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Optimal Buyer-Seller Inventory Models in Supply Chain

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Abstract**—**In many recent works, several authors presented the usefulness of inventory models on two member's simple buyer-seller in supply chain. The main object of the present paper is to investigate the optimal order interval and discount price such that the joint total cost is minimized during a finite planning horizon. The methodology presented here is based upon a simple algorithm and analysis on calculus. Our analysis consider negotiation factor between the buyer and the seller simultaneously utilize a joint saving-sharing scheme developed by Chakravarty and Martin (1988) to derive both the optimal discount price and the optimal order interval such that minimize the joint total cost of the buyer and the seller. The models are illustrated with a numerical example and compared the influence of different sharing values on related cost of buyer-seller system.

Keywords**—**Inventory, deterioration commodity, joint cost, quantity discount, time-varying demand.

1. INTRODUCTION

The subject of supply chain management has gained importance and popularity during the past two decades or so, due chiefly to its applications in numerous seemingly diverse fields of supply chain management (see, for details, Berry and Naim (1996), Spekman, Spear and Kamauff (2002) and Thoms and Griffin (1996)). Recently, many authors have explicitly obtained approximation or optimal cycle time of the planning horizon (see, for details, Rau, Wu and Wee (2003), Wee (1998) and Yang (2004), see also Chakravarty and Martin (1988)).

Monahan (1984) developed an initial formulation of a fixed order quantity discount pricing model for a single buyer which maximizes the supplier's incremental net profit and cash flow without making the buyer any worse off, and possibly better off. Lee and Rosenblatt (1986) generalized Monahan's model somewhat with respect to order policy and addressed an overlooked issue of inadequately constrained discount price. Lal and Staelin (1984) also developed a fixed order quantity decision model with a discount scheme aimed at certain seller's and potential buyer's benefits. Banerjee (1986) produced a more through analysis of a two-participant joint economic lot size problem. Recently, Chakravarty and Martin (1988) provided the vendor with the means for optimal determining both the discount price and replenishment interval under periodic review for any desired joint saving-sharing scheme between the seller and buyer.

This paper extends the research of Chakravarty and Martin (1988) from considering the problem of simultaneously determining the discount price and the replenishment interval and treated the demand rate as constant to the linear demand (increases or decreases) with time for a deterioration items. We consider the cooperation and noncooperation situation at two-stage single buyer-seller system. In cooperation situation, the seller offering an optimal discount price and the buyer control an optimal order interval to reduce the joint cost. In noncooperation situation, the price undiscounted and the buyer only controls when to replenish stock and how much stock to replenish to minimize his total relevant cost simultaneously utilize a joint saving-sharing scheme developed by Chakravarty and Martin (1988) to share the joint cost savings as variances in α between buyer and seller. We derive bounds for the optimal cycle time with those bounds; a simple algorithm to get the optimal cycle time will be developed. A numerical example is used to explain the model and solution algorithm.

2. ASSUMPTION AND NOTATION

2.1 The mathematical model is developed on the basis of the following assumption

1. Only consider single buyer and seller situation.
2. The system operates only on a prescribed period of H years.
3. Deterioration of units is considered only after they have been received into inventory and there is no replacement of deteriorate units.
4. Shortage is not allowed.
5. Consider providing quantity discount and undiscounted situation.
6. Only consider single commodity inventory in the planning horizon.

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7. The demand rate of the buyer is a function of time in the planning horizon.
8. The replenishment rate of seller is infinite with zero lead time.

2.2 The parameters of inventory model are designated as follows

H: planning horizon
m: number of replenishment in the planning horizon
P: undiscounted unit price
p_i : discount price during i period, $i = 1,2,...,m$
θ: deterioration rate
r: buyer's holding cost per unit per unite time
K: seller's order processing cost
S: seller's setup cost
A: buyer's order processing cost
BS: buyer's savings per unit time
SS: seller's savings per unit time
α: sharing rate for joint cost saving of buyer-seller system
I_i : total inventory during i period, $i = 1,2,...,m$
T_i : order interval, $i = 1,2,...,m$
t_i :the time when inventory is used up in any period, $i = 1,2,...,m$ $t_0 = 0$; $t_m = H$
$T_i = t_i - t_{i-1}$, $i = 1,2,...,m-1$
$t_i = t_{i-1} + T_i$; $0 < t_i < H$, $i = 1,2,...,m-1$
$T_0 = 0$; $T_m = H - t_{m-1}$
$D(t)$: demand rate which is a function of time
$D(t) = a + bt$, $0 \le t \le H$, where $a \ge 0$ and $a + bt \ge 0$; a and b are constant, when b>0, demand increases with time, when b<0, demand decreases with time.
$B(p_i, T_i)$: buyer's relevant total cost during i period, $i = 1,2,...,m$.
$S(p_i, T_i)$: seller's relevant total cost during i period, $i = 1,2,...,m$.
$J(p_i, T_i)$: joint relevant total cost of buyer and seller during i period, $i = 1,2,...,m$.
$Q_i(t)$: inventory level at any time during i period.

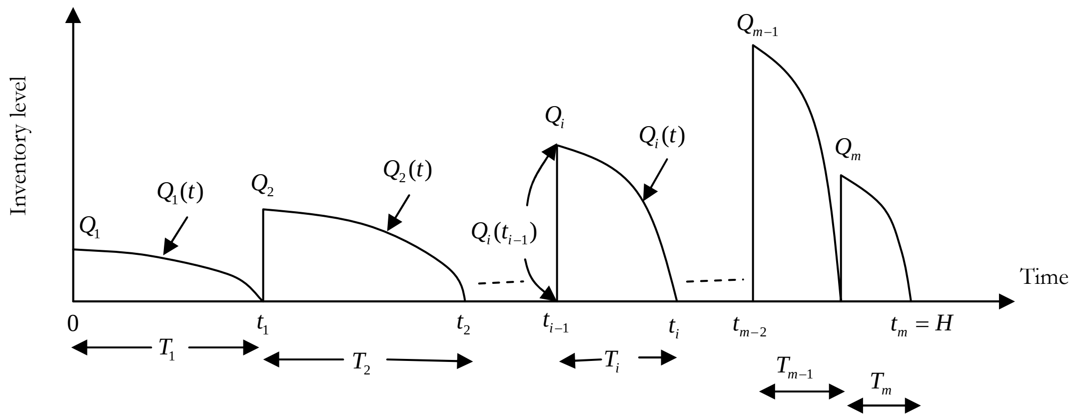

Figure 1. Inventory system with increasing demand rate.

3. MATHEMATICAL MODELING AND ANALYSIS

According to the assumption when the demand rate is linear increases with time, the relation figure of inventory level and time is depicted in figure 1.

