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# An inventory model for growing items with credit period offers, green subsidy, and profit sharing

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***Abstract:*** In this developed model, the packer processes the received lot from the grower and delivers the lot to the retailer in the non-processing period. Packer offers a credit period to retailers, and he shares his profit during the credit period. Demand is multi-variate. Carbon emissions are considered, and green product manufacturing is motivated by green subsidies. The joint total profit function is obtained, and the numerical and sensitivity analysis is given to support the results.

**Keywords–**Multi-echelon supply chain, Growing items, Trade credit, Profit sharing, Carbon emissions.

**AMS subject classification–**90B05, 90B06, 90B50

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## 1. INTRODUCTION

NABARD and the Ministry of Micro, Small, and Medium-Scale Enterprises provide the primary funding scheme to assist farmers and promote poultry farming activity. The scheme helps to strengthen and support the poultry industry by offering jobs or entrepreneurship opportunities in underserved areas of India, thereby contributing to the country's economic growth. The venture capital funding encourages food producers in non-traditional states and thus benefits the people by creating job opportunities in underserved areas. It also enhances the production of poultry products in the growing market in the national market and is in high demand in the national market, as well as the productivity of units through proper training and technological advancement. Also, through poultry dressing and branding outputs, it provides quality meat to consumers by FAO standards, maintains hygiene practices, and improves hygienic sales of poultry products in urban areas, particularly neighborhood societies. Enhances efficiency by manufacturing different facilities for rearing poultry species with high market potentials, such as quails, ducks, and turkeys. In this article, the packer and the retailer are involved in the developed model. The packer processes the obtained slaughtered meat from the grower, and some greenhouse gas emissions are involved during this process. The packer makes environmentally friendly green products, and the retailer's demand is advertisement dependent. A green subsidy for making green products is given to the packer.

## 2. RELATED LITERATURE

### 2.1 Growing items inventory supply chain

Economic order(production) quantity (EOQ/EPQ) models for manufacturing products have typically been provided. Various EOQ/EPQ models have been proposed in the literature, each incorporating certain significant properties of a specific category of items. Growing inventory items include poultry and cattle. Hidayat's *et al.*(2020) proposed scheme modifies three presumptions of the classical EOQ,i.e., purchased objects do not proliferate, infinite capacity, and an unlimited budget. Inventory models for growing items that take into account quality aspects, allowable shortages with complete back ordering, and holding costs during both the growth and consumption periods, a model of nonlinear programming is developed (Alfares and Afzal, 2021). Gharaeiand Almehdawe (2020) introduced Economic Growing Quantity (EGQ), a new group of inventory models focused on growing commodities in agricultural businesses like fishing, livestock, and cattle. For a growing item, an EGQ inventory model takes into account the probability density functions of lifespan and mortality. It also takes into account the growing activities of live and dead objects.

Abbasi *et al.* (2022) developed a theory of growing economic order quantity was developed in this work, which was presented using a fictitious numerical example. It was supposed that a corporation acquires day-old chicks, feeds and

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raises them until they reach the required weight, and then sells them after quality control. In this supply chain, a buyer orders commodities such as animals and poultry, which develop and achieve their optimal weight over time. Shortages are not permitted, and order numbers must be integer values. A two-tiered sustainable supply chain model with a supplier–retailer scenario is studied in this study (Chandramohan and Ramasamy, 2023). The supplier's primary responsibility is to breed newborn animals by a biological growth plan. Carbon emissions are calculated based on the transportation of killed items to the store. An integrated inventory model is developed (Khedlekar and Singh, 2022)to optimize the performance of the entire food supply chain. The processing echelon's goal is to convert live growing materials into processed food products. Once processed, the goods are vulnerable to deterioration at both the processing and retail levels. Cárdenas-Barrón*et al.*(2020) research considers an economic order quantity (EOQ) stock model with nonlinear dependent demand and nonlinear holding cost. It is designed from the retailer's perspective, with the supplier providing a trade credit period. The standard concept of zero-ending inventory level is relaxed in this work. Giri*et al.*(2018) worked on the manufacturer-retailer supply chain model for inventory products with trade credit offers and profit sharing strategy; this model benefits the manufacturer and retailer well. Since it benefits in a useful manner, we tried to implement this strategy in the three-echelon supply chain model for growing and deteriorating items (Mashud*et al.*, 2021), with selling price-dependent demand and trade credit offers also with profit-sharing by the retailer to the processor developed. Gharei and Almehdawe (2020) developed an inventory model for growing items to find the optimal replenishment cycle for the retailer to gain profit and minimize the total expense of the firm. Khedlekar and Singh (2022), Nigwal *et al.* (2022), and Khedlekar*et al.* (2023) worked on the optimal replenishment inventory model for the items consisting of various realistic demand patterns and worked on preservation technology investment to reduce the deterioration rate of the products.

