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Optimal Schedule Adjustment for Expected Aircraft Shortage in Multi-Fleet Operations

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Abstract**—**This research develops several network models for carriers that both efficiently and effectively adjust schedule resulting from the expected aircraft shortage for the operation of multiple fleets as well as non-stop and one-stop flights. These models are formulated as pure network flow problems or multi-commodity network flow problems. The former are solved using the network simplex method while the latter are solved using a Lagrangian relaxation-based algorithm. A case study regarding the international operations of a major Taiwan airline is presented.

Keywords**—**Expected aircraft shortage, Schedule adjustment, Time-space network, Multi-commodity network flow problem, Lagrangian relaxation

1. INTRODUCTION

Flight schedule perturbations may occur in an airline's regular operations due to numerous factors. For example, canceled or delayed flights may be caused by congestion at airports, poor meteorological conditions, equipment breakdown, late or absent crew members, poorly calculated block times and service times at certain airports, irregular onboard or package-handling processes, late transfers between flights, or sudden war Teodorovic (1988). Besides these factors which can typically be characterized as real time events, some expected events, for example, regular maintenance checks on aircraft or special uses of aircraft in the near future, may also result in schedule perturbations. These expected events, different from the real time events, are usually known some time (a few days or a week) before the events really happen. Thus, if airline carriers can effectively plan their temporary schedules (or handle schedule perturbations early) before the expected events happen, they may reduce their loss of profit and maintain a better level of service.

Aircraft maintenance checks are one typical expected event in short-term airline operations, as they are necessary Etschmaier and Mathaisel (1985). Airlines always adopt maintenance policies that conform with the government regulations. For example, other than the everyday routine inspections, many carriers have four other types of inspections on aircraft. The first major check (an 'A' check) occurs every 65 flight hours or about once a week. An 'A' check involves a visual inspection of all major systems such as landing gear, engines, and control surfaces. 'B' checks are performed every 300-600 hours and entail a thorough visual inspection plus the lubrication of all moving parts such as horizontal stabilizers and ailerons. 'C' and 'D' checks are done about once every one to four years respectively, and require taking the aircraft out of service for up to a month at a time Wallich (1986).

Although carriers have usually considered maintenance constraints in their fleet routing and flight scheduling in short-term operations Yan and Young (1996), the maintenance of aircraft may differ from their schedules, due to the complicated operation environment Teodorovic (1988). Thus, the carrier has to reschedule the dates of the maintenance checks for aircraft in short-term operations. Usually an 'A' check may not cause severe schedule perturbations. Carriers can do these short checks overnight or utilize lengthy ground times. However, for a longer check, for example, a B-check, a C-check, or a D-check, the aircraft usually has to be sent into the base plant for a longer time, resulting in a temporary shortage of aircraft and, therefore, a schedule perturbation.

There is normally one maintenance base (usually Taipei) for international airline carriers in Taiwan. The available slots that can be used for maintenance checks (particularly for B, C, and D-checks) are always limited. Therefore, the maintenance check has to be rescheduled early, so that the aircraft can be sent to the base plant in time for maintenance. According to a major Taiwan airline, such checks (especially B or C-checks) may occur from time to time in its operations. They often result in perturbations in flight schedules, not only during the period that the aircraft is in maintenance, but both before and after. Typically, if the adjustment to the flight schedule is too early, it may cause unnecessary schedule perturbations and a decreased level of service. However, if the adjustment to the flight schedule is too late, it may be infeasible to send the aircraft to the maintenance base in time. If the carrier uses the aircraft overtime, then it may cause safety problems. Because of this, when to start adjusting a flight schedule is an important issue for airline carriers, in their short-term operations.

According to a major Taiwan airline carrier, the available time slots at its base plant are very limited. The plant and the time slot for a scheduled-check are usually given before

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a schedule adjustment is made. The current process for adjusting the flight schedules of Taiwan carriers is inefficient and ineffective from a system perspective, especially for large flight networks. The process is as follows. Given the aircraft as well as the location of the plant and the time slot for maintenance, the planners start by choosing a suitable strategy (for example, flight cancellations, flight delays, ferrying of flights) based on the regular, planned schedule, projected (with booked) demand on all flights and the allocation of the currently available airplanes. The starting time for adjusting a schedule is always arbitrarily chosen. It is typically made to be as early as possible to avoid an infeasible solution. The detailed flight/fleet schedule is then adjusted by a trial-and-error process until a feasible solution is found. For example, if a flight cannot be served by its scheduled aircraft, then the flight can be suitably delayed so that a holding aircraft (or an incoming aircraft) can be rescheduled to serve this flight. However, if there is no holding aircraft (or no incoming aircraft), then the flight can be canceled, or served by an aircraft obtained from another station using a ferry flight. Such a process generally involves a series of local adjustments (usually by hand) to aircraft routes and their related flights. As network size grows, the process is more inefficient for finding the optimal solution. Usually, a feasible solution is only obtained under the real time constraint. The draft schedule is then sent to the operating division for the application of other constraints (for example crew constraints). The schedule will be executed if it is feasible, otherwise it will be returned to the planning division for revisions. The process is repeated until both the planning and operating divisions are satisfied with the revised schedule.