If $Q_i(t)$ represents the inventory level at any time during i period, and the following differential equation can be used to describe the behavior of the system, where $t_{i-1} \le t \le t_i$.

$$\frac{d}{dt} Q_i(t) + \theta Q_i(t) + (a + bt) = 0 \tag{1}$$

$$Q_i(t) = e^{-\theta t}\left[\int -(a+bt)e^{\theta t}dt + c\right]$$
$$= \frac{b - a\theta - b\theta t}{\theta^2} + e^{-\theta t} \cdot c$$

From figure 1, we know that $Q_i(t_i) = 0$

Substituting $t = t_i$ into (1) and solving for c .

$$Q_i(t_i) = \frac{b - a\theta - b\theta t_i}{\theta^2} + e^{-\theta t_i} \cdot c = 0$$

$$c = \frac{a\theta + b\theta t_i - b}{\theta^2} \cdot e^{\theta t_i}$$, therefore

$$Q_i(t) = e^{-\theta t}\left(\frac{a\theta + b\theta t_i - b}{\theta^2} \cdot e^{\theta t_i}\right) - \left(\frac{a\theta + b\theta t - b}{\theta^2}\right) \tag{2}$$

Assumes $t = t_{i-1}$, the starting inventory level during i period is

$$Q_i = Q_i(t_{i-1})$$
$$= e^{-\theta t_{i-1}}\left[\frac{e^{\theta t_i}(a\theta + b\theta t_i - b)}{\theta^2}\right] - \left(\frac{a\theta + b\theta t_{i-1} - b}{\theta^2}\right)$$
$i = 1,2,...,m$ (3)

And the total inventory during i period can be derived from (2) as

$$I_i(t)=\int_{t_{i-1}}^{t_i} Q_i(t)dt \ , \ t_{i-1}\le t\le t_i \ , \ i=1,2,...,m$$
$$=\int_{t_{i-1}}^{t_i}\left(\frac{a\theta+b\theta t_i-b}{\theta^2}\cdot e^{\theta t_i}\cdot e^{-\theta t}+\frac{b-a\theta-b\theta t}{\theta^2}\right)dt \tag{4}$$

Because of $Q_i(t_i)=0$ $(i=1,2,...,m)$ and $T_i=t_i-t_{i-1}$, therefore the (4) can be represented as

$$I_i(t)$$
$$=\int_{t_{i-1}}^{t_{i-1}+T_i}\left(\frac{a\theta+b\theta t_i-b}{\theta^2}\cdot e^{\theta t_i}\cdot e^{-\theta t}+\frac{b-a\theta-b\theta t}{\theta^2}\right)dt \tag{5}$$

When $i=1$, $t_1=T_1=T$ (in order to convenient, therefore assumes $T_1=T$) and substituting $t_0=0$ into (5), can derive the total inventory during first period.

$$I_1=\int_0^T (e^{-\theta t}\cdot M-\frac{a\theta+b\theta t}{\theta^2})dt,$$
$$\text{where } M=\frac{a\theta+b\theta T-b}{\theta^2}\cdot e^{\theta T}$$
$$=\frac{-M}{\theta}(e^{-\theta T}-1)-(\frac{a\theta T+\frac{1}{2}b\theta T^2-bT}{\theta^2}) \tag{6}$$
$$=\frac{1}{\theta}(\frac{a\theta+b\theta T-b}{\theta^2})\cdot e^{\theta T}(1-e^{-\theta T})$$
$$-(\frac{a\theta T+\frac{1}{2}b\theta T^2-bT}{\theta^2})$$
$$=(\frac{a\theta+b\theta T-b}{\theta^3})(e^{\theta T}-1)-(\frac{a\theta T+\frac{1}{2}b\theta T^2-bT}{\theta^2})$$

The demand of buyer at i period is

$$D_i=\int_{t_{i-1}}^{t_i}(a+bt)dt$$
$$=at_i+\frac{1}{2}bt_i^2-at_{i-1}-\frac{1}{2}bt_{i-1}^2 \tag{7}$$

When $i=1$ and, assumes the $t_i=t_1=T_1=T$.

Under periodic review, the buyer relevant total average cost is

$$\frac{B(p_1,T)}{T}=\frac{A}{T}+\frac{p_1Q_1}{T}+\frac{p_1rI_1}{T}$$, where p_1 is the discount unit price, $0<p_1\le P$ (8)

The seller relevant average total cost is

$$\frac{S(p_1,T)}{T}=\frac{S+K}{T}+\frac{(P-p_1)Q_1}{T} \tag{9}$$

Such that the buyer-seller relevant average total joint cost is

$$\frac{J(p_1,T)}{T}=\frac{A+S+K}{T}+\frac{PQ_1}{T}+\frac{p_1rI_1}{T} \ . \tag{10}$$

3.1 Noncooperation Situation - Undiscounted price model

3.1.1 Prove the existence of buyer's minimum cost at T^*

In the undiscounted price case, $p_1=P$ and only T is a decision variable. The buyer will minimize his costs with an optimal order interval

Assumes the $\dfrac{B(P,T)}{T}=B(T)$ (11)

The necessary condition of minimizing the buyer's relevant cost in first period is

$$B'(T)=0 \tag{12}$$

The buyer's minimum relevant cost existence, the reason as follows:
Because of P, Pr is constant, assumes the $Pr=H$ ($p_1=P$) and substitute for it into (1)
Substituting $i=1$ into (3) can derive Q_1 which the starting inventory level of first period

$$Q_1=\left[\frac{e^{\theta T}(a\theta+b\theta T-b)}{\theta^2}-\frac{a\theta-b}{\theta^2}\right] \tag{13}$$

Substituting (6) and (13) into (8) and simplifying, we have

$$B(T)=\frac{A}{T}+\frac{P\left[\frac{e^{\theta T}(a\theta+b\theta T-b)}{\theta^2}-\frac{a\theta-b}{\theta^2}\right]}{T}$$
$$+\frac{H\left[\frac{a\theta+b\theta T-b}{\theta^3}\left(e^{\theta T}-1\right)-\left(\frac{a\theta T+\frac{1}{2}b\theta T^2-bT}{\theta^2}\right)\right]}{T} \tag{14}$$
$$=\frac{A}{T}+P\left[e^{\theta T}\left(\frac{a}{\theta T}+\frac{b}{\theta}-\frac{b}{\theta^2 T}\right)-\frac{a}{\theta T}+\frac{b}{\theta^2 T}\right]$$
$$+H\left[\left(\frac{a}{\theta^2 T}+\frac{b}{\theta^2}-\frac{b}{\theta^3 T}\right)\left(e^{\theta T}-1\right)-\left(\frac{a}{\theta}+\frac{bT}{2\theta}-\frac{b}{\theta^2}\right)\right]$$

Differentiating (14)

$B'(T)=\dfrac{1}{\theta^3T^2}\left[f(T)\right]$, where $f(T)$ is

$$\begin{aligned} f(T) = {} & -\theta^3 A + P(a\theta^3 Te^{\theta T} + b\theta^3 T^2 e^{\theta T} \\ & -b\theta^2 Te^{\theta T} - a\theta^2 e^{\theta T} + b\theta e^{\theta T} + a\theta^2 - b\theta) \\ & +H(-a\theta e^{\theta T} + be^{\theta T} + a\theta - b + a\theta^2 Te^{\theta T} \\ & +b\theta^2 T^2 e^{\theta T} - b\theta Te^{\theta T} - \frac{1}{2} b\theta^2 T^2) \end{aligned}$$

And $f(0) = -\theta^3 A < 0,\ f(\infty) = \lim_{T\to\infty}\left(f(T)\right) = \infty$.
Differentiating $f(T)$, we have

$$\begin{aligned} f'(T) = {} & P(a\theta^3 e^{\theta T} + a\theta^4 Te^{\theta T} + 2b\theta^3 Te^{\theta T} \\ & +b\theta^4 T^2 e^{\theta T} - b\theta^2 e^{\theta T} - b\theta^3 Te^{\theta T} \\ & -a\theta^3 e^{\theta T} + b\theta^2 e^{\theta T}) + H(-a\theta^2 e^{\theta T} \\ & +b\theta e^{\theta T} + a\theta^3 e^{\theta T} + a\theta^3 Te^{\theta T} + 2b\theta^2 Te^{\theta T} \\ & +b\theta^3 T^2 e^{\theta T} - b\theta e^{\theta T} - b\theta^2 Te^{\theta T} - b\theta^2 T) \\ = {} & P(a\theta^4 Te^{\theta T} + b\theta^3 Te^{\theta T} + b\theta^4 T^2 e^{\theta T}) \\ & +H(a\theta^3 Te^{\theta T} + b\theta^3 T^2 e^{\theta T} + b\theta^2 Te^{\theta T} - b\theta^2 T) \\ = {} & P(a\theta^4 Te^{\theta T} + b\theta^3 Te^{\theta T} + b\theta^4 T^2 e^{\theta T}) \\ & +H\left[a\theta^3 Te^{\theta T} + b\theta^3 T^2 e^{\theta T} + b\theta^2 T(e^{\theta T} - 1)\right] \end{aligned}$$

