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A Procedure for Full Ranking of DEA Units with Secondary Frontier

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Abstract**—**There is increasing interest in fully ranking the performance of organizational units with multiple inputs and outputs. We smooth the congenital differences of efficient and inefficient units. The concept of the secondary frontier is to eliminate the effects of super-efficiency and inefficiency. The pseudo secondary frontier is constructed to lie between the efficient and inefficient units. Location of the projection points of each unit on the secondary frontier is determined and each has the same efficiency score. A common set of weights of those projection points is determined. Units then are ranked by the measured distances to the secondary frontier. The procedure is illustrated by the example of the ranking of profitability performances of the 29 companies in Taiwan's semiconductor industry.

Keywords**—**Data envelopment analysis, Performance analysis, Ranking, Secondary frontier, Common weights analysis

1. INTRODUCTION

There is increasing interest in fully ranking the performance of organizational decision-making units (DMUs) with multiple inputs and outputs. This study proposes a procedure for full ranking of DMUs by using the concept of the secondary frontier and common weights analysis. Charnes, Cooper and Rhodes (CCR) (1978) introduced Data Envelopment Analysis (DEA) that assesses the *comparative* or *relative* efficiency of homogeneous operating DMUs such as schools, hospitals, or sales outlets. The DMU assessment uses a set of resources referred to as input indices which are transformed into a set of outcomes, referred to as output indices. Usually the weighted sum of outputs divided by the weighted sum of inputs is defined as the efficiency of the transformation process.

DEA separates DMUs into two categories: efficient DMUs and inefficient ones. The relative efficiency measurement of an inefficient unit is reference to some set of efficient DMUs that are compared with each other. We cannot in general derive by means of DEA some absolute measure of efficiency unless we make additional assumptions that the comparisons include a 'sufficient' number of DMUs which are efficient in some absolute sense. Each DMU in the efficient category is assigned a set of weights or indices so that its relative efficiency score is equal to one. DEA may not provide enough information to rank the efficient DMUs on the frontier. If one further wants to understand which is best, we need another indicator to discriminate among the efficient DMUs.

Banker and Gifford (1988), Banker et al. (1989), Andersen and Petersen (1993) and Cook (1992) et al. developed procedures for ranking the efficient DMUs in the DEA. A super-efficiency DEA model, in which a DMU under evaluation is excluded from the reference set, was developed. Banker and Chang (2000) have demonstrated that the use of the super-efficiency model for ranking efficient DMUs is inappropriate.

However, as argued in Cooper and Tone (1997), the original efficiency value will generally be determined from different facets. This means that these values are derived from comparisons involving performances of different sets of DMUs.

Doyle and Green (1994) developed a scale utilizing the cross-efficiency matrix by ranking the average efficiency ratios of each DMU in the runs of all the other runs. Ganley and Cubbin (1992) considered the common weights for the DMUs by maximizing the sum of efficiency ratios of all of them and ranking each one. Cooper and Tone (1997) ranked the DMUs according to scalar measures of inefficiency in DEA, based on the slack variables. Sinuany–Stern et al. (1994) introduced several approaches for ranking DMUs within the DEA context, including a two-stage linear discriminating analysis.

Research about the idea of common weights and rankings has developed gradually in recent years. Cook et al. (1990) first proposed the idea of common weights in DEA and Roll et al. (1991) were first to use the context of applying DEA to evaluate highway maintenance DMUs. Cook and Kress (1990, 1991) gave a subjective ordinal preference ranking by developing common weights through a series of bounded DEA runs by closing the gap between the upper and lower limits of the weights.

Liu and Peng (2004) proposed Common Weights Analysis (CWA) to determine a set of indices for common weights to rank efficient DMUs of DEA. Employing the set of common weights, the *absolute* efficiency score of each DMU in the *efficient* category is recomputed. In other words, they set up an *implicit absolute* efficient frontier, also called a 'benchmark'. The efficiency score of the benchmark equals 1. The sum of the absolute efficiency

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scores of the DMUs in the efficient category is maximal with the set of common weights of indices. In other words, the sum of efficiency gaps of the efficient DMUs to the *benchmark* is minimal. They assume all the DMUs in the efficient category are weighted equally weighted in determining the set of common weights of indices.

Because the scores of efficiency and inefficiency (even super-efficiency) cannot be ranked, the concept of a secondary frontier eliminates the effect of super-efficiency and inefficiency. We smooth the congenital differences of efficiency and inefficiency, and use the same measure (minimize the gaps of the benchmark) of performance for the DMUs on the secondary frontier. To restore the original congenital efficiency of DMUs, we multiply the calculated score of DMUs on the secondary frontier by the original efficiency score. Finally, we rank all of the DMUs.

The rest of the paper is organized as follows. Section 2 briefly describes the concept of our five-phase procedure. In Section 3, the mathematical models of each phase are illustrated by reference to the ranked profitability performances of Taiwan 29's semiconductor companies. The paper ends with a summary and conclusions in Section 4.