Due to deregulation, the flight networks for Taiwan carriers have recently grown. Thus, it is increasingly difficult for the traditional approach to adequately handle such events. Perturbations will generally affect not only the flights and the scheduled routing for an aircraft, but also many other flights, and fleet routing as well because of complicated network operations. The poor scheduling of flights, or an entire fleet, may result in a substantial loss of profit, decreased levels of service, or even safety problems for the airline carriers. Thus, it would be helpful for carriers to have systematic computerized models to handle schedule perturbations efficiently and effectively, so as to reduce losses resulting from the expected aircraft maintenance.

Few optimization models exist for solving maintenance related problems. Feo and Bard (1989) introduced a large-scale mixed integer programming formulation for long-term maintenance scheduling problems. The model is used to locate maintenance stations and to develop flight schedules that better meet the cyclical demands for maintenance. Biro' et al. (1992) introduced an operative model to adjust the flight schedules in short-term operations. In particular, they used the node coloring technique in a graph to adjust the sequence of flights, in order to meet maintenance requirements. Although, the model was useful for arranging the flight sequences, many practical scheduling adjustments, such as flight cancellations, flight delays, and the ferrying of idle aircraft, were difficult to use to efficiently adjust flight schedules, especially in a complicated network operation.

Concerning past research on airline schedule perturbations e.g. Jedlinsky (1967), Etschmaier and Rothstein (1973), Deckwitz (1984), Teodorovic and Guberinic (1984), Gershkoff (1987), Teodorovic and Stojkovic (1990), Jarrah et al. (1993), Mulvey and Ruszczynski (1995), Yan and Yang (1996), Du and Hall (1997), Kenyon and Morton (2003), List et al. (2003) not much work has been devoted to the problem of perturbations involved in executing a planned airline schedule. In the past, research did not handle expected aircraft maintenance. Besides, the aforementioned models were all focused on the operations of single fleet. Note that airplanes of different types can support each other through a new routing with a temporary flight schedule if the schedule is perturbed temporarily. For example; some idle larger-size aircraft can serve flights scheduled for small-size aircraft; some flights can be delayed so that larger-size aircraft can be rescheduled to serve these flights if it is profitable from the system aspect. Consequently, there is not yet a scheduling model that formulates multi-fleet operations and all the practical scheduling rules for handling perturbations that result from the expected aircraft maintenance, in a systematic and combined framework.

In this research, we develop several network models to help carriers handle schedule perturbations resulting from the expected aircraft maintenance. Other expected events can be dealt with in the future. The maintenance types focused on in this research are those that have to be done in the plant, particularly, the 'B' checks (done in a day), 'C' checks (done in a week) or short-term progressive maintenance (done within a week). As to 'A' checks, because they can be done flexibly either overnight or using a long ground time, it may not be necessary to use our models to handle these. Moreover, the model developed in this research may not be suitable for 'D' checks, since 'D' checks are typically carried out in a month, and by then the timetable may be changed. Thus, instead of adjusting a temporary schedule, a new schedule may be designed always, incorporating the 'D' checks.

The scope of this research is focused on the operations of multiple fleets, as well as one-stop and non-stop flights. The operations involved in other types of multi-stop flights are left to be addressed in the future. In addition, since the current fleet sizes of Taiwan airline carriers are small and it is unusual for a carrier to have more than one aircraft rescheduled for a 'B' or 'C' check at the same time, we focused on only one case of aircraft maintenance for simplification. Although the model developed in this research needs to be modified to handle cases where more than one aircraft is rescheduled for 'B' or 'C' checks, we believe modification to be easy, a subject also left for future research. Similar to most fleet routing models e.g. Yan and

Yang (1996), Yan and Young (1996), the constraints of crew scheduling are excluded in the modeling to facilitate problem solving. The rest of this paper is organized as follows: we first introduce the modeling approaches including the single-fleet time-space network, the basic model and several strategic models. These models are then formulated as integer programs and their solutions are developed hereafter. Finally, a case study is performed to test the models in practice.

2. MODELING APPROACHES

In the modeling, we first introduce the single-fleet time-space network, then extend it to the basic model, and finally develop strategic models based on the basic model.

2.1 The single-fleet time-space network

We suggest using a time-space network (as shown in Figure 1) to formulate the single-fleet network. The network is indeed an extension of the one in Yan and Yang (1996), which is used for handling schedule perturbations caused by the breakdown of aircraft. In Figure 1, the horizontal axis represents airport locations, while the vertical axis represents the time duration. Each node represents a specific airport at a specific time. Each arc represents an activity for an airplane. There are four types of arcs; 1) flight arcs, 2) overnight arcs, 3) ground arcs and 4) position arcs. A flight arc represents a non-stop flight or a one-stop flight. To reflect additional charges for canceling flights, the modification of the arc cost of a flight arc is the same as in Yan and Yang (1996). A ground arc represents the holding of airplanes at an airport in a time window. An overnight arc represents the holding of airplanes overnight at an airport. A position arc represents a ferry flight between two airports. All of these arcs are made to be the same as those in Yan and Yang (1996).