Because of $b\theta^2 T(e^{\theta T} - 1) > 0$, that is $f'(T) > 0$, therefore $f(T)$ is strictly increasing and there exists unique T^*, such that $f(T) < 0$ for $T \in (0, T^*)$, and $f(T) > 0$ for $T \in (T^*, \infty)$, the function $f(T)$ is depicted in Figure 2.

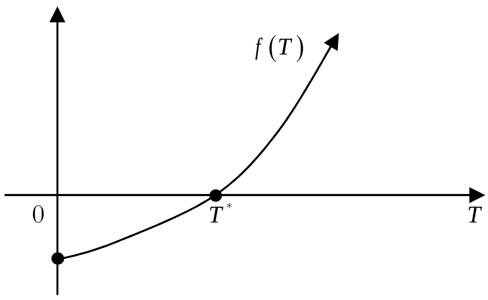

Figure 2. The figure of function $f(T)$.

Because of $B'(T) = \frac{1}{\theta^3 T^2}\left[f(T)\right]$, when $f(T^*) = 0$, namely $B'(T^*) = 0$, therefore $B'(T) < 0$ for $T \in (0, T^*)$, that is, the function $B(T)$ is strictly decreasing on $(0, T^*)$, and $B'(T) > 0$ for $T > T^*$, that is, the function $B(T)$ is strictly increasing on (T^*, ∞) depicted in Figure 3.

Therefore, we prove the existence of buyer's minimum cost $B(T)$ at $T = T^*$, the buyer's minimum cost is $B(T^*)$

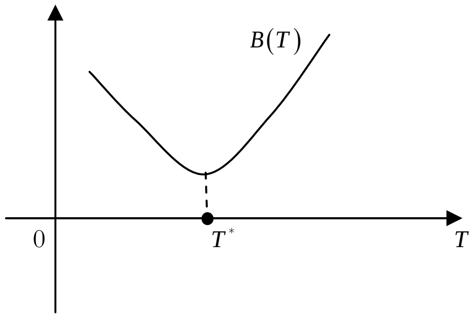

Figure 3. The figure of function $B(T)$.

3.1.2 The method of solving T^*.

First we solve the upper bound T_u^* and the lower bound T_l^* of T^*, then, solving T^* by dichotomy.

1. The solution of upper bound T_u^* as follows:

$$\begin{aligned} f(T) = {} & -\theta^3 A + P(a\theta^3 Te^{\theta T} + b\theta^3 T^2 e^{\theta T} - b\theta^2 Te^{\theta T} - a\theta^2 e^{\theta T} \\ & +b\theta e^{\theta T} + a\theta^2 - b\theta) + H(-a\theta e^{\theta T} + be^{\theta T} + a\theta - b \\ & +a\theta^2 Te^{\theta T} + b\theta^2 T^2 e^{\theta T} - b\theta Te^{\theta T} - \frac{1}{2} b\theta^2 T^2) \\ = {} & -\theta^3 A + P\left[\begin{aligned} & a\theta^2\left(\theta Te^{\theta T} - e^{\theta T} + 1\right) \\ & +b\theta e^{\theta T}\left(\theta^2 T^2 - \theta T + 1\right) - b\theta \end{aligned}\right] \\ & +H\left[\begin{aligned} & a\theta\left(\theta Te^{\theta T} - e^{\theta T} + 1\right) + be^{\theta T}\left(\theta^2 T^2 - \theta T + 1\right) \\ & -b - \frac{1}{2} b\theta^2 T^2 \end{aligned}\right] \\ \geq {} & -\theta^3 A + P\left[\begin{aligned} & a\theta(0) + b\theta e^{\theta T}\left(\frac{\theta^2 T^2}{2} + \frac{\theta^2 T^2}{2} - \theta T + 1\right) \\ & -b\theta \end{aligned}\right] \\ & +H\left[\begin{aligned} & a\theta(0) + be^{\theta T}\left(\frac{\theta^2 T^2}{2} + \frac{\theta^2 T^2}{2} - \theta T + 1\right) \\ & -b - \frac{1}{2} b\theta^2 T^2 \end{aligned}\right] \\ > {} & -\theta^3 A + P\left[b\theta e^{\theta T}\left(\frac{\theta^2 T^2}{2} + e^{-\theta T}\right) - b\theta\right] \\ & +H\left[be^{\theta T}\left(\frac{\theta^2 T^2}{2} + e^{-\theta T}\right) - b - \frac{1}{2} b\theta^2 T^2\right] \\ = {} & -\theta^3 A + P\left(\frac{b\theta^3 T^2}{2} e^{\theta T} + b\theta - b\theta\right) \\ & +H\left(\frac{b\theta^2 T^2}{2} e^{\theta T} + b - b - \frac{1}{2} b\theta^2 T^2\right) \\ \geq {} & -\theta^3 A + P\left[\frac{b\theta^3 T^2}{2}(1)\right] + H\left[\frac{b\theta^2 T^2}{2}(1) - \frac{1}{2} b\theta^2 T^2\right] \\ = {} & -\theta^3 A + \frac{Pb\theta^3}{2} T^2 + H(0) \\ = {} & -\theta^3 A + \frac{Pb\theta^3}{2} T^2 = F(T) \quad (say) \end{aligned}$$

Thus the positive root of $F(T) = 0$ is T_u^*, namely

$$T_u^* = \sqrt{\frac{2A}{Pb}} \tag{15}$$

Because of $f(T)$ is increasing function, therefore $T^* < T_u^*$, the relationship between $f(T)$ and $F(T)$ is depicted in Figure 4.

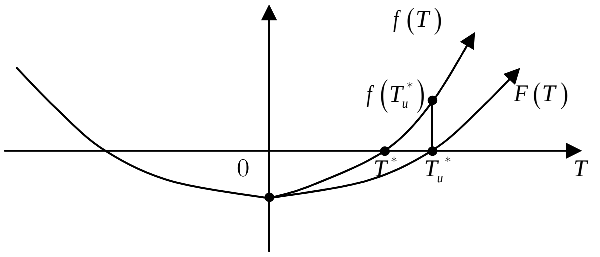

Figure 4. The figure of function $F(T)$.

