(δ_5)	1(δ_5)	0	1(δ_5)	0	0	0
	6	0	0	0	0	0	0	0	1(δ_1)	1(δ_1)	1(δ_2)

Therefore, Table 10 shows the optimal machine setups that are to be occurring in the different periods for items, x_{jt} of the PLC-PLSP-MM-CL. Furthermore, the optimal lot sizing q_{jt} and optimal inventory decisions I_{jt} of the various items by the PLC-PLSP-MM-CL model are shown in Tables 11 and 12. The results of the original PLSP-MM, PLC-PLSP-MM, and PLSP-MM-CL models are also obtained; but for space limitations, they will not be provided here in detail. But, Figures 4-7 provide the final lot sizing and scheduling of the four models in respective Gantt charts.

In the next subsections, comparisons of these models shall be made.

Table 10. Optimal machine setups to occur in different periods for the items, x_{jt}, of the PLC-PLSP-MM-CL

		x_{jt}									
Machine	Item	t =1	2	3	4	5	6	7	8	9	10
1	1	1	1	0	1	0	1	0	0	0	0
	2	0	0	0	0	0	0	0	0	0	1
	3	0	0	0	0	0	0	0	0	0	0
	4	1	0	0	0	0	0	1	0	0	0
	5	0	0	0	0	0	0	0	1	0	1
	6	0	0	1	0	1	0	0	1	0	0
2	1	0	0	0	0	0	0	0	0	0	0
	2	0	0	0	1	0	1	0	1	0	0
	3	0	1	0	0	1	0	1	0	0	0
	4	0	0	0	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0	1	1	0
	6	0	0	0	0	0	0	0	0	0	0
3	1	0	0	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0	0	0
	3	1	0	0	0	0	0	0	0	0	0
	4	1	0	1	0	1	0	0	0	1	0
	5	0	0	0	1	0	0	1	0	0	0
	6	0	0	0	0	0	0	0	1	0	1

Table 11. Optimal lot-sizing decisions q_{jt} of the items by the PLC-PLSP-MM-CL model

	q_{jt}									
Item	t = 1	2	3	4	5	6	7	8	9	10
1	9	10	11	25	35	40	50	0	0	0
2	0	0	0	10	20	35	30	55	25	20
3	9	11	10	25	35	40	50	0	0	0
4	12	7	11	35	55	80	75	55	25	20
5	0	0	0	30	30	0	35	57	23	20
6	0	0	10	20	30	35	0	57	23	20

Table 12. Optimal inventory decisions I_{jt} of the items by the PLC-PLSP-MM-CL

	I_{jt}									
Item	t =1	2	3	4	5	6	7	8	9	10
1	4	4	0	0	0	0	20	0	0	0
2	0	0	0	0	0	5	0	5	0	0
3	0	1	0	0	0	0	0	0	0	0
4	3	0	0	10	0	0	0	0	0	0
5	0	0	0	25	30	0	0	2	0	0
6	0	0	10	20	30	35	0	0	0	0

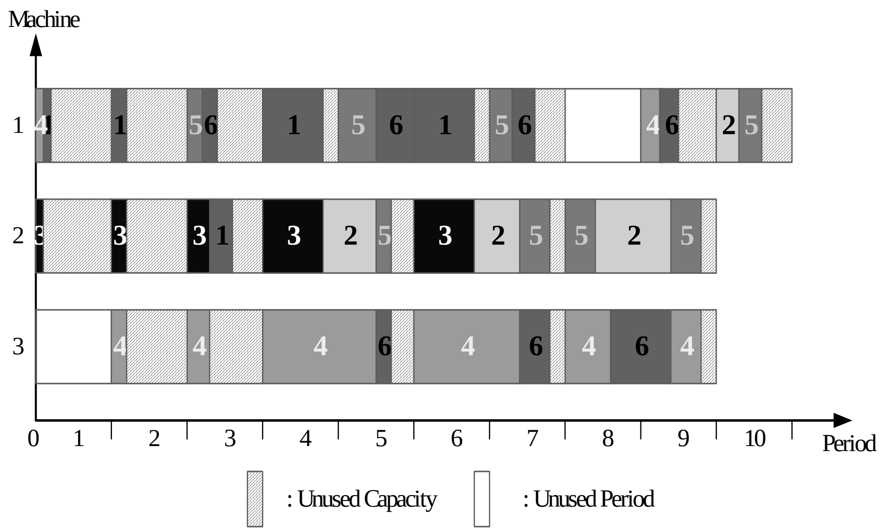

Figure 4. Optimal lot sizing and scheduling by the PLC-PLSP-MM-CL.