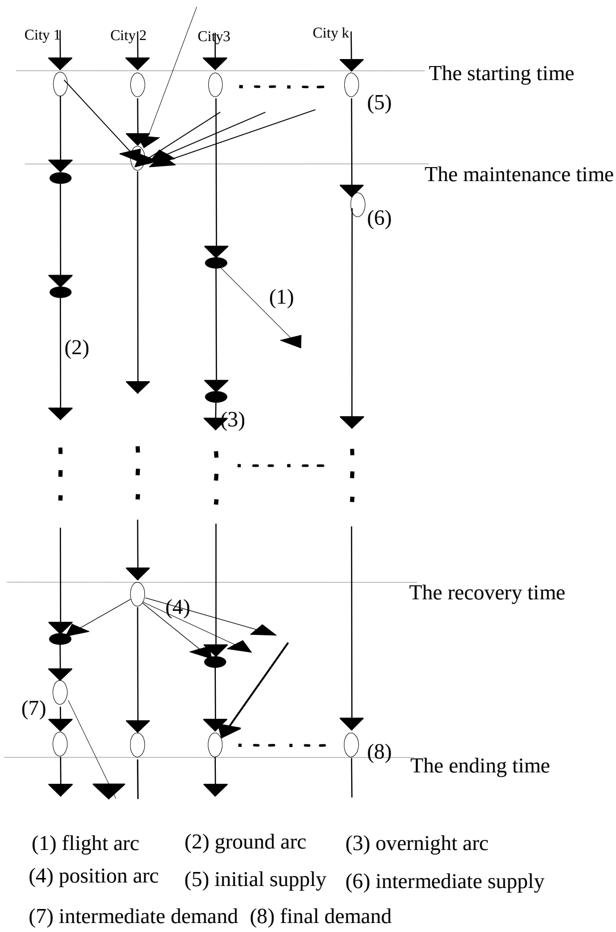


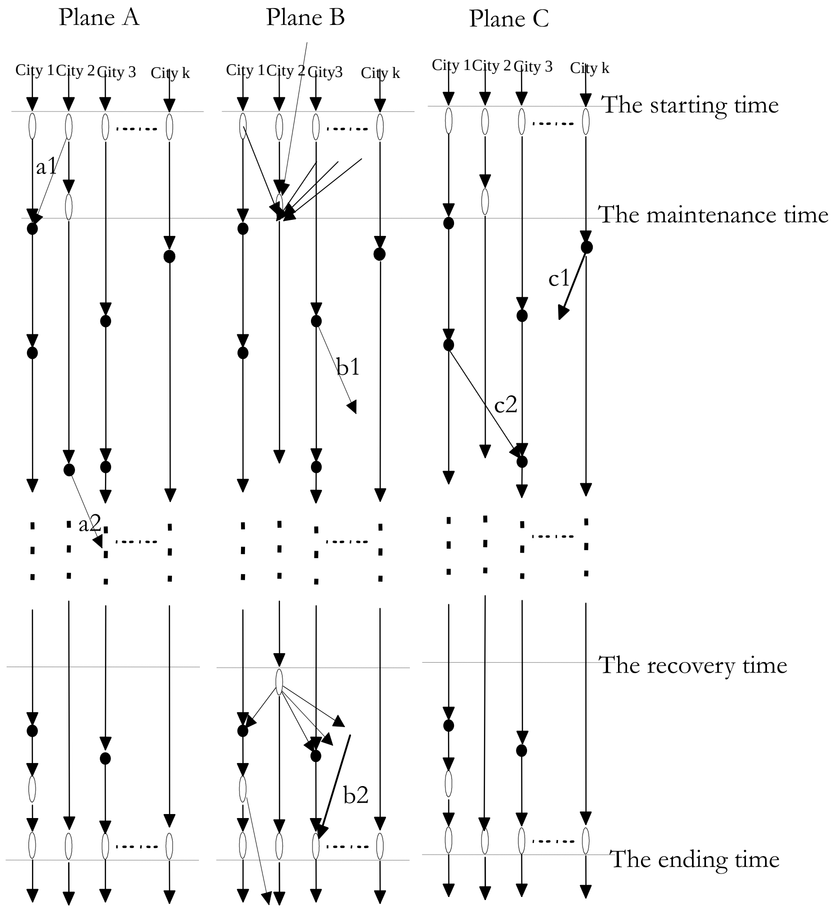
Figure 1. The single-fleet time-space network.

The basic network contains a perturbed period, from a starting time to an ending time. The starting time is the earliest time that the system starts the perturbations. The starting time is designed as follows. In order to send the specified aircraft to the maintenance time/space point in time (the time is when the aircraft starts its maintenance and the space is where the maintenance takes place), we add a position arc, from every other station to the maintenance time-space point. To minimize the perturbed period in the schedule, we set the starting time to be the latest time that the aircraft's original route meets the aforementioned position arcs. Thus, the aircraft can use at least one position flight to get to the maintenance base in time. Note that if the departure times of the aforementioned position arcs are ahead of the starting time, then these arcs should not be added into the network. It should be mentioned that the starting time can differ if the aircraft is different, because its original route also differs, as does the latest time that the route meets the position arcs. At the starting time, all airplanes located at airports, or in the air, are set as initial or intermediate node supplies. To determine the route of a specified aircraft, before finding the fleet routing, we find the shortest path from where the specified aircraft is located at the starting time to the maintenance time/space point. Accordingly, an intermediate node demand should be placed (i.e. the node supply should be minus one) at the time-space point where the airplane starts its route in the network, meaning that the specified airplane is extracted from the system at the starting time.