2. The solution of lower bound T_l^* as follows:

$$\begin{aligned} f(T) &= -\theta^3 A + P(a\theta^3 Te^{\theta T} + b\theta^3 T^2 e^{\theta T} - b\theta^2 Te^{\theta T} - a\theta^2 e^{\theta T} \\ &\quad + b\theta e^{\theta T} + a\theta^2 - b\theta) + H(-a\theta e^{\theta T} + be^{\theta T} + a\theta - b \\ &\quad + a\theta^2 Te^{\theta T} + b\theta^2 T^2 e^{\theta T} - b\theta Te^{\theta T} - \frac{1}{2}b\theta^2 T^2) \\ &= -\theta^3 A + P\begin{bmatrix} a\theta^2 e^{\theta T}\left(\theta T - 1 + e^{-\theta T}\right) \\ + b\theta e^{\theta T}\left(\theta^2 T^2 - \theta T + 1 - e^{-\theta T}\right) \end{bmatrix} \\ &\quad + H\begin{bmatrix} a\theta e^{\theta T}\left(\theta T - 1 + e^{-\theta T}\right) + \\ be^{\theta T}\left(\theta^2 T^2 - \theta T + 1 - e^{-\theta T} - \frac{1}{2}\theta^2 T^2 e^{-\theta T}\right) \end{bmatrix} \\ &< -\theta^3 A + P\begin{bmatrix} a\theta^2 e^{\theta T}\left(\theta T - 1 + 1 - \theta T + \frac{\theta^2 T^2}{2}\right) \\ + b\theta e^{\theta T}\left(\theta^2 T^2 - \theta T + 1 - 1 + \theta T\right) \end{bmatrix} \\ &\quad + H\begin{bmatrix} a\theta e^{\theta T}\left(\theta T - 1 + 1 - \theta T + \frac{\theta^2 T^2}{2}\right) \\ + be^{\theta T}\left(\theta^2 T^2 - \theta T + 1 - 1 + \theta T - \frac{1}{2}\theta^2 T^2 (0)\right) \end{bmatrix} \\ &= e^{\theta T}\left\{\begin{matrix} -\theta^3 Abe^{-\theta T} + P\left(a\theta^2\left(\frac{\theta^2 T^2}{2}\right) + b\theta(\theta^2 T^2)\right) \\ + H\left[a\theta\left(\frac{\theta^2 T^2}{2}\right) + b(\theta^2 T^2)\right] \end{matrix}\right\} \\ &= e^{\theta T}\begin{bmatrix} -\theta^3 A(1 - \theta T) + P\left(\frac{1}{2}a\theta^4 + b\theta^3\right)T^2 \\ + H\left(\frac{1}{2}a\theta^3 + b\theta^2\right)T^2 \end{bmatrix} \\ &= G(T) \quad (say) \end{aligned}$$

for all of $T > 0$.

The solution of $G(T) = 0$ as well as

$$\left(\frac{1}{2}a\theta + b\right)(P\theta + H)T^2 + A\theta^2 T - A\theta = 0$$

$$T = T_l^* = \frac{-A\theta^2 + \sqrt{A^2\theta^4 + 4\left(\frac{1}{2}a\theta + b\right)(P\theta + H)A\theta}}{2\left(\frac{1}{2}a\theta + b\right)(P\theta + H)} > 0$$

$$(H = Pr) \tag{16}$$

T_l^* is the solution of lower bound of T^*, the relationship between $G(T)$, $f(T)$ and $F(T)$ is depicted in Figure 5.

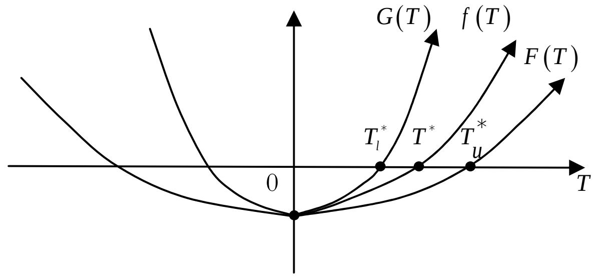

Figure 5. The figure of function $G(T)$.

The steps of algorithm:
(Note that: $f\left(T_u^*\right) > 0$, $f\left(T_l^*\right) < 0$)

The algorithm is given by Chung and Lin (1995) for solving the optimal order interval T^*.

Step1: Assumes a parameter ε, and let $\varepsilon > 0$. (more small more good)

Step2: Assumes the $T_l = T_l^*$, and $T_u = T_u^*$.

Step3: Assumes the $T_{opt} = \frac{T_l + T_u}{2}$.

Step4: If $\left|f\left(T_{opt}\right)\right| < \varepsilon$, implement step6, otherwise implement step5.

Step5:
If $f\left(T_{opt}\right) > 0$, let $T_u = T_{opt}$
If $f\left(T_{opt}\right) < 0$, let $T_l = T_{opt}$, then implement step 3.

Step6:
Exit the optimal order interval $T^* = T_{opt}$.
And hence Q_1^*

Substituting Q_1^* and T^* into (8) and (9) can derive the buyer-seller relevant average total cost.

The buyer relevant average total cost is

$$\begin{aligned} &\frac{B(P,T^*)}{T^*} \\ &= \frac{A}{T^*} + P\left[e^{\theta T^*}\left(\frac{a}{\theta T^*} + \frac{b}{\theta} - \frac{b}{\theta^2 T^*}\right) - \frac{a}{\theta T^*} + \frac{b}{\theta^2 T^*}\right] \qquad (17) \\ &+ Pr\left[\left(\frac{a}{\theta^2 T^*} + \frac{b}{\theta^2} - \frac{b}{\theta^3 T^*}\right)\left(e^{\theta T^*} - 1\right) - \frac{a}{\theta} - \frac{bT^*}{2\theta} + \frac{b}{\theta^2}\right] \end{aligned}$$

If the seller operates at T^*, his relevant average total cost will be

$$\frac{S(P,T^*)}{T^*}=\frac{S+K}{T^*} \tag{18}$$

3.2 Cooperation Situation - discount price model

3.2.1 Prove the existence of minimum average joint cost of buyer-seller at T^{**}

Assumes the $\dfrac{J(p_1,T)}{T}=J(T)$ (19)

Where p_1 is the discount unit price, $0<p_1\le P$

The unique minimum relevant joint cost of buyer-seller exists, the reason as follows:

Because of p_1, p_1r is constant, assumes the $p_1r=H$. Substitute for (6) and (13) into (10) and simplifying, we have

$$J(T)=\frac{A+S+K}{T}+\frac{P\left[\frac{e^{\theta T}(a\theta+b\theta T-b)}{\theta^2}-\frac{a\theta-b}{\theta^2}\right]}{T}$$
$$+\frac{H\left[\frac{a\theta+b\theta T-b}{\theta^3}\left(e^{\theta T}-1\right)-\left(\frac{a\theta T+\frac{1}{2}b\theta T^2-bT}{\theta^2}\right)\right]}{T}$$
$$=\frac{A+S+K}{T}+P\left[e^{\theta T}\left(\frac{a}{\theta T}+\frac{b}{\theta}-\frac{b}{\theta^2 T}\right)-\frac{a}{\theta T}+\frac{b}{\theta^2 T}\right] \tag{20}$$
$$+H\left[\left(\frac{a}{\theta^2 T}+\frac{b}{\theta^2}-\frac{b}{\theta^3 T}\right)\left(e^{\theta T}-1\right)-\left(\frac{a}{\theta}+\frac{bT}{2\theta}-\frac{b}{\theta^2}\right)\right]$$

Differentiating (20)

$$J'(T)=\frac{1}{\theta^3T^2}\left[g(T)\right],\text{ where} \tag{21}$$

$$\begin{aligned}g(T)=&-\theta^3\left(A+S+K\right)+P(a\theta^3Te^{\theta T}+b\theta^3T^2e^{\theta T}\\&-b\theta^2Te^{\theta T}-a\theta^2e^{\theta T}+b\theta e^{\theta T}+a\theta^2-b\theta)\\&+H(-a\theta e^{\theta T}+be^{\theta T}+a\theta-b+a\theta^2Te^{\theta T}\\&+b\theta^2T^2e^{\theta T}-b\theta Te^{\theta T}-\frac{1}{2}b\theta^2T^2)\end{aligned}$$