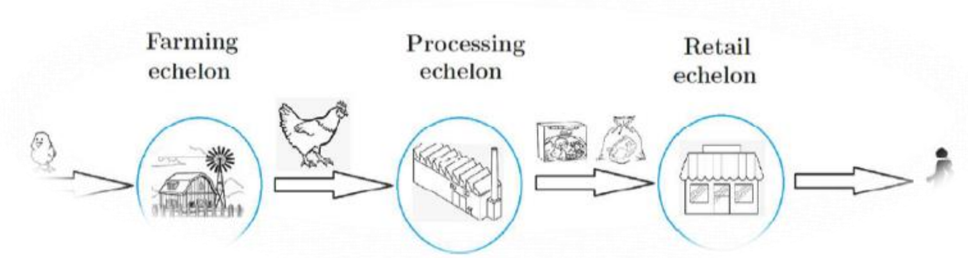

Figure 1: Representation of the Supply Chain

## 2.2 Contribution

The authors worked on a multi-echelon supply chain for growing and deteriorating items in this model. Consider a processor-retailer with a single growing item (Inventory model explanation in Fig 1). In this case, the processor's and retailer's demand rates are price-dependent. Because we have considered the emission cost for the processor's inventory cycle, the product emits carbon dioxide throughout the processing period. The processor also offers the retailer a trade credit period, and the retailer agrees to pay the processor a fixed percentage of his profit. In this model, it is a win-win situation for both the processor and the retailer. The retailer obtains the packed, processed product from the processor because every processed livestock product must have a limiting period of consumption. After all, the expiration rate has been factored into the retailer's inventory level. The optimal values and the joint total profit cost of the two-tiered system are determined, and the results are verified.

The table listed below contains the contributions of various authors regarding this study:

| Study | Echelon type | Items | Demand | Green Product |
|---|---|---|---|---|
| Abbasi*et al.* (2022) | EOQ | Growing items | Stock | × |
| Alfares and Afzal (2021) | EOQ | Growing items | Imperfect items | × |
| Hidayat*et al.* (2020) | EOQ | Growing items | Selling price | × |
| Cárdenas-Barrón*et al.* (2020) | EOQ | N.Growing | Stock | × |
| Khedelekar*et al.* (2023) | EOQ | N.Growing | Selling price | × |
| Khedelekar and Singh (2022) | EOQ | N.Growing | Selling price | × |
| Sebatjane and Adetunji (2022) | EOQ | Growing items | Selling price | × |
| Chandra and Ramasamy (2023) | Multi-echelon | Growing items | Selling price | × |
| Choudhry Mahata (2021) | Multi-echelon | Growing items | Selling price | × |
| Sebatjane (2022) | Multi-echelon | Growing items | Selling price | × |
| This model | Multi-echelon | Growing items | Price and Green product | ✓ |

## 3. NOTATIONS

| | Processor's Notations |
|---|---|
| $A_\gamma$ | Advertisement frequency |
| c,v,d,g | elasticity parameters for advertisement & credit period,greenness of the product |
| s | subsidy cost for green innovation |
| $P_r$ | Processing rate of the processor in period$T_{v_1}$. |
| $T_p$ | Period of theprocessor's echelon |
| $L_p$ | Processor's weight of the inventory |
| $p_p$ | Unit price of theprocessor's lot (decision variable) |
| $T_{v_1}$ | Processing time of the received inventory |
| $T_{v_2}$ | Non-processing time of the retailer |
| $\theta_p$ | Deterioration rate of the Processor over time$T_p$ |
| $I_{v_1}(t_1)$ | Inventory level of the processor in $0 \le t_1 \le T_{v_1}$ |
| $I_{v_2}(t_2)$ | Inventory level of the processor in time$0 \le t_2 \le T_{v_2}$ |
| $\delta$ | Profit ratio shared by the retailer to the processor |

| | Retailer's Notations |
|---|---|
| $I_r(t)$ | Inventory level of the retailer in $[0,T_r]$ |
| $L_r$ | Lot size of the retailer |
| $p_r$ | Unit price of the retailer's lot (decision variable) |
| $\theta_r(t)$ | Deterioration rate of the retailer |
| $c_d$ | Deterioration cost of the growing items in the inventory |
| $i_m$ | Rate of interest paid to the processor |
| $i_e$ | Interest earned from the source (bank) |
| $i_c$ | Interest Charged from the retailer by the source(bank) |
| $i_v$ | Opportunity lost sales cost for the processor |

## 4. ASSUMPTIONS

1. Two echelon inventory model for growing items is considered (Packer- Retailer)
2. The demand of the packer and retailer depends on the selling price, frequency of advertisement, green products, and credit period offered by the packer to the retailer.

$D_r$=(a-b$p_r$)) –Retailer's Demand pattern

where a and b are parameters regarding the fluctuation of the demand other than price $p_r$

$D_p = a - bp_p + vA_\gamma^c + dM + g\omega$) – Processor's demand pattern

where $\omega$ is the green product price's co-efficient and $g$ is the green product's price.

3. Shortages are not allowed.
4. If the payback time exceeds, the retailer pays the agreed profit share up to R.
5. If the retailer pays before S, the profit is just for that period. If the retailer fails to pay, the packer gains more. The packer may request the merchant to contribute a portion of revenue during S. Throughout this process; the retailer puts his profit in the bank.