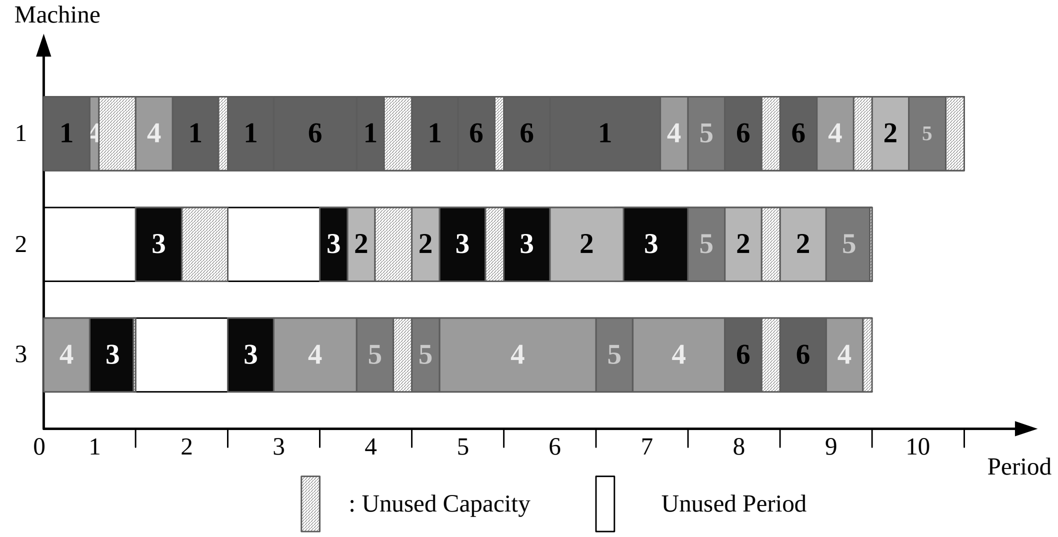

Figure 5. Lot sizing and scheduling by the original PLSP-MM.

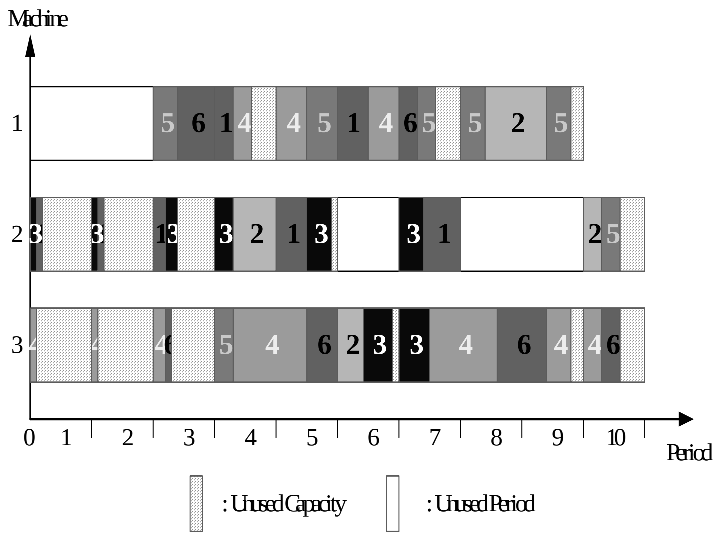

Figure 6. Optimal lot sizing and scheduling by the PLC-PLSP-MM.

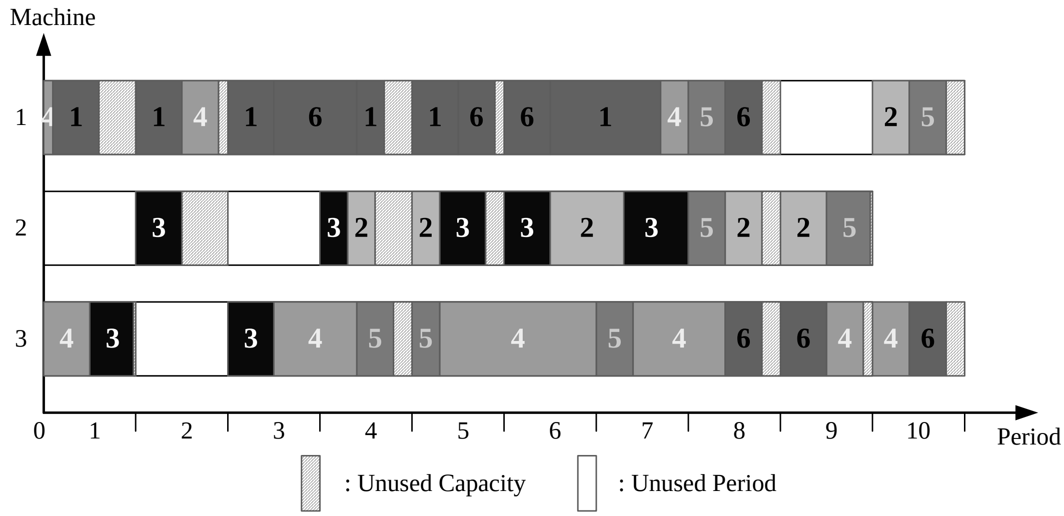

Figure 7. Optimal lot sizing and scheduling by the PLSP-MM-CL.

4.2 Comparison of the lot sizing scheduling costs of the models

Table 13 shows the setup cost, holding cost, and their total resulting with each model. The following important remarks can be made.