And

$$g(0)=-\theta^3\left(A+S+K\right)<0$$
$$g(\infty)=\lim_{T\to\infty}\left(g(T)\right)=\infty$$

Differentiating $g(T)$, we have

$$\begin{aligned}g'(T)=&P(a\theta^3e^{\theta T}+a\theta^4Te^{\theta T}+2b\theta^3Te^{\theta T}\\&+b\theta^4T^2e^{\theta T}-b\theta^2e^{\theta T}-b\theta^3Te^{\theta T}\\&-a\theta^3e^{\theta T}+b\theta^2e^{\theta T})+H(-a\theta^2e^{\theta T}\\&+b\theta e^{\theta T}+a\theta^3e^{\theta T}+a\theta^3Te^{\theta T}+2b\theta^2Te^{\theta T}\\&+b\theta^3T^2e^{\theta T}-b\theta e^{\theta T}-b\theta^2Te^{\theta T}-b\theta^2T)\\=&P(a\theta^4Te^{\theta T}+b\theta^3Te^{\theta T}+b\theta^4T^2e^{\theta T})\\&+H(a\theta^3Te^{\theta T}+b\theta^3T^2e^{\theta T}+b\theta^2Te^{\theta T}-b\theta^2T)\\=&P(a\theta^4Te^{\theta T}+b\theta^3Te^{\theta T}+b\theta^4T^2e^{\theta T})\\&+H\left[a\theta^3Te^{\theta T}+b\theta^3T^2e^{\theta T}+b\theta^2T(e^{\theta T}-1)\right]\end{aligned}$$

Because of $b\theta^2T(e^{\theta T}-1)>0$

That is $g'(T)>0$, hence $g(T)$ is strictly increasing and there is an unique T^{**} such that $g(T)<0$ for $t\in\left(0,T^{**}\right)$. And $g(T)>0$ for $t\in\left(T^{**},\infty\right)$. The function $g(T)$ is depicted in Figure 6.

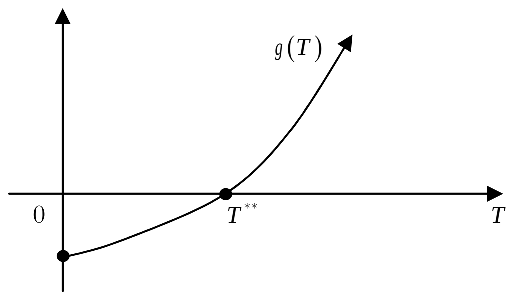

Figure 6. The figure of function $g(T)$.

Because of $J'(T)=\frac{1}{\theta^3T^2}\left[g(T)\right]$, when $g\left(T^{**}\right)=0$ namely $J'\left(T^{**}\right)=0$, therefore $J'(T)<0$ for $T\in\left(0,T^{**}\right)$, that is, the function $J(T)$ is strictly decreasing on $\left(0,T^{**}\right)$, and $J'(T)>0$ for $T>T^{**}$ that is, the function $J(T)$ is strictly increasing on $\left(T^{**},\infty\right)$ depicted in Figure.7.

Therefore, we proved the existence of the minimum joint cost of buyer-seller $J(T)$ at $T=T^{**}$, the minimum joint cost of buyer-seller is $J\left(T^{**}\right)$.

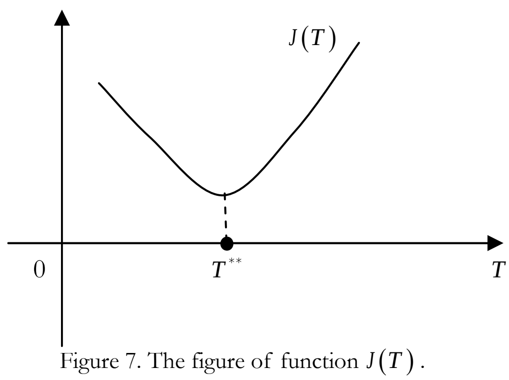

Figure 7. The figure of function $J(T)$.

3.2.2 The method of solving T^{**}

The algorithm of finding upper bound T_u^{**} and lower bound T_l^{**} of T^{**} is the same as finding T^*. Therefore, the solution of T^{**} and T_u^{**} are

$$T_u^{**} = \sqrt{\frac{2(A+S+K)}{p_1 b}} \tag{22}$$

$$T_l^{**} = \frac{-(A+S+K)\theta^2}{2\left(\frac{1}{2}a\theta+b\right)(p_1\theta+H)} + \frac{\sqrt{(A+S+K)^2\theta^4+4\left(\frac{1}{2}a\theta+b\right)(p_1\theta+H)(A+S+K)\theta}}{2\left(\frac{1}{2}a\theta+b\right)(p_1\theta+H)} > 0 \tag{23}$$

for $0 < T_l^{**} < T^{**} < T_u^{**}$

The algorithm is given by Chung and Lin (1995) for solving the optimal order interval T^{**}, and hence Q_1^{**}.

Substituting Q_1^{**} and T^{**} into (8), (9) and (10), then the buyer relevant total average cost is

$$\frac{B(p_1,T^{**})}{T^{**}} = \frac{A}{T^{**}} + \frac{p_1 Q_1^{**}}{T^{**}} + \frac{p_1 r I_1^{**}}{T^{**}} \tag{24}$$

If the seller operates at T^{**}, his relevant total average cost will be

$$\frac{S(p_1,T^{**})}{T^{**}} = \frac{S+K}{T^{**}} + \frac{(P-p_1)Q_1^{**}}{T^{**}} \tag{25}$$

The buyer-seller relevant average total joint cost is

$$\frac{J(p_1,T^{**})}{T^{**}} = \frac{A+S+K}{T^{**}} + \frac{PQ_1^{**}}{T^{**}} + \frac{p_1 r I_1^{**}}{T^{**}} \tag{26}$$

3.3 Probing into the relevant cost of buyer-seller in discount price and undiscounted price situation

In the undiscounted price case, $p_1 = P$ and only T is a decision variable. Due to $T^{**} > T^*$ therefore, $\frac{B(P,T^{**})}{T^{**}} > \frac{B(P,T^*)}{T^*}$ the buyer will need some enticement to operate at any order interval longer than T^*.

In the discount price case, the seller setting the unit price $p_1 < P$ inspire the buyer to place large order, but the seller total relevant cost will increase, represents

$$\frac{S(p_1,T^{**})}{T^{**}} > \frac{S(P,T^*)}{T^*}.$$

Base on foregoing analysis that the optimal scheme must look for the optimal price p_1^* substitute for p_1 and the optimal order interval T^{***} substitute for T^{**} to satisfy the condition of

$$\frac{B(p_1^*,T^{***})}{T^{***}} \le \frac{B(P,T^*)}{T^*} \text{ and } \frac{S(p_1^*,T^{***})}{T^{***}} \le \frac{S(P,T^*)}{T^*}.$$

Therefore define the cost saving of buyer-seller between the undiscounted price and discount price situation as follows:

Define the buyer's cost saving as

$$BS = \frac{B(P,T^*)}{T^*} - \frac{B(p_1^*,T^{***})}{T^{***}} \tag{27}$$

Define the seller's cost saving as

$$SS = \frac{S(P,T^*)}{T^*} - \frac{S(p_1^*,T^{***})}{T^{***}} \tag{28}$$

Relate (27) and (28) using the following relation equation is given by Chakravarty and Martin (1988)

$$SS = \alpha BS \tag{29}$$

Substitute for (27) and (28) into (29) and simplifying, the optimal discount price p_1^* is

$$p_1^* = \frac{\begin{pmatrix} T^{***}S + T^{***}K - T^*S - T^*K - T^*Q^{***}P \\ -\alpha AT^{***} - \alpha PQ^*T^{***} - \alpha P rI^*T^{***} + \alpha T^* A \end{pmatrix}}{T^*\left(Q^{***} + \alpha Q^{***} + \alpha r I^{***}\right)} \tag{30}$$

3.3.1 Prove the existence of minimum average joint cost of buyer-seller at T^{***}

Substitute for p_1^* into (10), we can derive

$$\frac{J(p_1^*,T)}{T} = \frac{A+S+K}{T} + \frac{PQ_1}{T} + \frac{p_1^* r I_1}{T} \tag{31}$$

Assumes the $J^*(T) = \frac{J(p_1^*,T)}{T}$ (32)

When discount price is p_1^*,

The unique minimum relevant joint cost of buyer-seller exists, the reason as follows:

Because of p_1^*, $p_1^* r$ is constant, assumes the $p_1^* r = H$, and substitute for (6) and (13) into (31), we have

$$J^*(T) = \frac{A+S+K}{T} + \frac{P\left[\frac{e^{\theta T}(a\theta + b\theta T - b)}{\theta^2} - \frac{a\theta - b}{\theta^2}\right]}{T}$$

$$+\frac{H\left[\frac{a\theta+b\theta T-b}{\theta^3}\left(e^{\theta T}-1\right)-\left(\frac{a\theta T+\frac{1}{2}b\theta T^2-bT}{\theta^2}\right)\right]}{T}$$

$$=\frac{A+S+K}{T}+P\left[e^{\theta T}\left(\frac{a}{\theta T}+\frac{b}{\theta}-\frac{b}{\theta^2 T}\right)-\frac{a}{\theta T}+\frac{b}{\theta^2 T}\right] \qquad (33)$$
$$+H\left[\left(\frac{a}{\theta^2 T}+\frac{b}{\theta^2}-\frac{b}{\theta^3 T}\right)\left(e^{\theta T}-1\right)-\left(\frac{a}{\theta}+\frac{bT}{2\theta}-\frac{b}{\theta^2}\right)\right]$$

Differentiating (33)

$$J^{*\prime}(T)=\frac{1}{\theta^3T^2}\left[h(T)\right], \text{ where} \qquad (34)$$

$$\begin{aligned}h(T)=&-\theta^3(A+S+K)+P(a\theta^3Te^{\theta T}+b\theta^3T^2e^{\theta T}\\&-b\theta^2Te^{\theta T}-a\theta^2e^{\theta T}+b\theta e^{\theta T}+a\theta^2-b\theta)\\&+H(-a\theta e^{\theta T}+be^{\theta T}+a\theta-b+a\theta^2Te^{\theta T}\\&+b\theta^2T^2e^{\theta T}-b\theta Te^{\theta T}-\frac{1}{2}b\theta^2T^2)\end{aligned}$$

And

$$h(0)=-\theta^3(A+S+K)<0,$$
$$h(\infty)=\lim_{T\to\infty}\left(h(T)\right)=\infty$$

Differentiating $h(T)$, we have

$$\begin{aligned}h'(T)=&P(a\theta^3e^{\theta T}+a\theta^4Te^{\theta T}+2b\theta^3Te^{\theta T}\\&+b\theta^4T^2e^{\theta T}-b\theta^2e^{\theta T}-b\theta^3Te^{\theta T}\\&-a\theta^3e^{\theta T}+b\theta^2e^{\theta T})+H(-a\theta^2e^{\theta T}\\&+b\theta e^{\theta T}+a\theta^3e^{\theta T}+a\theta^3Te^{\theta T}+2b\theta^2Te^{\theta T}\\&+b\theta^3T^2e^{\theta T}-b\theta e^{\theta T}-b\theta^2Te^{\theta T}-b\theta^2T)\\=&P(a\theta^4Te^{\theta T}+b\theta^3Te^{\theta T}+b\theta^4T^2e^{\theta T})\\&+H(a\theta^3Te^{\theta T}+b\theta^3T^2e^{\theta T}+b\theta^2Te^{\theta T}-b\theta^2T)\\=&P(a\theta^4Te^{\theta T}+b\theta^3Te^{\theta T}+b\theta^4T^2e^{\theta T})\\&+H\left[a\theta^3Te^{\theta T}+b\theta^3T^2e^{\theta T}+b\theta^2T(e^{\theta T}-1)\right]\end{aligned}$$

Because of $b\theta^2T(e^{\theta T}-1)>0$, that is, $h'(T)>0$, therefore $h(T)$ is strictly increasing and there exists unique T^{***}, such that $h(T)<0$ for $T\in\left(0,T^{***}\right)$ and $h(T)>0$ for $T\in\left(T^{***},\infty\right)$. The function $h(T)$ is depicted in Figure 8.

Because of $J^{*\prime}(T)=\frac{1}{\theta^3T^2}\left[h(T)\right]$, When $h\left(T^{***}\right)=0$, namely $J^{*\prime}\left(T^{***}\right)=0$, therefore $J^{*\prime}(T)<0$ for $T\in\left(0,T^{***}\right)$, that is, the function $J^*(T)$ is strictly decreasing on $\left(0,T^{***}\right)$, and $J^{*\prime}(T)>0$ for $T\in\left(T^{***},\infty\right)$, that is, the function $J^*(T)$ is strictly increasing on $\left(T^{***},\infty\right)$ depicted in Figure 9.

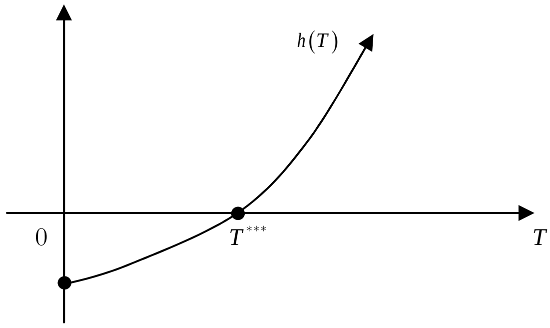

Figure 8. The figure of function $h(T)$.

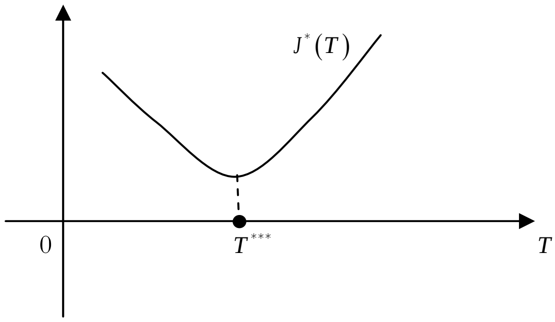

Figure 9. The figure of function $J^*(T)$.

Therefore, we proved the existence of the minimum joint cost of buyer-seller $J^*(T)$ at $T=T^{***}$, the minimum joint cost of buyer-seller is $J^*\left(T^{***}\right)$.