## 5. MODEL FORMULATION

The grower feeds the live newborn items up to the maturity stage $w_1$(reaches the maximum possible weight), then ships the grown items $L_g$ to the processor in each shipment. After some time, the lot in the processor's echelon depletes due to demand $D_p$ and deterioration $\theta_p$ and reaches zero in time $T_{v_2}$. After the processing period $(T_{v_1})$ ends the processor ships the lot $L_p$, to the retailer in the non-processing time $T_{v_2}$. In each shipment, the products in the processor's lot $L_p$ also depletes in the ratio $\theta_r$. After received by the retailer and reaches zero due to demand $D_r$ and deterioration. The process of the three supply chain players is explained in the upcoming sections.

### 5.1 Processor's Echelon

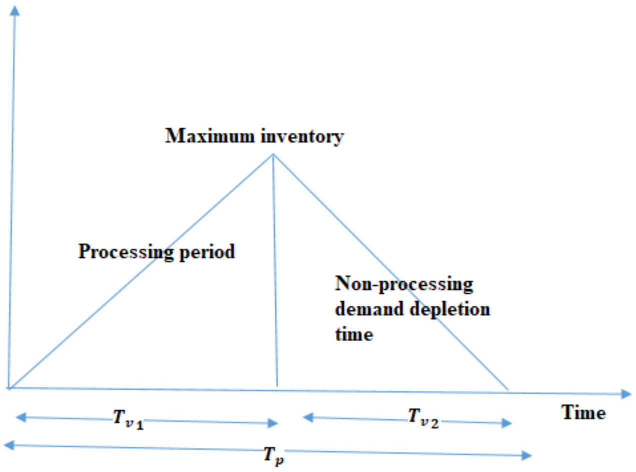

Figure 2: Processor's Inventory Cycle

Every $T_p$ time units, the processor receives an order for $L_p$ weight units of inventory. The processor delivers $N$ evenly proportioned shipments of processed inventory to the retailer in every $T_r$ time units based on the lot size of $L_p$ weight units. It can be separated into two pieces based on the processor's cycle time $T_p$, namely the preparation and non-preparation portions $T_{v_1}, T_{v_2}$ . The representation of the cycle preparation period and lot size is described in Figure 2.

$$I_p(t) = \int_0^{T_{v_1}} I_{v_1}(t_1)dt_1 + \int_0^{T_{v_2}} I_{v_2}(t_2)dt_2 \tag{1}$$

The prepared inventory is generated at rate $P_r$ , and deteriorates at a constant rate $\theta_p$ . The processor inventory is depleted owing to demand and deterioration, and it accumulates due to processing. As a result, the inventory level is shown here over the time window $[0, T_p]$ is, (where $D_p = a - bp_p + vA_\gamma^c + dM + g\omega$)

$$\frac{dI_{v_1}(t_1)}{dt_1} = (P_r - D_p) - \theta_p I_{v_1}(t_1), 0 \le t_1 \le T_{v_1} \tag{2}$$

Similarly, after the processing time of the cycle, the prepared inventory degrades due to supply and deterioration; even so, there is no gathering of the treated inventory in time $[0, T_{v_2}]$ .

$$\frac{dI_{v_2}(t_2)}{dt_2} = -D_p - \theta_p I_{v_2}(t_2), 0 \le t_2 \le T_{v_2} \tag{3}$$

The boundary conditions $I_{v_1}(0) = I_{v_2}(T_{v_2}) = 0$, is applied to find the inventory level of the processor's cycle,

$$I_{v_1}(t_1) = \frac{P_r - D_p}{\theta_p}\left(1 - e^{-\theta_p t_1}\right), 0 \le t_1 \le T_{v_1} \tag{4}$$

$$I_{v_2}(t_2) = \frac{D_p}{\theta_p}\left(e^{-\theta_p (T_{v_2} - t_2)} - 1\right), 0 \le t_2 \le T_{v_2} \tag{5}$$

The boundary conditions $I_{v_1}(T_{v_1}) = I_{v_2}(0)$, is used to find out the period of processing,
The processing amount is identical to the quantity received by the processor from the grower.

### 5.2 Carbon mission during the processing of Livestock

A common misconception is that chickens have no impact on climate change because, unlike cows, they do not emit methane during digestion. GHGs, such as CO2 from fossil fuels and nitrous oxide from fertilizer applications, are still released in the production of chicken feed. Furthermore, chicken manure emits nitrous oxide, which is even more powerful than biogas and has 298 times the overall heating capacity of CO2 over 100 years. Only half of the emissions from chicken production occur before slaughter. Chicken flesh is typically processed into a variety of products, including boneless, skinless meat and chicken nuggets; each of these stages requires a significant amount of energy and water, significantly increasing the GHG carbon output of chicken products (Giri *et al.* 2018).

We have included the carbon emission cost during the processing process because these emission costs may increase the total cost of the processor, which affects the processor's profit, and it may also change the total profit of the integrated system of the supply chain, because we have to introduce some carbon emission regulation policy to reduce the emission, such as carbon tax, cap and trade policy, carbon cap, and offset. We worked on carbon tax policy in our model, as inKhedlekar*et al.* (2023).

1. When the processor's lot size is smaller than demand during $T_p$ $(D_pT_p)$ , the quantity of degrading inventory throughout this processor's cycle, and carbon emission on degradation is defined. When the processor's degradation cost of $c_d$ per weight unit is taken into account as,

$$DC_{pr} = \frac{c_d+c_d'}{T_p}\left(L_p - D_pT_p\right) = \frac{c_d+c_d'}{T_p}\left(P_rT_{v_1} - D_pT_p\right) \tag{6}$$

where $c_d'$, is the emission parameter for degradation cost.