2. PROCEDURE FRAMEWORK

In this paper we present a new procedure for full ranking of DMUs. The procedure includes five phases. The concept of the secondary frontier is to eliminate the effects of super-efficiency and inefficiency. The psuedo secondary frontier constructed lies between the efficient and inefficient DMUs. Location of the projection points of each DMU on the secondary frontier is determined and each has the same efficiency score. A common set of weights of those projection points is determined. DMUs then are ranked by the measured distances to the secondary frontier.

In Phase I, a DEA model is employed to distinguish the efficient DMUs (*eDMUs*) and inefficient DMUs (*iDMUs*). In this research, we employ Slack Based Measurement (SBM) (Tone, 2001) for the involved DEA models. In contrast to the radial models, CCR and BCC (Banker et al., 1984) which are based on the proportional reduction (enlargement) of input (output) vectors and which do not take account of slacks, the SBM deals directly with input excess and output shortfall. SBM is non-radial and deals with input/output slacks directly. The SBM returns an efficiency measure between 0 and 1 and gives unity if and only if the DMU concerned is on the frontiers of the production possibility set with no input/output slacks. In that respect, SBM differs from traditional radial measures of efficiency that do not take account of the existence of slacks (Tone, 2002).

The data set appearing in Tone (2002), Table 1, is used to illustrate our procedure. In Figure 1, the seven DMUs A, B, …, and G are plotted on the graph with axes, inputs X_1 and X_2. DMUs are separated into *eDMU*s C, D, and E, and *iDMU*s A, B, F, and G. The primary efficient frontier is constructed by *eDMU*s C, D, and E.

Table 1. Data from Tone (2002)

	X_1	X_2	Y
A	4	3	1
B	7	3	1
C	8	1	1
D	4	2	1
E	2	4	1
F	10	1	1
G	12	1	1

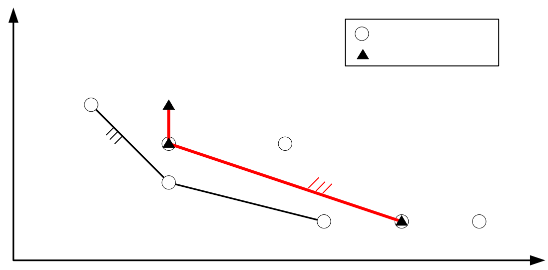

Figure 1. The primary and the secondary frontiers.

In Phase II, the super-SBM model is employed to locate the projection point of each *eDMU* on the foremost (primary) frontier. The super-SBM model (Tone, 2002) is to minimize a sort of weighted l_1 distance from an efficient DMU to the production possibility set excluding the DMU. The measure is thus in sharp contrast to other methods proposed so far. The model differs from contemporary ones based on radial measure, e.g. the super-efficiency model (Andersen & Petersen, 1993), in that the former deals directly with slacks in inputs/outputs while the latter does not take account of the existence of slacks. Their projection points are treated as pseudo DMUs called *peDMU*s. Each *eDMU* reduces its super-efficiency, $\rho_o^{II*}>1$, to 1 by expanding its inputs and contracting its outputs to reach the secondary frontier. This procedure smoothed the data of each *eDMU* to *peDMU* so that it nearly has no super-efficiency more than 1. We hoped the secondary frontier would provide smoother faces to the *iDMU*s inside for efficiency assessment. *iDMU*s may project different faces of the secondary frontier that are 'smoothed'. In our example, expanding the inputs of *eDMU*s C, D, and E are projected on the secondary frontier at C', D', and E', respectively, which are the *peDMU*s. C' and D' coinciding with points F and A. In this case, the secondary frontier is the piece-wised heavy line constructed by C', D', and E'.

In Phase Ⅲ, the SBM model can be employed to measure the efficiency score of every *iDMU*. According to the efficiency score, ρ_o^{III*}, the *iDMU* can be classified into *piDMU*s and *i2DMU*s. Some *iDMU*s may possess ρ_o^{III*} equal to 1, their projection on themselves named as *piDMU*s. For those *iDMU*s possess ρ_o^{III*} less than 1, their projection points are named as *i2DMU*s. All the DMUs that construct the secondary frontier, *peDMU*s and *piDMU*s are called *e2DMU*s. We illustrate the process via the example. Figure 1 depicts *iDMU*s A, B, F, and G projected on the secondary frontier at A', B', F' and G' respectively. *piDMU*s A' and F' are located on the same position of A and F, respectively. *i2DMU*s B' and G' are located on the position of A and F, respectively. *e2DMU*s A', C', D', E', and F' are located on the secondary frontier. After Phase III, the projection points of all the original DMUs on the secondary frontier A', B', C', D', E', F', and G' have an efficiency score equal to 1. The purposes of Phase II and Phase III are to 'smooth' the data of original DMUs which have nearly same efficiency.

A linear programming model is used in Phase IV to determine a common set of weights of the indices for all the projection points, *e2DMU*s, on the secondary efficient frontier. Using the obtained common set of weights for performance indices, the efficiency score of each projection point is recomputed. The sum of recomputed scores should be the maximum. The recomputed efficiency scores of A', B', C', D', F' and G' are equal to 1 while E' is less than 1.

In Phase V, the efficiency score of each DMU equals to the efficiency score of its projection point multiplies its measured distance to the secondary frontier. DMUs are then ranked according to their obtained efficiency scores.