3.3.2 The method of solving T^{***}

Substitute for T^{***} into (31), the relevant average joint cost of buyer-seller will be

$$J^*\left(T^{***}\right)=\frac{J(p_1^*,T^{***})}{T^{***}}$$
$$=\frac{A+S+K}{T^{***}}+\frac{PQ_1^{***}}{T^{***}}+\frac{p_1^*rI_1^{***}}{T^{***}} \qquad (35)$$

Differentiating $J^*\left(T^{***}\right)$ with respect to T^{***}, setting the resulting expression equal to zero, and solving for optimal interval T^{***}. And then, substituting T^{***} into (30) for finding optimal discount price p_1^* (See Appendix).

Assumes the $T^*=T^{***}$ and substitute T^* and p_1^* into (8), (9) and (10), the relevant cost during the first period can be found for the buyer, the seller as well as their joint relevant cost, their total cost during the planning horizon can be found as

$$\sum_{i=1}^{m} B(p_i^*, T_i^*) = \sum_{i=1}^{m} A + \sum_{i=1}^{m} p_i^* Q_i^* + r\sum_{i=1}^{m} p_i^* I_i^* \tag{36}$$

$$\sum_{i=1}^{m} S(p_i^*, T_i^*) = \sum_{i=1}^{m} S + \sum_{i=1}^{m} K + \sum_{i=1}^{m} (P - p_i^*) Q_i^* \tag{37}$$

$$\sum_{i=1}^{m} J(p_i^*, T_i^*) = \sum_{i=1}^{m} (A + S + K) + \sum_{i=1}^{m} P Q_i^* + r\sum_{i=1}^{m} p_i^* I_i^* \tag{38}$$

Note that: the a need adjust with time on the go, such as $D(t_{i-1}) = a + bt_{i-1}$, $t_i = t_{i-1} + T_i^*$, $i = 1, 2, ..., m$.

4. NUMERICAL EXAMPLE

A product has a planning horizon of five years. The seller's manufacture setup cost is \$500/setup; the order processing cost is \$200/order. The buyer's demand increases with time, the demand function is 10+3600t. His order processing cost is \$300/order, the unit price of item is \$1.5/unit, and the buyer's holding cost is 40% of the unit cost of item, the rate of deterioration is 0.2.

Table 1. Relevant total cost of the buyer-seller with different α values

Sharing value α	Buyer's total cost $\frac{B(p_1^*, T^{***})}{T^{***}}$	Seller's total cost $\frac{S(p_1^*, T^{***})}{T^{***}}$	Joint total cost $\frac{J(p_1^*, T^{***})}{T^{***}}$
0.001	1167.257	2342.403	3509.660
0.01	1175.037	2335.587	3510.624
0.1	1244.589	2274.662	3519.251
0.3	1359.673	2173.868	3533.541
0.4	1403.254	2135.703	3538.957
0.5	1440.312	2103.253	3543.565
1	1569.192	1989.996	3559.188
10	1842.624	1751.058	3593.682
100	1893.276	1706.726	3600.002
1000	1898.757	1701.929	3600.686

According to foregoing numerical can get the optimal order interval T^* in undiscounted model, substituting T^* into (17) and (18), such that the buyer's undiscounted minimum cost strategy results in $\frac{B(P, T^*)}{T^*}$ = 1925.764; With the seller operation at T^* his relevant cost is $\frac{S(P, T^*)}{T^*}$ = 2346.568.

The numerical example demonstrates that the optimal discount price p_1^* and optimal order interval T^{***} is able to satisfy the condition of $\frac{B(p_1^*, T^{***})}{T^{***}} \le \frac{B(P, T^*)}{T^*}$ and $\frac{S(p_1^*, T^{***})}{T^{***}} \le \frac{S(P, T^*)}{T^*}$ (see Table1.). Furthermore, the global minimum $\frac{J(p_1^*, T^{***})}{T^{***}}$ of \$3509.66 is achieved at α =0.001, where $\frac{B(p_1^*, T^{***})}{T^{***}}$ is also minimized (see Table1.).

The seller, however, realizes without benefit at that level. Indeed, $\frac{S(p_1^*, T^{***})}{T^{***}}$ is minimized as α approaches infinity, where T^{***} approaches T^{**} and p_1^* approaches P.

Table 2. Resulting values for different number of order when α=1

No. of order i	Order period Ti	Order epoch ti	Discount price pi	Order quantity Qi
1	0.532996	0.532996	0.849941	554.8127
2	0.490547	1.023544	0.899896	1456.678
3	0.458304	1.481848	0.948344	2175.348
4	0.431822	1.913670	0.997253	2766.146
5	0.409409	2.323079	1.046925	3262.160
6	0.390033	2.713112	1.097529	3685.027
7	0.372998	3.086110	1.149214	4049.592
8	0.357825	3.443935	1.202085	4366.616
9	0.344154	3.788090	1.256280	4643.956
10	0.331721	4.119811	1.311917	4887.640
11	0.320330	4.440140	1.369077	5102.487
12	0.309817	4.749958	1.427888	5292.201
13	0.300060	5.050018	1.488449	5793.920

5. CONCLUDING REMARKS

In this paper, a mathematical model is developed for the optimal order interval and the optimal quantity discount price. The numerical example demonstrates that adjust the optimal order interval and the optimal quantity discount price can benefit both the buyer and the seller in cooperation situation. But minimize the joint cost of buyer-seller seems unable to benefit equally both the buyer and the seller. Therefore, how to determine a best negotiated policy to more equitably distribute cost savings is very important. We provide the procedures and a joint cost saving-sharing scheme to balance the mutual benefits of the buyer-seller. But we think that regards sharing value as the level of contribution of the buyer-seller to joint system seems fairer than an instrument of negotiation seems to be interesting and worthy of further investigation.

ACKNOWLEGEMENTS

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APPENDIX

Solving T^{***} and p_1^* for numerical example

(The numerical solution calculate by Maple)

b:=3600;theta:=0.2;P:=1.5;r:=0.4;A:=300;

K:=200;S:=500;alpha:=1;a:=10;T*:=0.2983079387:

Q_1^* := evalf(((exp(1)^(theta*T*))*(a*theta+b*theta*T*-b)/theta^2)-((a*theta-b)/theta^2));

$$Q_1^* = 169.76739$$

I_1^*:=evalf(((a*theta+b*theta*T*-b)/theta^3)*(exp(1)^(theta*T*)-1)-((a*theta*T*+(1/2)*b*theta*T*^2-b*T*)/theta^2));

$$I_1^* = 33.0329$$

$B\left(P,T^*\right)/T^*$:=evalf(A/T*+P*Q/T*+P*r*l/T*); Let $B\left(T^*\right)=B(P,T^*)/T^*$

$$B(T^*) = 1925.764455$$

$S\left(P,T^*\right)/T^*$:=evalf((S+K)/T*); let $S\left(T^*\right)=S(P,T^*)/T^*$

$$S(T^*) = 2346.568459$$

I_1^{***}:=evalf(((a*theta+b*theta*T***-b)/theta^3)*(exp(1)^(theta*T***)-1)-((a*theta*T***+(1/2)*b*theta*T***^2-b*T***)
/theta^2));

$$I_1^{***} = 125(-3598+720T^{***})\left(e^{0.2T^{***}}-1\right)+89950T^{***}-9000T^{***^2}$$

Q_1^{***}:=evalf(((exp(1)^(theta*T***))*(a*theta+b*theta*T***-b)/theta^2)-((a*theta-b)/theta^2));

$$Q_1^{***} = 25e^{0.2T^{***}}\left(-3598+720.07\right)+89950$$

$B(p_1^*,T^{***})/T^{***}$:=evalf(simplify(A/T***+p$_1^*$*Q$_1^*$/T***+p$_1^*$*r*I$_1^*$/T***)); let $B\left(T\right)=B(p_1^*,T^{***})/T^{***}$

$$B(T) = -\frac{10\begin{pmatrix}-30+26985\,p_1^*e^{0.2T^{***}}-5400\,p_1^*e^{0.2T^{***}}T^{***}\\-26985\,p_1^*+2\,p_1^*T^{***}+360\,p_1^*T^{***^2}\end{pmatrix}}{T^{***}}$$