2. The processor's holding cost for the processed inventory per unit time,

$$HC_{pr} = \frac{h_p+h_p'}{T_p}\int_0^{T_{v_1}} I_{v_1}(t_1)dt_1 + \int_0^{T_{v_2}} I_{v_2}(t_2)dt_2 \tag{7}$$

$$= \frac{h_p+h_p'}{T_p}\left[\frac{(P_r-D_p)T_{v_1}^2}{2}\left(1-\frac{\theta_pT_{v_1}}{3}\right) + \frac{D_pT_{v_2}^2}{2}\left(1+\frac{\theta_pT_{v_2}}{3}\right)\right] \tag{8}$$

$h_{\mathrm{p}}'$- carbon emission cost for holding the inventory.

3. The carbon emission due to processing is, where $e_p$, emission cost for the processing period per unit item,

$$CE_p = e_pP_rT_{v_1} \tag{9}$$

4. The processor's opportunity cost due to trade credit offer is,

$$OLC_p = \frac{i_pp_p}{T_p}\int_0^R D_pdt = \frac{i_pp_pD_pR}{T_p} \tag{10}$$

where $i_p$ is the opportunity loss cost due to the credit period offer.

5. The green subsidy cost offered by the government for making green, innovative products to make the environment better; the subsidy amount given by the Government is directly proportional to the ratio of the product's greenness level, and $\omega$ greenness level of the product. The subsidy cost is

$$GC = P_rs\omega T_{v_1} \tag{11}$$

**Case IA:**
When $R < M$
Here, the profit-sharing ratio of the processor is the same for the upcoming case IC.

The Profit shared is $PF_{p_{11}} = PF_{p_{13}} = \delta(p_r - p_p)D_rM$ (12)

Then, the total profit of the processor in this case is given as

$$TP_{pIA}(T,p_p,R) = (p_r-p_p)D_p - \frac{K_{pr}}{T_p} - \frac{p_g\alpha_1gw_1}{T_p} - e_pP_rT_{v_1} - \frac{c_d+c_d'}{T_p}\left(P_rT_{v_1} - T_p\right) + i_m(R-M)p_pD_p + GC - \frac{h_p+h_p'}{T_p}\left[\frac{(P_r-D_p)T_{v_1}^2}{2}\left(1-\frac{\theta_pT_{v_1}}{3}\right) + \frac{D_pT_{v_2}^2}{2}\left(1+\frac{\theta_pT_{v_2}}{3}\right)\right] - \frac{i_pp_pD_pR}{T_p} + \frac{\delta(p_r-p_p)D_pM}{T_p} \tag{13}$$

**Case IB:**
When $R < M$
Here, the profit-sharing ratio of the processor is the same for the upcoming case II C.

The Profit shared is $PF_{p_{12}} = PF_{p_{23}} = \delta(p_r - p_p)D_rR$ (14)

$$TP_{p_{IB}}(T,p_p,R)=(p_r-p_p)D_p-\frac{K_{pr}}{T_p}-\frac{p_g\alpha_1gw_1}{T_p}-e_pP_rT_{v_1}-\frac{c_d+c_d^{`}}{T_p}\left(P_rT_{v_1}-D_pT_p\right)$$
$$+\frac{\delta(p_r-p_p)D_pR}{T_p}-\frac{h_p+h_p^{`}}{T_p}\left[\frac{(P_r-D_p)T_{v_1}^2}{2}\left(1-\frac{\theta_pT_{v_1}}{3}\right)+\frac{D_pT_{v_2}^2}{2}\left(1+\frac{\theta_rT_{v_2}}{3}\right)\right]-\frac{i_pp_pD_pR}{T_p}+GC \quad (15)$$

**Case IC:**
When $M < T$

$$TP_{p_{IC}}(T_p,p_p,R)=(p_r-p_p)D_p-\frac{K_{pr}}{T_p}-\frac{p_g\alpha_1gw_1}{T_p}-e_pP_rT_{v_1}-\frac{c_d+c_d'}{T_p}\left(P_rT_{v_1}-T_p\right)+i_m(R-M)p_pD_p+GC$$
$$-\frac{h_p+h_p'}{T_p}\left[\frac{(P_r-D_p)T_{v_1}^2}{2}\left(1-\frac{\theta_pT_{v_1}}{3}\right)+\frac{D_pT_{v_2}^2}{2}\left(1+\frac{\theta_pT_{v_2}}{3}\right)\right]-\frac{i_pp_pD_pR}{T_p}+\frac{\delta(p_r-p_p)D_pM}{T_p} \quad (16)$$

**Case II A:**
When $R > T$ Here, the profit sharing ratio of the processor is the same for the upcoming case II B.

The Profit shared is $PF_{p_{21}} = PF_{p_{22}} = \delta(p_r-p_p)D_rT_r$ (17)

$$TP_{p_{IIA}}(T_p,p_p,R)=(p_r-p_p)D_p-\frac{K_{pr}}{T_p}-\frac{p_g\alpha_1gw_1}{T_p}-e_pP_rT_{v_1}-\frac{c_d+c_d'}{T_p}+\delta(p_r-p_p)D_p+GC-$$
$$\frac{h_p+h_p'}{T_p}\left[\frac{(P_r-D_p)T_{v_1}^2}{2}\left(1-\frac{\theta_pT_{v_1}}{3}\right)+\frac{D_pT_{v_2}^2}{2}\left(1+\frac{\theta_pT_{v_2}}{3}\right)\right]-i_pp_pD_pR \quad (18)$$