3. RANKING THE PROFITABILITY PERFORMANCES OF SEMICONDUCTOR COMPANIES IN TAIWAN

In recent years, the financial performance of a company is important to investors. From an investor's point of view, how to choose an investment target (company) by its technical efficiency performance is a very important issue. In Taiwan, a large proportion of the industrial output value is in the semiconductor industry. The World Semiconductor Trade Statistics (WSTS) estimated that sales of the global semiconducting industry were US$ 166.4 billion in 2003. The Taiwan Semiconductor Industry Association (TSIA) put the total value of the local industry at NT$ 818.8 billion (about US$ 24.6 billion) in the same year. We use the procedure developed to rank the profitability performances of top 29 public semiconductor companies in Taiwan.

Traditionally one used financial ratios, such as return on investment (ROI), return on sales (ROS) and earnings per share (EPS), to characterize a company's performance. Although accounting and financial ratios provide important and useful information for benchmarking performance, there are, in fact, many other factors to consider, e.g., assets, revenue, market value, investments, number of employees, and market share, etc. (Zhu, 1999) Today it is recognized that a business is a complex phenomenon and its performance is a multi-dimensional construct characterized by more than a single criterion.

We use the following five indices (Zhu, 1999) to assess the profitability performances of the 29 Taiwanese semiconductor companies.

X_1: The actual number of employees at the end of the year.

X_2: Total assets, including buildings, equipment, inventory, capital and accounts receivable ($100 million New Taiwan (NT) dollars).

X_3: Equity is the sum of all capital stock, paid-in capital, and retained earnings at the company's year-end ($100 million NT dollars).

Y_1: Revenues, including net operating income for products and services for the whole year. It excludes non-operating income, such as interest and grants ($100 million NT dollars).

Y_2: Profits after the cumulative effects of accounting charges ($100 million NT dollars).

(One hundred NT dollars approximately may exchange to three US dollars at 2004.)

Note that performance is increasing in Y_1 and Y_2 indices; that is, a higher value of the measure indicates superior performance along the dimension being considered. These are named as to-be-maximized indices, as the so called output indices in DEA literature. In contrast to the to-be-maximized indices, a lower value of X_1, X_2, and X_3 indices translates into superior performance along the dimension considered. They are named as to-be-minimized indices, as the so called input indices. We use the term DMUs to denote the companies. X_i denotes ith input ($i = 1, 2, \ldots, m$) and Y_r denotes rth output ($r = 1, 2, \ldots, s$). x_{ij} and y_{rj} denotes the values of DMU_j on indices X_i and Y_r, respectively. Let R represent the set formed by the 29 DMUs, $R = \{1, 2, 3, \ldots, 29\}$. The data of the 29 companies listed in Table 2 appeared in the CommonWealth 1000 (*CommonWealth*, May 1st, 2004). The names of the companies are attached in the appendix.

Table 2. Data of the 29 companies

DMU	X_1	X_2	X_3	Y_1	Y_2
1	16000	3964.17	3292.14	2019.04	472.59
2	9000	3201.14	2322.42	848.62	140.2
3	640	419.87	360.35	380.64	165.22
4	10138	765.84	451.23	314.88	27.43
5	3659	707.97	593.41	259.49	-11.13
6	3239	847.64	491.76	284.29	-14.48
7	9150	540.59	296.98	273.83	28.39
8	2000	296.41	216.65	203.86	-16.51
9	3428	576.05	292.47	173.95	-81.98
10	801	269.01	194.63	167.25	2.04
11	739	116.23	140.3	110.98	20.07
12	276	88.93	62.47	109.05	21.26
13	687	204.11	178.07	92.78	27.92
14	2858	199.58	131.95	67.87	7.73
15	522	56.06	28.65	64.71	0.05
16	123	63.41	46.86	53.33	10.2
17	1514	70.67	45.49	51.86	9.27
18	411	53.76	38.63	46.17	7.57
19	881	46.54	27.16	39.75	3.74
20	1613	57.14	39.84	39.14	3.69
21	430	45.14	37.54	37.67	10.24
22	288	79.29	52.49	28.57	6.01
23	182	15.13	10.79	20.11	4.21
24	715	29.89	19.68	17.67	4.18
25	172	17.35	13.69	17.33	2.24

26	176	341.24	99.74	15.98	5.09
27	161	32.18	29.92	15.18	4.02
28	623	43.34	27.4	13.48	-2.12
29	106	8.64	6.59	11.97	2.37
Sum	**70532**	**13157.32**	**9549.3**	**5779.45**	**859.51**

3.1 Phase I

The following [SBM] model is employed to assess the relative efficiency of DMU_o, o is an element of the set R. The decision variables s_{io}^- and s_{ro}^+ are the improvement (slack) of DMU_o on the indices X_i and Y_r, respectively. The decision variables' λ_j are weights of the reference DMUs.