$S\left(p_1^*,T^{***}\right)/T^{***}$:=evalf(simplify((S+K)/T***
+(P-p$_1^*$)*Q$_1^{***}$/T***)); , let $S\left(T\right)=S\left(p_1^*,T^{***}\right)/T^{***}$

$$S(T) = -\frac{25\begin{pmatrix}-5425+5397e^{0.2T^{***}}-1080e^{0.2T^{***}}T^{***}\\-3598\,p_1^*e^{0.2T^{***}}+720\,p_1^*e^{0.2T^{***}}T^{***}+3598\,p_1^*\end{pmatrix}}{T^{***}}$$

BS:=evalf(simplify(B(T*)-B(T)));

$$BS = \frac{1}{T^{***}}\left(5\times10^{-6}\begin{pmatrix}3.85152891\times10^{8}T^{***}-6\times10^{7}+5.397\times10^{10}\,p_1^*e^{0.2T^{***}}\\-1.08\times10^{10}\,p_1^*e^{0.2T^{***}}T^{***}-5.397\times10^{10}\,p_1^*+4\times10^{6}\,p_1^*T^{***}\\+7.2\times10^{8}\,p_1^*T^{***^2}\end{pmatrix}\right)$$

SS:=evalf$\left(\text{simplify}\left(\text{S}\left(\text{T}^*\right)\text{-S}\left(\text{T}\right)\right)\right)$;

$$SS=\frac{1}{T^{***}}\left(10^{-6}\begin{pmatrix}2.346568459\times10^{9}T^{***}-1.35625\times10^{11}+1.34925\times10^{11}e^{0.2T^{***}}-2.7\times10^{10}e^{0.2T^{***}}T^{***}\\-8.995\times10^{10}p_1^{*}e^{0.2T^{***}}+1.8\times10^{10}p_1^{*}e^{0.2T^{***}}T^{***}+8.995\times10^{10}p_1^{*}\end{pmatrix}\right)$$

M:=evalf(simplify(SS-alpha*BS));

$$M=-\frac{1}{T^{***}}\left(4\times10^{-6}\begin{pmatrix}-1.05201001\times10^{8}T^{***}+3.383125\times10^{10}-3.373125\times10^{10}e^{0.2T^{***}}+6.75\times10^{9}e^{0.2T^{***}}T^{***}\\+8.995\times10^{10}p_1^{*}e^{0.2T^{***}}-1.8\times10^{10}p_1^{*}e^{0.2T^{***}}T^{***}-8.995\times10^{10}p_1^{*}+5\times10^{6}p_1^{*}T^{***}+9\times10^{8}p_1^{*}T^{***2}\end{pmatrix}\right)$$

P_1^{*}:=simplify(solve(M=0,p_1^{*}));

$$p_1^{*}=-\frac{2\times10^{-7}\begin{pmatrix}-1.05201001\times10^{8}T^{***}+3.383125\times10^{10}\\-3.373125\times10^{10}e^{0.2T^{***}}+6.75\times10^{9}e^{0.2T^{***}}T^{***}\end{pmatrix}}{17990e^{0.2T^{***}}-3600e^{0.2T^{***}}T^{***}-17990+T^{***}+180T^{***2}}$$

And then, substitute p_1^{*} into $J(p_1^{*},T^{***})/\mathrm{T}^{***}$, solving T^{***}

$J(p_1^{*},T^{***})/\mathrm{T}^{***}$:=simplify((A+S+K)/T***+P*Q$_1^{***}$/T***+p$_1^{***}$*r*I$_1^{***}$/T***);

let $J(T)=J(p_1^{*},T^{***})/\mathrm{T}^{***}$

$$J(T)=\frac{\left(8\times10^{-5}\begin{pmatrix}5.07047752\times10^{10}T^{***}+6.103072399\times10^{11}T^{***2}\\-1.831956228\times10^{13}e^{0.2T^{***}}T^{***}+9.129362812\times10^{13}e^{0.2T^{***}}\\-4.551188906\times10^{13}e^{0.4T^{***}}+1.8214875\times10^{13}e^{0.4T^{***}}T^{***}\\-5.970194099\times10^{11}e^{0.2T^{***}}T^{***2}-1.8225\times10^{12}e^{0.4T^{***}}T^{***2}\\+1.215\times10^{11}e^{0.2T^{***}}T^{***3}-9.46809009\times10^{8}T^{***3}\\-4.578173906\times10^{13}\end{pmatrix}\right)}{\left(17990e^{0.2T^{***}}-3600e^{0.2T^{***}}T^{***}-17990+T^{***}+180T^{***2}\right)T^{***}}$$

N:=simplify(diff(J(T),T***));

$$N=\frac{\left(2\begin{pmatrix}-1.965749512\times10^{13}e^{0.6T^{***}}T^{***}+3.662539125\times10^{9}T^{***}-7.868826\times10^{11}e^{0.6T^{***}}T^{***3}+5.497064455\times10^{11}T^{***2}\\+5.243698215\times10^{12}e^{0.6T^{***}}T^{***2}-1.974251541\times10^{13}e^{0.2T^{***}}T^{***}+9.863943425\times10^{13}e^{0.2T^{***}}-5.2488\times10^{9}e^{0.4T^{***}}T^{***5}\\+1.7496\times10^{8}e^{0.2T^{***}}T^{***6}+3.063741974\times10^{11}e^{0.4T^{***}}T^{***3}-2.171837232\times10^{10}e^{0.2T^{***}}T^{***4}+4.805568\times10^{10}e^{0.4T^{***}}T^{***4}\\-8.8600405\times10^{8}e^{0.2T^{***}}T^{***5}+3.275035537\times10^{13}e^{0.6T^{***}}-3.294453943\times10^{13}-4.39425\times10^{9}T^{***4}\\+5.2488\times10^{10}e^{0.6T^{***}}T^{***4}+6.32498763\times10^{8}T^{***3}+2.163543955\times10^{11}e^{0.2T^{***}}T^{***2}+2.184404301\times10^{11}e^{0.2T^{***}}T^{***3}\\+3.939634799\times10^{13}e^{0.4T^{***}}T^{***}-6.017530735\times10^{12}e^{0.4T^{***}}T^{***2}-9.844525015\times10^{13}e^{0.4T^{***}}\end{pmatrix}\right)}{\begin{pmatrix}17990e^{0.2T^{***}}-3600e^{0.2T^{***}}T^{***}\\-17990+T^{***}+180T^{***2}\end{pmatrix}^{2}T^{***2}}$$

T***:=solve(N=0,T***);

$$\mathrm{T}^{***}=0.532996469$$

substitute T^{***} into (30),

$$p_1^* = -\frac{2\times10^{-7}\begin{pmatrix}-1.05201001\times10^8 T^{***} + 3.383125\times10^{10}\\ -3.373125\times10^{10} e^{0.2T^{***}} + 6.75\times10^9 e^{0.2T^{***}} T^{***}\end{pmatrix}}{\begin{pmatrix}17990e^{0.2T^{***}} - 3600e^{0.2T^{***}} T^{***}\\ -17990 + T^{***} + 180T^{***2}\end{pmatrix}}$$

$$= 0.8499413409$$

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