**Case II B:**
When $M > T$

$$TP_{p_{IIB}}(T_p,p_p,R)=(p_r-p_p)D_p-\frac{K_{pr}}{T_p}-\frac{p_g\alpha_1gw_1}{T_p}-e_pP_rT_{v_1}-\frac{c_d+c_d'}{T_p}\left(P_rT_{v_1}-T_p\right)+\delta(p_r-p_p)D_p+GC-$$
$$\frac{h_p+h_p'}{T_p}\left[\frac{(P_r-D_p)T_{v_1}^2}{2}\left(1-\frac{\theta_pT_{v_1}}{3}\right)+\frac{D_pT_{v_2}^2}{2}\left(1+\frac{\theta_pT_{v_2}}{3}\right)\right]-i_pp_pD_pR+i_m(R-M)p_pD_p \quad (19)$$

**Case II C:**
When $R < T_p$

$$TP_{p_{IIC}}(T,p_p,R)=(p_r-p_p)D_p-\frac{K_{pr}}{T_p}-\frac{p_g\alpha_1gw_1}{T_p}-e_pP_rT_{v_1}-\frac{c_d+c_d'}{T_p}\left(P_rT_{v_1}-DT_p\right)+\frac{\delta(p_r-p_p)D_pR}{T_p}+GC-$$
$$\frac{h_p+h_p'}{T_p}\left[\frac{(P_r-D_p)T_{v_1}^2}{2}\left(1-\frac{\theta_pT_{v_1}}{3}\right)+\frac{D_pT_{v_2}^2}{2}\left(1+\frac{\theta_pT_{v_2}}{3}\right)\right]-\frac{i_pp_pD_pR}{T_p} \quad (20)$$

### 5.3 Retailer's Echelon

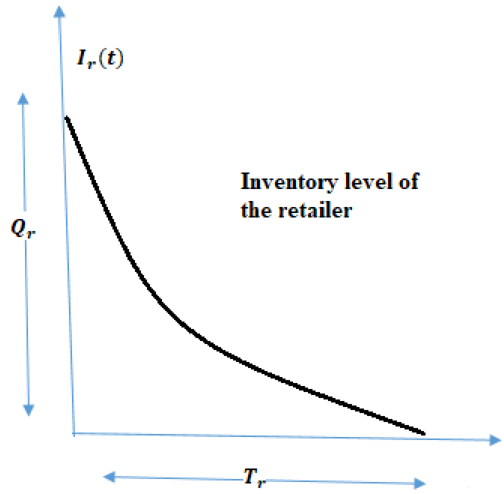

Figure 3: Retailer's Inventory Cycle

In the retailer's cycle (Figure 3), $L_r$ weight units of treated stock time $T_r$ units are delivered to the store. The treated inventory deteriorates at $\theta_r(t)$, which is time-dependent. All the livestock the retailer has is going to expire (i.e.) not worth human consumption. Therefore, the expiration rate of the product is $u$, where $0 \le \theta_r \le 1$, as inHidayat*et al.* (2020).

$$\theta_r(t) = \frac{1}{1+u-t}, 0 \le u \le T_r \le u \tag{21}$$

As a result, the processed inventory is decreased during the replenishment cycle due to both customer needs and deterioration. Then, the processed inventory throughout the time interval $[0, T_r]$ becomes, (where $D_r = (a - bp_r)$)

$$\frac{dI_r(t)}{dt} = -D_r - \theta_r(t) I_r(t) 0 \le t \le T_r \text{ where } \theta_r(t) = \frac{1}{1+u-t}, 0 \le t \le u \tag{22}$$

The retailer's demand (inventory) at any time $t$, can be solved using the given boundary condition, $I_r(T_r) = 0$, and the inventory level of the retailer is given as,

$$I_r(t) = D_r(1 + u - t)\, ln\left(\frac{1+u-t}{1+u-T}\right) 0 \le t \le T_r \tag{23}$$

The prior order size received by the store at the opening of each loop is,

$$L_r = I_r(0) = D_r(1 + u)\, ln\left(\frac{1+u}{1+u-T}\right) \tag{24}$$

1. During the retailer's cycle of duration $T_r$, the quantity of deteriorating inventory is described as the order quantity $(L_r)$, less than the demand during $T_r$. Taking the firm's degradation cost of $c_d$ per weight unit into account, the firm's deterioration cost per unit time is,

$$DC_r = \frac{c_d(1+u)\, ln\left(\frac{1+u}{1+u-T}\right)}{T}\left[D_r(1 + u)\, ln\left(\frac{1+u}{1+u-T}\right) - D_r T_r\right] \tag{25}$$