Similar to the case in Yan and Yang (1996), the specified airplane can be returned to service after the recovery time (i.e. after finishing the maintenance), so the ending time that is when the fleet resumes its normal operation is determined to be the same as that in Yan and Yang (1996). Final or intermediate node demands are set according to the fleet allocation at airports or in the air, at the ending time. Since the specified airplane should be reintroduced into the system, an intermediate node supply should be placed at the time/space point corresponding to the maintenance base and the recovery time when the specified airplane is again ready for service.

2.2 The basic model

The basic model is developed based on the single-fleet time-space network. An example of the basic model is shown in Figure 2, assuming that there are three fleets in operation, each corresponding to a type of aircraft. Since this research simplifies the case to the expected maintenance of one aircraft, only one network (a type B network in the example) in the basic model is designed with position arcs. Other types of networks (type A and type C) are designed to be similar to type B without position arcs, because there are no absent aircraft involved.



a1, a2: plane A (small type) flight arc
b1, b2: plane B (median type) flight arc
c1, c2: plane C (large type) flight arc

Figure 2. The basic model network.

The integer program formulating the basic model is shown below;

Min

$$Z=\sum_{n\in M}\left(\sum_{i,j\in A^n\setminus F^n} C_{ij}^n X_{ij}^n+\sum_{i,j\in F^n}\left(C_{ij}^n-CA_{ij}^n\right)X_{ij}^n+\sum_{i,j\in F^n} CA_{ij}^n\right) \quad (1)$$

St.

$$\sum_{j\in O(i)} X_{ij}^n-\sum_{k\in I(i)} X_{ki}^n=b_i^n \qquad \forall i\in N^n,\forall n\in M \quad (2)$$

$$0\le X_{ij}^n\le U_{ij}^n \qquad \forall (i,j)\in A^n,\forall n\in M \quad (3)$$

$$X_{ij}^n\in I \qquad \forall (i,j)\in A^n,\forall n\in M \quad (4)$$

where

n : the number of aircraft types

M : the set of all aircraft types

N^n : the set of all nodes in the nth type network

A^n : the set of all arcs in the nth type network

F^n : the set of all flight arcs in the nth type network

$O(i)$: the set of head nodes for arcs emanating from node i

$I(i)$: the set of tail nodes for arcs pointing into node i

b_i^n : the node supply/demand of node i in the nth type network. Note that if a specified airplane is taken out from the system for maintenance or other reasons at a node, then its node supply/demand need to be minus one in calculation. On the contrary, for a node if an airplane is returned to the system, then the node supply/demand need to be plus one.

$C_{ij}^n, X_{ij}^n, U_{ij}^n$: arc (i,j) cost, flow and upper bound in the nth type network respectively

CA_{ij}^n: the cancellation cost for flight (i,j) in the nth type network.

The objective of this model is to "flow" all node supplies to all node demands in each network at a minimum cost. Since revenue is formulated as negative cost, the objective function (1) is equivalent to a maximization of the system profit. Constraint (2) ensures flow conservation constraint. Constraint (3) ensures that all arc flows are within their upper and lower bounds, and constraint (4) ensures that all arc flows are integers. Note that the basic model can be used to evaluate the cancellation of flights.