(1) The effect of PLC concern: Examining the two pairs of model's costs, original PLSP-MM model with the PLC-PLSP-MM and PLSP-MM-CL with the PLC-PLSP-MM-CL model (Table 13) indicates that both pairs differ in whether the PLC is considered or not and that not only the holding cost but also the setup cost are improved by the PLC considerations. Particularly a greater saving can be even given in the holding cost. The pairs of comparison of models' costs are summarized in Table 14.

(2) The combined effect of cash flow and setup-cost learning: Although the two models, original PLSP-MM model and PLSP-MM-CL (and similarly PLC-PLSP-MM model and PLC-PLSP-MM-CL) may not be directly compared due to the reason of different cost bases, one should still observe that the improvement of the total cost due to the PLC consideration combined with the cash flow and setup cost learning (i.e., PLSP-MM-CL → PLC-PLSP-MM-CL) is greater than that with only the PLC factor (i.e., original PLSP-MM → PLC-PLSP-MM) in Table 14. This indicates that the 'CL' has also an effect on the model and therefore should be taken into consideration in the model.

Table 13. Optimal cost of the models

Model	Setup Cost	Holding Cost	Total Cost
Original PLSP-MM	310	685	995
PLC-PLSP-MM	300	526	826
PLSP-MM-CL	182	682	864
PLC-PLSP-MM-CL	172	473	645

Table 14. Comparison of the optimal cost improvement of the models

	Reduction		
	Setup Cost	HoldingCost	Total Cost
Original PLSP-MM →	310 → 300	685 → 526	995 → 826
PLC-PLSP-MM	(-3.2%)	(-23.2%)	(-17.0%)
PLSP-MM-CL →	182 → 172	682 → 473	864 → 645
PLC-PLSP-MM-CL	(-5.5%)	(-30.6%)	(-25.3%)

4.3 Comparison of deviations between scheduled outputs of the models and demands

This subsection further compares the deviations between the scheduled output and items' demands by the models. This task can be done either in a single-period-by-single-period manner or by a cumulated manner. For space limitations, only the cumulated results are provided here. In Table 15, first the demands of the items in each period are summarized. Therefore, the cumulated demands and cumulated scheduled outputs for all items (in units of capacity) up to each period by the four models are shown in Table 16. The sum of square differences, SSD, between the cumulated demands and outputs of each model can be shown in Table 17. Similar remarks to that in Section 4.2 can be made. The SSDs by the PLC-PLSP-MM-CL model and PLC-PLSP-MM model are much smaller than that by the PLSP-MM-CL and PLSP-MM, respectively. Furthermore, although the SSD by the PLSP-MM-CL is higher than that by the PLSP-MM, the SSD by the PLC-PLSP-MM-CL is smaller than that by PLC-PLSP-MM. So, again we observe that the PLC-PLSP-MM model produces a lot-sizing schedule that is better matching the products' demands in the PLCs than that by the PLSP-MM model. And the combined effect of the cash flow and setup cost learning curve (i.e., the PLC-PLSP-MM-CL model) again reinforces this result. The extension of the PLSP-MM by the PLC-PLSP-MM-CL model provides benefits to the PLSP problem with the product-life-cycle considerations.

Table 15. Demands of the items for each period

	Demand									
Item	t =1	2	3	4	5	6	7	8	9	10
1	5	10	15	25	35	40	30	20	0	0
2	0	0	0	10	20	30	35	50	30	20
3	5	10	15	25	35	40	30	20	0	0
4	5	10	15	35	55	70	65	70	30	20
5	0	0	0	10	20	30	35	50	30	20
6	0	0	0	10	20	30	35	50	30	20

Table 16. Cumulated demands and scheduled outputs of the models

	t=1	2	3	4	5	6	7	8	9	10
Cumulated Demands	15	45	90	205	390	630	860	1120	1240	1320
Cumulated Outputs:										
Original PLSP-MM	15	45	140	370	550	780	960	1120	1280	1320
PLC-PLSP-MM	30	58	100	245	450	680	920	1144	1280	1320
PLSP-MM-CL	15	45	175	385	610	770	960	1120	1240	1320
PLC-PLSP-MM-CL	30	58	100	245	450	680	920	1144	1240	1320

Table 17. Sum of square differences (SSD) between cumulated demands and scheduled outputs by each model

	Original PLSP-MM	PLC-PLSP-MM	PLSP-MM-CL	PLC-PLSP-MM-CL
SSD	89425	13970	117625	12370

5. CONCLUSIONS

This paper has presented an extended PLSP-MM model based on the product life cycle concept. It has introduced the proper PLC production capacity-allocation constraints and lot sizing constraints for the products for different stages as the additional constraints. The results of the example showed that the benefits of the extended model provided not only reduced both the setup and holding costs but also that the lot sizing scheduling was significantly much better matching the products' demands in the PLCs than the PLSP-MM model. Also, in the model the setup cost learning and cash flow were also considered. The results also showed that these factors should be included in the model.

Further research from this research may apply a similar or re-developed approach to the models of other lot sizing and scheduling problems such as that reviewed in this paper or (Drexl and Kimms, 1997).

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