2. The holding cost of the retailer's inventory after received from the processor is given as,

$$HC_r = \frac{h_r}{T_r}\int_0^{T_r} I_r(t)dt = \frac{h_r D_r}{T_r}\left(\frac{(1+u)^2}{2} ln\left(\frac{1+u}{1+u-T}\right) - \frac{(1+u)}{2} T_r + \frac{1}{4} T_r^2\right) \tag{26}$$

### 5.4 Profit sharing and Interest earned and paid by the Retailer

In this section, there are various possibilities for the retailer to share their profit with the packer, and the optimal period changes regarding the credit period offered and revenue sharing period are discussed in various cases in this section are given below:

**Case I:** $M \le T_r$

**Case IA:** When $M \le R \le T_r$
The sales revenue of the retailer in this case is given as $SR_1 = p_r D_r M$.
According to the agreement, the retailer gives the ratio of profit to the processor,

$$PF_r = \delta(p_r - p_p) D_r M \tag{27}$$

to the manufacturer from his sales revenue.
In case $R > M$ , then the retailer pays the interest amount on purchase cost at the rate $i_p$ , then. The total interest earned by the retailer $IE_{R1} = i_e p_r \int_0^R D_r t dt = \frac{i_e p_r D_r R^2}{2}$

According to this, in time $R$, the retailer has to deposit the purchase price to the processor along with his share in profit and interest. It may lead to two other situations in the inventory cycle, they are,

**Case I.A1**
If the total paid profit share is more than the revenue of the retailer at that time since the amount has been adjusted from the source with an interest rate $i_c$. In the end, the interest paid by the retailer in the cycle is,

$$IP_{R_{I.1}} = i_c(T_r - R)\left[\left(p_pD_rT_r - p_rD_rR\right) + \delta\left(p_r - p_p\right)D_rM + i_m(R - M)p_pD_rT_r - \frac{i_ep_rD_rR^2}{2}\right] \tag{28}$$

The retailer's entire profit in this case is,

$$TP_{R_{1.1}} = \begin{pmatrix} p_rD_rM - \frac{K_r}{T_r} - \frac{p_p\alpha_1gw_1}{T_r} - \frac{c_d(1+u)\,ln\left(\frac{1+u}{1+u-T}\right)}{T}\left[D_r(1+u)\,ln\left(\frac{1+u}{1+u-T}\right) - D_rT_r\right] \\ -\frac{\delta(p_r-p_p)D_rM}{T_r} - i_m(R-M)p_pD + \frac{i_ep_rD}{2T_r}(T_r-R)^2 - i_c(T_r-R) \\ \left[(p_pD_rT_r - p_rD_rR) + \delta(p_r-p_p)D_rM + i_m(R-M)p_pD_rT_r - \frac{i_ep_rD_rR^2}{2}\right] \\ +\frac{h_rD_r}{T_r}\left(\frac{(1+u)^2}{2}ln\left(\frac{1+u}{1+u-T}\right) - \frac{(1+u)}{2}T_r + \frac{1}{4}T_r^2\right) \end{pmatrix} \tag{29}$$

**Case I.A2**
What if the retailer earns more profit than to be paid(shared)
In this case, the retailer keeps the additional amount, and at the end of the cycle, the interest earned by the retailer is,

$$IE_{R_{1.2.1}} = i_e(T_r - R)\left[\left(p_rD_rR - p_pD_rT_r\right) - \delta\left(p_r - p_p\right)D_rM - i_m(R - M)p_pD_rT_r + \frac{i_ep_rD_rR^2}{2}\right] \tag{30}$$

In the time $(T_r - R)$, the retailer has his revenue to the same source, and at the end of the cycle, the interest earned is,

$$IE_{R_{1.2.2}} = i_ep_r\int_0^{T_r-R} D_rtdt = \frac{i_ep_rD_r(T_r-R)^2}{2} \tag{31}$$

**Case IB:** When $R \leq M \leq T_r$
The fraction of the profit the processor gets when this chance flows, $PF_{r2} = \delta(p_r - p_r)D_rR$.
Here, as the payment is made in advance, there is no need to pay any interest.

$$\text{Interest earned up to the payment time } IE_{pt} = \frac{i_ep_rD_rR^2}{2} \tag{32}$$

$$\text{I. e.}, \text{after the payment and until the end } IE_{pe} = \frac{i_ep_rD_r(T_r-R)^2}{2} \tag{33}$$

$\text{The I. C by the processor is}$

$$IC_{p1} = i_c\left[(p_pT_r - p_rR)D + \delta(p_r - p_p)D_rR - \frac{i_ep_rD_rR^2}{2}\right] \tag{34}$$

Similarly, the interest earned in this case is,

$$IE_1 = i_e\left[(p_rR - p_pT_r)D_r - \delta(p_r - p_p)D_rR + \frac{i_ep_rD_rR^2}{2}\right](T_r - R) \tag{35}$$

**Case IC**: When $M \leq T_r \leq R$
The retailer pays at the end of the cycle, the sales revenue of the retailer is, $p_rD_rT_r$
∴ The profit sharing in this case for the retailer is $\delta(p_r - p_p)D_rM$.

$$IP_r = i_m(R - M)p_pD_rT_r \tag{36}$$

Here, there is no discussion of taking any bank loan or from any other source. Then, the interest earned by the retailer in this case is, $i_ep_r\int_0^T D_rtdt = \frac{i_ep_rD_rT_r^2}{2}$ .Interest earned by the retailer is,

$$IE_{R3} = \frac{i_ep_rD_rT_r^2}{2} + \left(p_rD_rT_r + \frac{i_ep_rD_rT_r^2}{2}\right)(R - T_r)i_e \tag{37}$$

Hence, the total profit for the retailer at the end of the cycle is,

**Case II**: When $T_r \leq M$
Similar to the above case I, we consider the other three various options for the total profit of the retailer when $T_r \leq M$.