[SBM]

$$\min\ \rho_o^{I*} = \frac{1-(1/m)\sum_{i=1}^{m} s_{io}^-/x_{io}}{1+(1/s)\sum_{r=1}^{s} s_{ro}^+/y_{ro}}$$

$$s.t.\ x_{io} = \sum_{j\in R} x_{ij}\lambda_j + s_{io}^-,\ i=1,\ \ldots,\ m,$$

$$y_{ro} = \sum_{j\in R} y_{rj}\lambda_j - s_{ro}^+,\ r=1,\ \ldots, s,$$

$$\lambda_j,\ s_{io}^-,\ s_{ro}^+ \ge 0,\ j\in R,\ i=1,\ \ldots,\ m,\ r=1,\ \ldots, s.$$

[SBM] can be transformed into [SBM_t]. t_o^I is a supplemental decision variable to solve the SBM model. ε is an Archimedean infinitesimal constant.

[SBM_t]

$$\min\ \rho_o^{I*} = t-(1/m)\sum_{i=1}^{m} ts_{io}^-/x_{io}$$

$$s.t.\ 1 = t+(1/s)\sum_{r=1}^{s} ts_{ro}^+/y_{ro}$$

$$x_{io} = \sum_{j\in R} x_{ij}\lambda_j + s_{io}^-,\ i=1,\ \ldots,\ m,$$

$$y_{ro} = \sum_{j\in R} y_{rj}\lambda_j - s_{ro}^+,\ r=1,\ \ldots, s,$$

$$\lambda_j,\ s_{io}^-,\ s_{ro}^+ \ge 0,\ j\in R,\ i=1,\ \ldots,\ m,\ r=1,\ \ldots, s,$$

$$t_o^I \ge \varepsilon > 0.$$

We define $S_{io}^- = ts_{io}^-, S_{ro}^+ = ts_{ro}^+$, and $\Lambda_j = t\lambda_j$. [SBM_t] becomes the following linear program in $t_o^I, S_{io}^-, S_{ro}^+$:

[Computing SBM Model]

$$\rho_o^{I*} = \min\ t_o^I - (1/m)\sum_{i=1}^{m} S_{io}^-/x_{io},$$

$$s.t.\ 1 = t_o^I + (1/s)\sum_{r=1}^{s} S_{ro}^+/y_{ro},$$

$$t_o^I x_{io} = \sum_{j\in R} x_{ij}\Lambda_j + S_{io}^-,\ i=1,\ \ldots,\ m,$$

$$t_o^I y_{ro} = \sum_{j\in R} y_{rj}\Lambda_j - S_{ro}^+,\ r=1,\ \ldots, s,$$

$$\Lambda_j,\ S_{io}^-,\ S_{ro}^+ \ge 0,\ j\in R,\ i=1,\ \ldots,\ m,\ r=1,\ \ldots, s,$$

$$t_o^I \ge \varepsilon > 0.$$

If the optimal solution of this model $\rho_o^{I*}=1$, DMU_o is efficient, otherwise $\rho_o^{I*}<1$, is inefficient. We separate all 29 DMUs into *eDMU*s and *iDMU*s, respectively. In our application, *eDMU*s are **#*3*, #*12*, #*15*, #*23*, and #*29***; and *iDMU*s are the remaining 24 DMUs.

3.2 Phase II

The following [SuperSBM] model is employed to measure the super-efficiency of each *eDMU*, say DMU_o. It measures the possible *contraction* of all output indices, Y_r, and the possible *expansion* of all input indices, X_i for DMU_o without becoming dominated by (a convex combination of) the other DMUs. It is possible to differentiate between *eDMU*s. The decision variables $\bar{x}_{io}$ and $\bar{y}_{ro}$ are the expanded and contracted values on indices X_i and Y_r after adjustment, respectively.

[SuperSBM]

$$\rho_o^{II*} = \min\ \frac{1/m\sum_{i=1}^{m}\bar{x}_{io}/x_{io}}{1/s\sum_{r=1}^{s}\bar{y}_{ro}/y_{ro}}$$

$$s.t.\ \bar{x}_{io} \ge \sum_{j\in R,\ j\ne o} x_{ij}\lambda_j,\ i=1,\ \ldots,\ m,$$

$$\bar{y}_{ro} \le \sum_{j\in R,\ j\ne o} y_{rj}\lambda_j,\ r=1,\ \ldots, s,$$

$$\bar{x}_{io} \ge x_{io},\ i=1,\ \ldots,\ m,$$

$$\bar{y}_{ro} \ge 0,\ r=1,\ \ldots, s,$$

$$\lambda_j \ge 0,\ j\in R,\ j\ne o.$$

The fraction program [SuperSBM] can be transformed into linear programming as

[Computing SuperSBM Model]

$$\rho_o^{II*} = \min\ \frac{1}{m}\sum_{i=1}^{m}\frac{\tilde{x}_{io}}{x_{io}},$$

$$s.t.\ 1 = \frac{1}{s}\sum_{r=1}^{s}\frac{\tilde{y}_{ro}}{y_{ro}}$$

$$\tilde{x}_{io} \ge \sum_{j\in R,\ j\ne o} x_{ij}\Lambda_j,\ i=1,\ \ldots,\ m,$$

$$\tilde{y}_{ro} \le \sum_{j\in R,\ j\ne o} y_{rj}\Lambda_j,\ r=1,\ \ldots, s,$$

$$\tilde{x}_{io} \ge t_o^{II} x_{io},\ i=1,\ \ldots,\ m,$$

$$\tilde{y}_{ro} \le t_o^{II} y_{ro},\ r=1,\ \ldots, s,$$

$$\tilde{y}_{ro} \ge 0,\ r=1,\ \ldots, s,$$

$\Lambda_j \geq 0, \ \ j \in R, \ \ j \neq o,$

$t_o^{II} > 0.$

The location of the projection point of the efficient DMU_o on the secondary frontier, *peDMU*$_o$ ($\bar{x}_{io}^*, \bar{y}_{ro}^*$), is computed by the optimal solutions $\tilde{x}_{io}^*$, $\tilde{y}_{ro}^*$, t_o^{II*}.