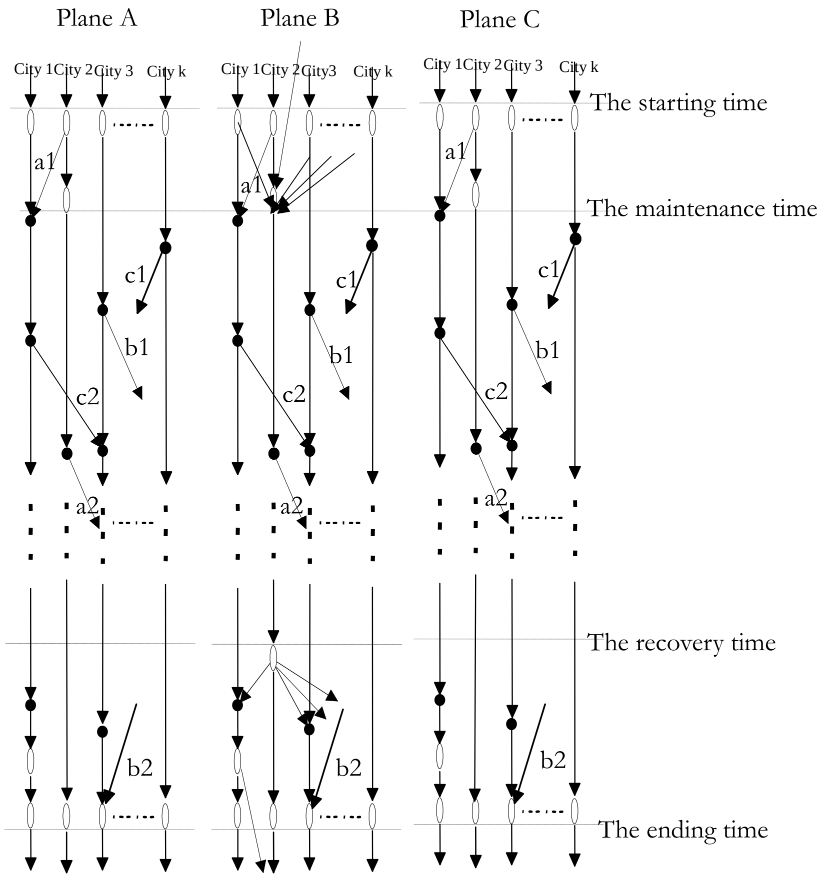
2.3 The strategic models

To make the basic model more useful, four practical scheduling rules are incorporated to develop strategic models, in particular; (a) the swap of aircraft types (b) flight delays, (c) the modification of one-stop flights and (d) the ferrying of idle aircraft. The strategy of "the swap of aircraft types" is used for evaluating the swap of aircraft types for some flights for the best use of all aircraft in network operations. The strategy of "flight delays" is used for the evaluation of delays for certain flights or all flights in the network. he strategy of "the modification of one-stop flights" is used for evaluating the cancellation of one-stop flight segments for some or all one-stop flights when the aircraft are too few for service, due to a schedule perturbation. The strategy of "the ferrying of idle aircraft" is used to evaluate the ferrying of idle aircraft to where and when the system needs them for the best routing.

Modifications for rules (b) and (d) can be referred to Yan and Yang (1996); while modifications for rule (c) can be referred to Yan and Young (1996). We note that additional side constraints should be added to the basic model if rule (b) or (c) is incorporated to form a strategic model Yan and Yang (1996); Yan and Young (1996). However, if only rule (d) is incorporated into the basic model, then no additional side constraints but additional arcs are added to the basic network Yan and Yang (1996).

Rule (a) is suitable for a multi-fleet operation. An example of the modifications for this rule is shown in Figure 3; assume that there are three types of aircraft (three fleets) in operation where the capacity of type A is the smallest and that of type C is the largest. Since larger aircraft can serve smaller-type flights, smaller-type flight arcs can be added into larger-type fleet networks. For example, as shown in Figure 3, type A flight arcs (e.g. "a1" and "a2") can be added into the type B and type C networks. Type B flight arcs (e.g. "b1" and "b2") can be added into the type C network. Some of the type B flight arcs (e.g. "b1" and "b2") may be added into the type A network. If passengers not able to get on the type A aircraft (because of fewer seats), they can be reaccommodated on other suitable flights or alternate modes to the same destinations. Similarly, some type C flight arcs (e.g. "c1" and "c2") may be added into the type A or type B network. Note that, similar to flight cancellations, additional charges may be incurred for reaccommodating passengers who are unable to get on the smaller-sized aircraft, on suitable flights or alternate modes to the same destinations. Also note that, when using another type of aircraft to serve a flight, this type of aircraft should be feasible in terms of the flight mileage and the associated airport facilities.

If flight swaps happen when using another type of aircraft to serve flights, then swap costs (for example, the additional costs of switching gates or switching crew members) should be included when modeling. After all, the smaller-type flight arc cost in the larger-type fleet network (e.g. "a1" in the type C fleet network) equals the larger-type of aircraft's flight expenses, plus the swap cost, minus the on-board passenger revenue. Similarly, the larger-type flight arc cost in the smaller-type fleet network (e.g. "b1" in the type A fleet network) equals the smaller-type of aircraft's flight expenses, plus the swap cost, minus the on-board passenger revenue, plus the additional charges acquired for passengers not getting on the smaller-type aircraft.



a1, a2: plane A (small type) flight arc
b1, b2: plane B (median type) flight arc
c1, c2: plane C (large type) flight arc

Figure 3 Network modifications for the swap of aircraft types.

It should be mentioned that although modifications of the basic model for rules (b), (c) and (d) in each fleet network are referred to Yan and Yang (1996), and Yan and Young (1996), if rule (a) is applied together with rule (b), then, because at most one departure time is assigned for a flight, a side constraint should be introduced for a flight arc and its alternate flight arcs among all associated fleet networks to ensure that at most one flight is served. Similarly, if rule (a) is applied together with rule (c), then the additional side constraint mentioned in Yan and Young (1996) should be extended across all the associated fleet networks. Besides, to ensure that each non-stop flight (which is modified from a one-stop flight) is served at most once, a side constraint should be introduced for each non-stop flight across all the associated fleet networks. We note that models containing rule (a) can be characterized as multi-fleet scheduling models; otherwise they can be characterized as single-fleet scheduling models, because the networks in any of them are independent of each other.

Carriers may create different models based on the basic model and the selected scheduling rules, and then choose the best one for application. Because the problem size in actual operations is usually large, we also developed an automatic data process in our research to help users readily apply the models in actual operations. In particular, with the user's choice of strategies, including the swap of possible aircraft types, possible delayed flights, one-stop flights for possible modifications, or OD pairs for possible ferrying, this process can automatically build model input, develop the strategic model, optimize the model, and create the final output, all within the computer environment. We also note that when users apply the framework in actual operations a decision support system (DSS) incorporating the automatic data process, the optimization algorithm and a user-friendly interface would be better. Such a DSS can be a direction of future research.