**Case II A**
The profit share for the processor in this case is,

$$PF_{IIA} = \delta(p_r - p_p)D_rT_r \tag{38}$$

In this case, there is no interest to be paid because the retailer pays before the trade credit period.

Interest earned by the retailer up to time $T = \frac{i_e p_r D_r T_r^2}{2}$ (39)

The interest is the same as in the previous case $i_e(R - T_r)\left(p_rD_rT_r + \frac{i_e p_r D_r T_r^2}{2}\right)$ (40)

The retailer deposits his remaining amount in the bank for $(M - R)$ period, and the interest earned with this is,

$$i_e \int_0^{M-R} \left[(p_r - p_p)D_rT_r - \delta(p_r - p_p)D_rT_r + \frac{i_e d_r D_r T_r^2}{2} + \left(p_rD_rT_r + \frac{i_e p_r D_r T_r^2}{2}\right)\right] i_e(R - T_r)dt - i_e(M - R)\left(\left[(p_r - p_p)D_rT_r - \delta(p_r - p_p)D_rT_r + \frac{i_e d_r D_r T_r^2}{2} + \left(p_rD_rT_r + \frac{i_e p_r D_r T_r^2}{2}\right)\right] i_e(R - T_r)\right) \tag{41}$$

**Case IIB**: When $T_r \leq M \leq R$
Here, the profit sharing to the processor is the same as the sub-case.
Then interest earned up to the time$(R - T_r)$ *is*

$$i_e\left[\frac{p_rD_rT_r(R - T_r)}{2} + \frac{i_e p_r D_r T_r^2}{2}(R - T_r)\right] \tag{42}$$

The interest earned by the processor after the credit period is $i_m(R - M)p_pD_rT_r$
The interest paid in this period is null.

**Case II C:** When $R \leq T_r \leq M$
The profit share by the retailer is

$$\delta(p_r - p_p)D_rR \tag{43}$$

The interest paid to the source(bank) is

$$i_c(T_r - R)\left[(p_pT_r - p_rR)D_r + \delta(p_r - p_p)D_rR - \frac{i_e p_r D_r R}{2}\right] \tag{44}$$

The interest earned by the retailer in time $(T_r - R)$ is

$$i_e(T_r - R)\left[(p_rR - p_pT_r)D_r - \delta(p_r - p_p)D_rR + \frac{i_e p_r D_r R^2}{2}\right] \tag{45}$$

The interest earned up to the trade credit period is

$$i_e(M - T_r)\left[(p_r - p_p)D_rT_r - \delta(p_r - p_p)D_rR - i_c(T_r - R)\left((p_pT_r - p_rR) + \delta(p_r - p_p)D_rR - p_rD_rR - \frac{i_e p_r D_r R^2}{2}\right) + \frac{i_e p_r D_r}{2}(T_r - R)^2\right] \tag{46}$$

The profit function of the retailer can be stated as similar to the profit function of the retailer represented in equation (16).

### 5.5 Integrated Supply Chain

Considering all of the instances covered in the preceding subcategories, the efficient supply chain's profit margin per unit of time is provided by,

$$TPG(T, p, R) = TP_{pij}(T_p, p_p, R) + TP_{rij}(T_r, p_r, R) \tag{47}$$

$where\ i = I, II, j = A, B, C$

$$= \begin{cases} TPG_1(T,p,R) & if, M \le R \le T \\ TPG_2(T,p,R) & if, M \le R \le T \\ TPG_3(T,p,R) & if, R \le M \le T \\ TPG_4(T,p,R) & if, R \le M \le T \\ TPG_5(T,p,R) & if, M \le T \le R \\ TPG_6(T,p,R) & if, T \le R \le M \\ TPG_7(T,p,R) & if, T \le M \le R \\ TPG_8(T,p,R) & if, R \le T \le M \\ TPG_9(T,p,R) & if, R \le T \le M \end{cases} \tag{48}$$

In the total profit function, the decision variables $p, T$, represent the corresponding price of the retailer and processor, $p_r, p_p$, and the period $T$ as $T_r, T_p$.

## 6. SOLUTION METHODOLOGY

By applying the same solution procedure followed by Gharaei and Almehdawe (2021), we have found the optimal values of the profit function and the concavity of the profit function also satisfied. Due to the complexity of finding the derivatives concerning all the decision variables, and it seems hard to derive them analytically, we have adopted the solution methodology followed by Gharaei and Almehdawe (2021), which maximizes the profit values of the Integrated Supply Chain. We implemented the algorithm developed by Nigwal *et al.* (2022) to find the optimal values in all the profit functions mentioned above.