$$\begin{cases} \bar{x}_{io}^* = \tilde{x}_{io}^* / t_o^{II*}, i = 1, ..., m, \\ \bar{y}_{ro}^* = \tilde{y}_{ro}^* / t_o^{II*}, r = 1, ..., s. \end{cases} \tag{1}$$

The super-efficiency score of *DMU$_o$*, ρ_o^{II*}, is not less than 1. The efficiency score is also the distance of *eDMU$_o$* to the secondary frontier. Each *eDMU* reduces the super-efficiency by expanding the inputs and contracting the outputs to reach its projection point on the secondary frontier. The amount of input expansion (g_i^+) and the amount of output contraction (g_r^-) are computed by equation (2). So we can eliminate the effect of super-efficiency. The efficiency score of its projection point, *peDMU$_o$*, is exactly equal to 1. All the *peDMU*s construct the secondary frontier. The six *peDMU*s, #***3'***, #***12'***, #***15'***, #***23'***, and #***29'*** are typed in bold-italics in Table 3.

$$\begin{aligned} g_i^+ &= \bar{x}_{io}^* - x_{io}, \quad i = 1, ..., m, \\ g_r^- &= y_{ro} - \bar{y}_{ro}^*, \quad r = 1, ..., s. \end{aligned} \tag{2}$$

Table 3. The results

DMU	Projection point on the secondary frontier					Efficiency scores					Rank
	X_1	X_2	X_3	Y_1	Y_2	ρ_o^{I*}	ρ_o^{II*}	ρ_o^{III*}	ρ_o^{IV*}	ρ_o^{V*}	
1'	12303.06	3964.17	2784.68	3490.74	947.69	0.316		0.467	1	0.467	14
2'	9000	2899.89	2037.07	2553.56	693.26	0.146		0.233	1	0.233	25
3'	***897.57***	***419.87***	***360.35***	***380.64***	***73.13***	***1***	***1.572***		***0.974***	***1.532***	***1***
4'	1993.59	642.35	451.23	565.64	153.56	0.130		0.183	1	0.183	26
5'	2372.63	283.50	168	259.49	-11.13	0.431		0.444	0.840	0.373	20
6'	2655.77	322.43	189.57	284.29	-14.48	0.512		0.529	0.816	0.431	16
7'	4718.67	392.27	296.98	521.39	109.15	0.190		0.260	0.980	0.255	23
8'	2000	277.79	162.75	203.86	-16.51	0.872		0.896	0.713	0.639	9
9'	3428	368.15	230.34	424.95	0.33	0.624		0.663	1	0.663	8
10'	801	258.09	181.30	227.27	61.70	0.034		0.061	1	0.061	29
11'	532.47	116.23	82.70	110.98	28.54	0.417		0.636	0.995	0.633	10
12'	***276***	***88.93***	***62.47***	***78.31***	***21.26***	***1***	***1.164***		***1***	***1.164***	***2***
13'	633.47	204.11	143.38	179.73	48.80	0.333		0.493	1	0.493	13
14'	582.97	187.84	131.95	165.41	44.91	0.125		0.173	1	0.173	27
15'	***522***	***56.06***	***35.07***	***64.71***	***0.05***	***1***	***1.075***		***1***	***1.075***	***3***
16'	***123***	***63.41***	***46.86***	***53.33***	***10.20***	***0.587***		***1***	***0.991***	***0.991***	***6***
17'	722.78	60.09	45.49	79.86	16.72	0.384		0.464	0.980	0.455	15
18'	166.85	53.76	37.76	47.34	12.85	0.433		0.584	1	0.584	11
19'	431.54	35.87	27.16	47.68	9.98	0.326		0.390	0.980	0.382	19
20'	633.01	52.62	39.84	69.94	14.64	0.205		0.268	0.980	0.263	22
21'	140.09	45.14	31.71	39.75	10.79	0.518		0.686	1	0.686	7
22'	231.91	74.72	52.49	65.80	17.86	0.235		0.347	1	0.347	21
23'	***182***	***15.13***	***11.45***	***20.11***	***4.21***	***1***	***1.021***		***0.980***	***1.001***	***5***
24'	312.69	25.99	19.68	34.55	7.23	0.363		0.417	0.980	0.409	18
25'	94.68	17.35	12.44	17.33	4.33	0.407		0.559	0.993	0.555	12
26'	176	90.73	67.05	76.31	14.60	0.100		0.169	0.991	0.168	28
27'	99.87	32.18	22.61	28.34	7.69	0.293		0.419	1	0.419	17
28'	161.27	22.71	12.45	13.48	-2.12	0.402		0.412	0.591	0.244	24
29'	***106***	***9.03***	***6.59***	***11.97***	***2.37***	***1***	***1.015***		***1***	***1.015***	***4***
Weights	$v_1^* = 1$	$v_2^* = 6.211$	$v_3^* = 12.281$	$u_1^* = 20.103$	$u_2^* = 1$						

Note: *peDMU*s and *e2DMU*s are typed in bold-italics. *peDMU*s are ***3'***, ***12'***, ***15'***, ***23'***, and ***29'***; *e2DMU*s are ***3'***, ***12'***, ***15'***, ***16'***, ***23'***, and ***29'***.