3. SOLUTION METHODS

The aforementioned models are formulated as pure network flow problems or multi-commodity network flow problems. In particular, the basic model and the strategic model using only rule (d) are formulated as pure network problems. The others are multi-commodity network flow problems, which are characterized as NP-hard problems Garey and Johnson (1979). Since these models are designed to have optimal solutions, in actual applications they do not generate infeasible or unbounded solutions. Referring to Yan and Yang (1996), we suggest using the network simplex method to solve the pure network flow problems due to its demonstrated efficiency. We modified the Lagrangian relaxation-based algorithm developed by Yan and Yang (1996) to solve the multi-commodity network flow problems. In particular, we used the subgradient method developed by Camerini et al. (1975), instead of the one suggested by Fisher (1981), along with the Lagrangian heuristic, the network simplex method, and the Lagrangian relaxation technique to develop the algorithm. After much testing of our problems, we found that our algorithm performed better than the one developed by Yan and Yang (1996).

The solution steps are summarized as follows:

Step 0: Set the initial Lagrangian multipliers.

Step 1: Use the technique of Lagrangian relaxation to relax the side constraints with the Lagrangian multipliers to form a Lagrangian problem. Obviously, the Lagrangian problem can be decomposed into several independent pure network flow problems. Optimally solve the Lagrangian problem using the network simplex method to get a lower bound. Update the lower bound.

Step 2: Apply the Lagrangian heuristic Yan and Yang, (1996) to find an upper bound and update the upper bound.

Step 3: If the gap between the lower bound and the upper bound is within a 0.1% error, or the number of iterations reaches 500, stop the algorithm.

Step 4: Adjust the Lagrangian multipliers using the subgradient method Camerini et al. (1975).

Step 5: Set $k = k + 1$. Go to Step 1.

Since the solutions obtained are all arc flows and are incapable of expressing the route of each airplane, we used the same flow decomposition method Yan and Yang (1996) to decompose the arc flows into arc chains, with each arc chain representing an airplane's route in the perturbed period. Note that the arc chains may not be unique. The planners can choose several solutions and send them to other divisions for the application of the other operating constraints, for example, crew availability. Thus, a satisfactory solution for all divisions could be relatively easy to find.

4. CASE STUDY

To demonstrate our models, we use a case study based on data from a major Taiwan airline's international operations. There are 24 cities involved in its operations. The flight timetable, used for the testing, is rotated once a week and includes 273 flights. About 20 percent of them are one-stop flights; the others are non-stop flights. There are several types of aircraft involved in its operations, including B737's, AB3's, AB6's, MD11's, B74L's, B744's and B747's. For simplification, we had three types of aircraft in this case study; type A indicates B737's (3 airplanes) with 120 seats, type B includes AB3's, AB6's, MD11's and B74L's (17 airplanes) with an average of 269 seats and type C includes B744's and B747's (6 airplanes) with an average of 403 seats.

For ease in testing, all the cost parameters were set according to the airline's reports and the Taiwan government regulations, with reasonable simplifications. Note that according to the airline carrier; the swap cost for different aircraft types in its operations is very small compared to the flight cost. For simplification, the swap cost between different fleets is assumed to be zero. Besides, since the airline has contracts with other airlines for transporting passengers under irregular operations, without additional charges, for simplification, we assume that the additional charges, for reaccommodating passengers not getting on the smaller-sized aircraft for suitable flights to the same destinations, are zero. Note that the cost typically affects the test results. As the case study is only for demonstration purposes at the current stage, the evaluation of the application of this framework to actual operations is left to future work.

For the simplification of strategy (b), we only add an alternate flight arc after each flight arc, each alternate flight denoting a delay of 30 minutes (in other words, we do not allow a delay of more than 30 minutes in this test). For strategy (d), we add position arcs between every OD pair every 12 hours from the starting time to the recovery time. We assume that an airplane is rescheduled for a 'B' check at Taipei, starting at 12:00 AM on Wednesday, and will be ready for service 24 hours later. Consequently, the starting time, by definition, is 7:00 AM on Wednesday, the maintenance beginning time is 12:00 AM, the recovery time is 12:00 AM on Thursday and the ending time by definition is 15:00 PM on Friday. Note that the starting time (Wednesday 7:00 AM) is the latest time that the schedule has to be adjusted; otherwise, the specified airplane cannot be sent to Taipei before Wednesday 12:00 AM. Also, the fleet will return to its normal operation after Friday 15:00 PM at the latest.