### 6.1 Example. (Basic Numerical Data)

$P_r = 3500$ units ; $K_r = \$.2500; K_p = \$.2500; h_r = \$.1; p_r = \$.50; h_p = 0.5; p_p = \$.30; \alpha_1 = 0.9; g =$ 179 units ; $C_d = \$.2; c_g = \$.1; p_v = \$.10; \alpha = 51;\ \beta = 5; M = 0.1$ yr ; $i_v = \$.0.1; i_c\$. = 0.18; \gamma = 0.12; \delta = 0.4$ ; $\beta_1 = 25$ units ; $i_e = \$.0.14; i_m = \$.0.15; w_0 = 8.5$ kg; $w_1 = 30$ grams ; $T_g = 0.36$ years ; $\theta_r = (1 - \alpha_1); T_{v_2} = Tv1 - Tp; R = 0.45$ years; $a = 200$ units ; $b = 0.15; T_r = 0.39$ year ; $h'_p = \$.0.1;\ c'_d = \$.0.6; e_p = \$.1$;. These are the parameters used in the numerical example to verify the developed model. The optimal values are obtained, and the results are verified.

## 7. NUMERICAL ANALYSIS

Optimal results on Profit for different subcases

| Cases | R | T | Processor | Retailer | Total Profit in Rs. |
|---|---|---|---|---|---|
| IA1 | 0.3209 | 0.3925 | 83,481 | 65,569 | **1,49,050** |
| IA2 | 0.1045 | 0.3467 | 82,985 | 65,511 | **1,48,496** |
| IB1 | 0.1016 | 0.4014 | 81,894 | 66,438 | **1,48,832** |
| IB2 | 0.2102 | 0.3997 | 82,165 | 66,153 | **1,48,318** |
| IC | 0.4204 | 0.4015 | 80,987 | 39,545 | **1,20,532** |
| IIA | 0.4213 | 0.3990 | 79,721 | 65,306 | **1,45,027** |
| IIB | 0.4509 | 0.39896 | 78,789 | 65,137 | **1,43,926** |
| IIC1 | 0.3017 | 0.3578 | 82,956 | 64,653 | **1,47,609** |
| IIC2 | 0.2897 | 0.3486 | 80,879 | 63,546 | **1,44,425** |

### 7.1 Discussion

Here, we compared some of the results obtained by changing the parameters of demand and credit period. By changing the constant demand a, value the lot size of the system varies, and the profit oscillates accordingly.The constant demand pattern may change due to the seasonal demand and change in the price of the product, and the price change may cause a delay in the depletion of the demand, and it can increase the time length of the inventory system. If we change the rate of b, it directly affects the lot size and profit, for a higher $b$ value, the demand is low compared to the other values of $b$ , which may decrease the net profit of the system. Moreover, if the credit period increases, the payment time and the cycle length will increase, and it decreases the value of the profit of the entire system.

From the obtained optimal values, it is stated clearly that the case IB1 has maximum profit, and also, the credit period offered is less than the period of the entire system, but the processor gains less profit than the retailer. In general, the higher the selling price gives more profit to the retailer than the processor, and the entire profit value may hike; it seems a bit less to the processor.

In case A1, the retailer earns more profit than in case A2,but the period seems a bit higher than A1, and also the whole system's profit hikes more in A2.

In case B1, the credit period offered is lower than the period of the cycle, and the profit of the processor and retailer is comparatively a bit low compared to case B2, but the period offered is low, and in case B2, the the period T is bit lower than in B1, the profit of the retailer and processor seems a bit improved compared to case B1.

In case C1, the credit period offered is higher than the period, which improves the total profit of the processor and retailer andaffectsthe whole system's profit.

In case IIA, the retailer earns a profit comparably lower than the other cases obtained above, but the processor earns a profit more than the other cases; undoubtedly, it is one of the highest profits of the system compared to other cases obtained.

In case IIB, the credit period offered is a bit higher than it has been offered in other cases; it also lowered the profit values of the processor and retailer, and it is the second lowest profit value of the system.

Similar to case IIB, the period of the system is lower compared to all other values of the cases given, and the R-value is a bit low; because of the lower period, the profit values of the retailer and processor oscillate and reduce the system's profit.

The credit period offered here is less than the period of the system, but the period is the last one, and the profit of the system also oscillates but does not affect much care.

## 8. MANAGERIAL INSIGHTS

Through this developed model, it is suggested that the processor should focus on the processing period reduction by availing the minimum time-consumingequipment. Then it is the livestock product so the customers can be attracted by following the green manufacturing procedures and promoting the green product itself the promotion of the product so it may reduce the unnecessary promotional efforts expense overall.

## 9.CONCLUSION

A multi-echelon supply chain for growing items with product expiry is developed in this study. The grower begins his cycle by maturing immature items to a specific stage and selling them to the processor after they reach the maximum growing stage; the processor then slaughters the matured livestock, preserves it, and packs it into a specific form. During the packing process, it emits carbon, which costs the processor in inventory costs, and he has to pay the tax for carbon emissions. After the packing process, he sends the product to the retailer and offers it. There are six possibilities for these assumptions, which are discussed in the developed model. A solution procedure is obtained, and numerical analysis and sensitivity analysis are given to verify the sustainability of the model. Further, this model can be developed by considering stochastic demand patterns, advertisement patterns, a carbon cap, and trade regulations.

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