3.3 Phase III

We employ [SBM2] model to measure the efficiency score, ρ_o^{III*}, for each *iDMU*, say *DMU$_o$*, respects to set *R2* that is composed by *peDMU*s and *iDMU*s. The projection point of each *iDMU* on the secondary efficient frontier is located as well. ρ_o^{III*} is also the distance of the *iDMU$_o$* inside the secondary frontier.

As indicated in [SBM2], the reference set is *R2* with the data obtained from equation (1). For each DMU_o, the efficiency score, ρ_o^{III*} is less than 1.

[SBM2]

$$\rho_o^{III*} = \min \ t_o^{III} - (1/m)\sum_{i=1}^{m} S_{io}^{-}/x_{io}$$

$$s.t. \ 1 = t_o^{III} + (1/s)\sum_{r=1}^{s} S_{ro}^{+}/y_{ro}$$

$$t_o^{III} x_{io} = \sum_{j\in R2} \bar{x}_{ij}\Lambda_j + S_{io}^{-}, \ i = 1, \ldots, m,$$

$$t_o^{III} y_{ro} = \sum_{j\in R2} \bar{y}_{rj}\Lambda_j - S_{ro}^{+}, \ r = 1, \ldots, s,$$

$$\Lambda_j, \ S_{io}^{-}, \ S_{ro}^{+} \geq 0, \quad j \in R2, \ i = 1, \ldots, m, \ r = 1, \ldots, s,$$

$$t_o^{III} \geq \varepsilon > 0.$$

From the optimal solutions, S_{io}^{-*} and S_{ro}^{+*}, the data of projection point, *piDMU* $(\bar{x}_{io}, \bar{y}_{ro})$ are computed by the following equations:

$$\begin{cases} \bar{x}_{io} = x_{io} - S_{io}^{-*}, & i = 1, \ldots, m, \\ \bar{y}_{ro} = y_{ro} + S_{ro}^{+*}, & r = 1, \ldots, s. \end{cases} \tag{3}$$

[SBM2] measures the possible *expansion* of all output indices, Y_1 and Y_2, and the possible *contraction* of all input indices, X_1, X_2, and X_3 for DMU_o without becoming dominated by (a convex combination of) the other DMUs in the reference set *R2*. Some *iDMU*s may possess ρ_o^{III*} equal 1, their projection points are themselves, named as *piDMU*s. For those *iDMU*s possessing ρ_o^{III*} smaller than 1, their projection points are named as *i2DMU*s. ***#16'*** is an identified *piDMU*s. All the DMUs that locate on the secondary frontier, *peDMU*s and *piDMU*s, are called *e2DMU*s. In Table 3, the six *e2DMU*s are typed in bold-italics and the other 23 *i2DMU*s are also listed.

3.4 Phase IV

In Phase II and Phase III, the projection points of all DMUs, called pseudo DMUs, are identified. All the pseudo DMUs are on secondary frontier and their efficiency scores equal 1. In this circumstance, we can use the concept of CWA to find the fair measure to evaluate the performance of DMUs. The following [CWA-FP] model is used to determine the set of weights of the indices for all the points on the secondary efficient frontier, *e2DMU*s and *i2DMU*s. We denoted set E as the union of *e2DMU*s and *i2DMU*s. Using the obtained common set of weights for indices, the efficiency score of each point is recomputed. The sum of recomputed scores should be the maximum. Note, the data $\bar{x}_{ij}$ and $\bar{y}_{rj}$ are resulted in Phase II and Phase III. Decision variables, u_r and v_i denote the weights of performance indices Y_r and X_i, respectively.

[CWA-FP]

$$\min \quad \sum_{j\in E}\left(\Delta_j^O + \Delta_j^I\right)$$

$$s.t. \quad \frac{\sum_{r=1}^{s} \bar{y}_{rj}u_r + \Delta_j^O}{\sum_{i=1}^{m} \bar{x}_{ij}v_i - \Delta_j^I} = 1, \quad j \in E,$$

$$\Delta_j^O, \Delta_j^I \geq 0, \qquad j \in E,$$

$$u_r \geq \varepsilon > 0, \qquad r = 1, \ldots, s,$$

$$v_i \geq \varepsilon > 0, \qquad i = 1, \ldots, m.$$

The model minimizes the sum of the efficiency gaps of DMUs, $\Delta_j^O + \Delta_j^I$. In order to decrease the complexity, let $\Delta_j = \Delta_j^O + \Delta_j^I$.