Eighteen scenarios were done in this test. Scenario 1 (indicated as "Normal") denotes a normal operation. As in Gershkoff (1987), Scenario 2 ("SSP") applies the successive shortest paths to finding a series of aircraft routes from the starting time to the ending time. The number of canceled flights can thus be calculated after fleet assignment. To assure that the specified aircraft can be sent to the maintenance time/space point and the fleet can resume its normal operations after the ending time, ferry flights could be used. We note that the SSP method is easy to implement using our networks. In particular, the label correcting algorithm Ahuja (1993) is applicable for solving the fleet assignment. The SSP method used here is for the preliminary evaluation of our models. Scenarios 3 through 18 are associated with our strategic models. For example, Scenario "B" indicates the basic model (flight cancellations); "Ba" denotes the combined model for flight cancellations and the swap of aircraft types; "Bb" denotes the combined model for flight cancellations and flight delays; "Bc" denotes the combined model for flight cancellations and the modification of multi-stop flights; "Bd" denotes the combined model for flight cancellations and the ferrying of idle aircraft; and "Babcd" denotes the combined model for flight cancellations, the swap of aircraft types, flight delays, the modification of multi-stop flights and the ferrying of idle aircraft. Models "B" and "Bd" are pure network flow problems and the other models are multi-commodity network flow problems. We used the network simplex method to solve the pure network flow problems and applied the Lagrangian relaxation-based algorithm to solve the multi-commodity network flow problems. The flow decomposition method Yan and Yang (996) was used to decompose the link flows into arc chains in each fleet network, with each arc chain representing an airplane's route in the perturbed period.

Several C programs and an automatic data process were developed for; (1) the analysis of raw data (2) the building of the basic model (3) the development and solution of the strategic models and (4) the output of data. The case study was implemented on an HP735 workstation. Sixteen strategic models were tested with problem sizes of up to 4,285 nodes and 14,288 arcs. All of the results indicate that the framework could be useful for actual operations. The results are summarized in Table 1 and analyzed below.

Table 1. Results of all scenarios

strategy (1)	computation time (2)	# iteration (3)	objective value (Z^U) (4)	converged gap (5) $\lvert(Z^U-Z^L)/Z^U\rvert$	# nodes (6)			# arcs (7)			# side constraints (8)
	(sec)		(NT$)	%	A	B	C	A	B	C	
Normal	----	----	-96,529,295	----	----	----	----	----	----	----	----
SSP	0.55	1	-93,776,688	----	1,369	1,369	1,369	2,720	2,799	2,727	----
B	0.58	1	-93,896,259	0	1,369	1,369	1,369	2,720	2,799	2,727	85
Ba	36.51	45	-95,584,793	0	1,369	1,369	1,369	2,785	2,815	2,797	85
Bb	57.06	62	-94,540,748	0.03	1,369	1,369	1,369	2,728	2,855	2,741	85
Bc	0.86	1	-93,896,259	0	1,369	1,403	1,371	2,720	2,918	2,734	121
Bd	1.04	1	-94,314,642	0	1,369	1,369	1,369	3,761	4,395	5,160	85
Bab	115.73	102	-95,765,110	0	1,369	1,369	1,369	2,851	2,886	2,875	85
Bac	37.7	34	-95,584,793	0	1,397	1,403	1,405	2,883	2,934	2,906	121
Bad	77.83	57	-95,601,389	0	1,369	1,369	1,369	3,826	4,411	5,230	85
Bbc	77.14	68	-94,540,748	0	1,369	1,431	1,373	2,728	3,072	2,755	121
Bbd	119.28	59	-94,540,748	0	1,369	1,369	1,369	3,769	4,451	5,174	85
Bcd	1.28	1	-94,314,642	0	1,369	1,403	1,371	3,761	4,514	5,167	121
Babc	157.32	107	-95,765,110	0	1,419	1,431	1,435	3,026	3,103	3,089	121
Babd	260.44	112	-95,928,199	0	1,369	1,369	1,369	3,892	4,482	5,308	85
Bacd	72.33	45	-95,521,776	0.09	1,397	1,403	1,405	3,924	4,530	5,339	121
Bbcd	218.24	95	-94,540,748	0	1,369	1,431	1,373	3,769	4,668	5,188	121
Babcd	252.19	92	-95,928,199	0	1,419	1,431	1,435	4,067	4,699	5,522	121

Table 1. Results of all scenarios (continued)

scenario	# original flights (9)			# flights served by other fleet (10)		# canceled flights (11)			# delayed flights (12)			# modified multi-stop flights (13)			# ferry flights (14)			# canceled flights and flight segments (15)
	A	B	C	B/A	C/B	A	B	C	A	B	C	A	B	C	A	B	C	Total
Normal	8	62	15	----	----	0	0	0	0	0	0	0	0	0	0	0	0	0
SSP	8	62	15	----	----	0	7	0	----	----	----	----	----	----	0	1	0	7
B	8	62	15	----	----	0	4	0	----	----	----	----	----	----	0	1	0	4
Ba	8	62	15	4	0	0	5	0	----	----	----	----	----	----	0	1	0	1
Bb	8	62	15	----	----	0	2	0	0	1	0	----	----	----	0	1	0	2
Bc	8	62	15	----	----	0	4	0	----	----	----	0	0	0	0	1	0	4
Bd	8	62	15	----	----	0	3	0	----	----	----	----	----	----	0	2	0	3
Bab	8	62	15	4	4	0	5	4	0	2	0	----	----	----	0	1	0	1
Bac	8	62	15	4	0	0	5	0	----	----	----	0	0	0	0	1	0	1
Bad	8	62	15	4	0	0	4	0	----	----	----	----	----	----	1	1	0	0
Bbc	8	62	15	----	----	0	2	0	0	1	0	0	0	0	0	1	0	2
Bbd	8	62	15	----	----	0	2	0	0	1	0	----	----	----	0	1	0	2
Bcd	8	62	15	----	----	0	3	0	----	----	----	0	0	0	0	2	0	3
Babc	8	62	15	4	4	0	5	4	0	2	0	0	0	0	0	1	0	1
Babd	8	62	15	2	4	0	2	4	0	3	0	----	----	----	1	1	0	0
Bacd	8	62	15	2	0	0	3	0	----	----	----	0	0	0	1	1	0	1
Bbcd	8	62	15	----	----	0	2	0	0	1	0	0	0	0	0	1	0	2
Babcd	8	62	15	2	4	0	2	4	0	3	0	0	0	0	1	1	0	0