[CWA Step-1]

$$\Delta^* = \min \sum_{j\in E}\Delta_j$$

$$s.t. \quad \sum_{r=1}^{s}\bar{y}_{rj}u_r - \sum_{i=1}^{m}\bar{x}_{ij}v_i + \Delta_j = 0, \quad j \in E$$

$$u_r, v_i \geq \varepsilon > 0, \quad r = 1, \ldots, s; \ i = 1, \ldots, m,$$

$$\Delta_j \geq 0, \qquad j \in E.$$

[CWA Step-1] provides a unique optimal value Δ^*. The best alternative combination of the indices weights could be obtained by using the [CWA Step-2] proposed by Obata & Ishii (2003). Regarding output (to-be-maximized) data, it is better to adopt the smaller weights vector to obtain the product while the same product exists. This means that preference of the same product resulted from the data rather than from the weights. Similarly, it is better to use the larger weights vector for input (to-be-minimized) data. We have reduced the probability of alternative optimal weights as much as possible.

[CWA Step-2]

$$\min \quad \sum_{r=1}^{s} u_r - \sum_{i=1}^{m} v_i$$

$$s.t. \quad \sum_{r=1}^{s}\bar{y}_{rj}u_r - \sum_{i=1}^{m}\bar{x}_{ij}v_i + \Delta_j = 0, \quad j \in E,$$

$$\sum_{j\in E}\Delta_j = \Delta^*,$$

$$u_r, v_i \geq \varepsilon > 0, \quad r = 1, \ldots, s; \ i = 1, \ldots, m,$$

$$\Delta_j \geq 0, \qquad j \in E.$$

Using the unique optimal common weights $\left(v_i^*, u_r^*\right)$, the efficiency score of each original DMU_o on the secondary frontier is calculated according to equation (4). The results of Phases II and III are the projection points of all DMUs on the secondary frontier with efficiency scores equal to 1.

$$\rho_o^{IV*} = \sum_{r=1}^{s} u_r^* \bar{y}_{ro} \Big/ \sum_{i=1}^{m} v_i^* \bar{x}_{io}, \tag{4}$$

The commonest of weights of indices $(v_1^*, v_2^*, v_3^*, u_1^*, u_2^*)$ are listed in the last row of Table 3.

3.5 Phase V

In this phase we trace back the efficiency score of each company by multiplying the implicit absolutely efficient score of its projection point on the secondary frontier to the efficiency score obtained from Phases II or III.

$$\rho_o^{V*} = \begin{cases} \rho_o^{IV*} \times \rho_o^{II*}, & \text{if DMU}_o \text{ is an } eDMU; \\ \rho_o^{IV*} \times \rho_o^{III*}, & \text{if DMU}_o \text{ is an } iDMU. \end{cases} \quad (5)$$

The efficiency scores resulted by the five phases of every DMU_o are listed in Table 3. The rankings according to ρ_o^{V*} are also listed in the last column.

4. CONCLUSION

In this paper, we propose a new ranking method for the DMUs with indices for multiple outputs and inputs. We employ the procedure for the performance ranking of the 29 public semiconductor companies in Taiwan. One may further smooth the DMUs data set. Substituting *eDMU*s by their *peDMU*s results in Phase II, then executing Phases I and II again to identify the tertiary frontier. The data of the points on the tertiary frontier are smoothed once more. Their super-efficiencies are closer to 1. Repeating the process, the quaternary frontier is constructed. One would expect the super-efficiencies of the points on those layered frontier may be converged in a certain cycle.

In our framework, we use the concept of constant returns-to-scale (CRS) in SBM model. One may need to rank the DMUs by pure efficiencies. The single constraint $\sum \lambda_j = 1$ is added in Phases I~III.

One also can restrict the range of weights in Phase IV. For instance, the following constraints on the ratio of common weights of inputs are added to [CWA-FP] models.

$$l_{1,i} \le v_i / v_1 \le u_{1,i} \quad (i = 2, \cdots, m) \quad (6)$$

$l_{1,i}$ and $u_{1,j}$ are lower and upper bounds that the ratio v_i/v_1 may assume.

For distinguishing the DMUs, and locating the projections points on the secondary frontier in Phases I~III, we use slack-based measurement models. One may use radial-base models such as CCR and BCC for executing the process.

Using slack- and radial-based models, *peDMU*s may not always keep the status as efficient ones. If the *peDMU*s is inefficient, we should find the projection points to construct the secondary frontier in Phase III. Table 4 depicts the numerical example to illustrate the property. A, C, and B are *eDMU*s and their projection points (*peDMU*s) A', C' are inefficient and B' is efficient. Employing Phase III, the projection points of A', C', and F on the secondary frontier are A" (coincide with point D), C" (coincide with point E) and F' (coincide with point E). D and E are *i2DMU*s. F' is a *piDMU*. A", B', C", D, E, and F' are *e2DMU*s. The secondary frontier appears in Figure 2.

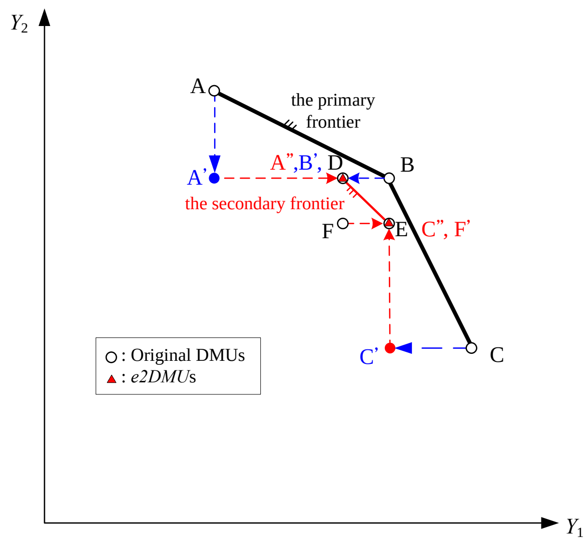

Figure 2. The primary and the secondary frontiers (slack-based).

We also construct the secondary frontier by a radial-based model (CCR-O/SuperCCR-O). A, B, and C are *eDMU*s, and their projection points are A' (coinciding with point B), B', and C' (coinciding with point B), respectively. The secondary frontier is converged on B, as shown in Figure 3. Obviously, using slack- and radial-based models would result in different secondary frontiers.

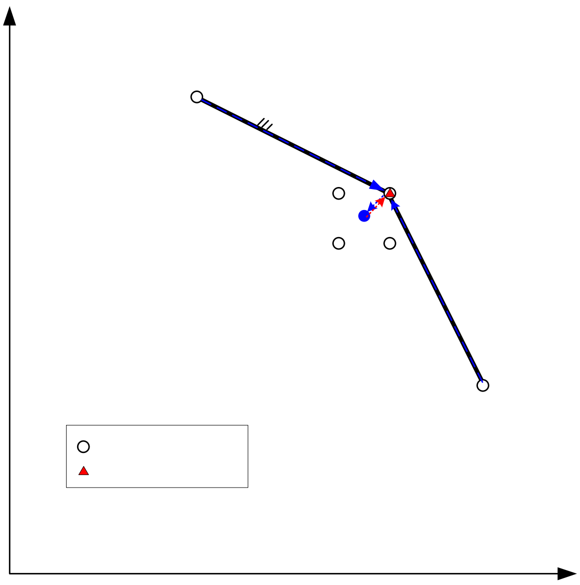

Figure 3. The primary and the secondary frontiers (radial-based).

Table 4. Data and results

	Original data					peDMUs					e2DMUs		
DMU	X_1	Y_1	Y_2	ρ_o^{I*} (SBM)	ρ_o^{II*} (Super-SBM)	X_1	Y_1	Y_2	ρ_o^{III*} (SBM)	(Super-SBM)	X_1	Y_1	Y_2
A	1	2	5	1	1.111	***1***	***2***	***4***	0.727		***1***	***3.5***	***4***
B	1	4	4	1	1.067	***1***	***3.5***	***4***	1	1	1	3.5	4
C	1	5	2	1	1.111	***1***	***4***	***2***	0.727		***1***	***4***	***3.5***
D	1	3.5	4	0.933					1	1	1	3.5	4
E	1	4	3.5	0.933					1	1.049	1	4	3.5
F	1	3.5	3.5	0.875					0.933		**1**	**4**	**3.5**

			peDMUs					e2DMUs		
DMU	(CCR-O)	(Super-CCR-O)	X_1	Y_1	Y_2	(CCR-O)	(Super-CCR-O)	X_1	Y_1	Y_2
A	1	1.25	***1***	***4***	***4***	1	1	1	4	4
B	1	1.067	***1***	***3.75***	***3.75***	0.938	0.938	**1**	**4**	**4**
C	1	1.25	***1***	***4***	***4***	1	1	1	4	4
D	0.933					1	1	**1**	**4**	**4**
E	0.933					1	1	**1**	**4**	**4**
F	0.875					0.875	0.875	**1**	**4**	**4**

APPENDIX

DMU\	Company
1	Taiwan Semiconductor Manufacturing Co., Ltd.
2	United Microelectronics Corp.
3	Mediatek Inc.
4	Advanced Semiconductor Engineering, Inc.
5	Winbond Electronics Corp.
6	Nanya Technology Corp.
7	Siliconware Precision Industries Co., Ltd.
8	VIA Technologies, Inc.
9	Macronix International Co., Ltd.
10	Silicon Integrated Systems Corp.
11	Sunplus Technology Corp.
12	Novatek Microelectronics Corp.
13	Realtek Semiconductor Co., Ltd.
14	King Yuan Electronics Co., Ltd.
15	ALI Corporation
16	Elite Semiconductor Memory Technology Inc.
17	Greatek Electronics Inc.
18	Elan Microelectronics Corp.
19	International Semiconductor Technology Ltd.
20	Lingsen Precision Industries, Ltd.
21	Faraday Technology Corporation
22	Taiwan Mask Corporation
23	Richtek Technology Corp.
24	Sigurd Microelectronics Co.
25	Integrated Technology Express Inc.
26	Sonix Technology Co., Ltd.
27	Weltrend Semiconductor Inc.
28	G.T.M Corp.
29	Sitronix Technology Corp

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