Note that the mark "----" in Table 1 indicates that the data is not available for that scenario.
Note that B/A in column (10) indicates the number of type B flights served by the type A fleet. Similarly, C/B denotes the number of type C flights served by the type B fleet.

(1) The algorithms performed very well, indicating they could be useful in practice. In particular, models "B" and "Bd" were optimally solved in about 1 second of CPU time. The other models converged within 0.1% of the error in at most 112 seconds of CPU time. Note that the CPU times for network generation and data output in each scenario are relatively short compared with model solutions and can be neglected. This shows that the network simplex algorithm should be efficient for solving pure network problems, like "B" and "Bd", and this research indicates that the Lagrangian relaxation-based algorithm could be efficient for solving the multi-commodity network flow problems. Compared with the efficiency of the traditional approach, the algorithms are superior.

(2) All of our models yield a higher profit than the SSP approach does. The best result (-95,928,199), for Models "Babcd" and "Babd", is closest to the profit achieved in normal operations (-96,529,295). Models "Babcd" and "Babd" cause much less profit loss (NT$ 601,096) than using the SSP approach (NT$ 2,752,607). Though the models, "B" and "Bc", are the worst among the strategic models, their objective (-93,896,259) is better than that of the SSP method (-93,776,688). The reason that our models outperform the SSP method could be that although the SSP method cancels a series of uneconomic (7 flights) flights, it does not consider the delaying of flights, the modification of multi-stop flights, the ferrying of idle aircraft or a combination of these in adjusting the schedule. Through the effective adjustment of flight schedules or fleet routes, fewer flights are canceled (for example, no flight is canceled in "Babcd"). Thus, higher profits are achieved.

We note that if more rules except (c) are incorporated to develop a strategic model, then the model will be more flexible for optimizing a temporary schedule, so that system profits can be improved more. For example, the result for "Babcd" is better than that for "Babc", which is better than "Bac". The reader can see other examples in Table 1. e find that flight cancellations, flight delays and ferry flights are suggested by many of the strategic models, as effective methods for schedule adjustment. We note that the strategy of (c) is not effective in this case. For example, the objectives of "B" and "Bc", "Ba" and "Bac", "Bb" and "Bbc", "Bd" and "Bcd", "Bab" and "Babc", or "Babc" and "Babcd", are same. Note that because there is a convergence error for "Bacd", the objective of "Bad" is slightly better than that of "Bacd". Since the strategy of (c) has been shown to be effective in Yan and Young (1996), more cases may need to perform to verify this, which could be a subject of future work.

We also find that the multi-fleet scheduling models (models containing strategy "a"), for example, "Ba", "Bab", "Bac", "Bad", "Babc", "Babd", "Bacd", or "Babcd" yield higher profits (more than 95,000,000) than the single-fleet scheduling models (models excluding strategy "a") (less than 95,000,000). From column (10), all eight multi-fleet scheduling models suggest that a fleet can temporarily serve other types of flights in scheduling, so as to improve the system profit. For example, four type B flights in Model "Ba" are served by the type A fleet. The result for Model "Ba" (-95,584,793) is better than that for the basic model "B" (-93,896,259), an improvement of NT$ 1,688,534 (about 1.77%). The reader can see other examples in Table 1. This implies that the swap of aircraft types is an important and effective strategy in schedule adjustment in the case.

(3) Other than the profit considerations, the degree of schedule perturbation may be a criteria for carriers to evaluate levels of service for all strategic models. Typically the number of canceled flights and delayed flights in the case study may serve as an index of schedule perturbation. In this study, since a flight has at most a delay of 30 minutes, its influence on the level of service could be omitted when compared to a canceled flight. Thus, in terms of the level of service, Models "Babcd" and "Babd" could be the best (no canceled flight) and SSP the worst (7 canceled flights). Note that the multi-fleet scheduling models canceled less flights (less than or equal to 1 flight) than the single-fleet scheduling models (more than 1). Consequently, considering both profit and level of service, the multi-scheduling models perform better than others. In particular, Models "Babcd" and "Babd" are the best of them all.

It should be noted that if a solution obtained from a strategic model is associated with a large degree of schedule perturbation, it might not be acceptable to carriers under real operating constraints (for example, crew availability). Then, carriers may choose another model to find a solution with fewer flight schedule perturbations, or they may make minor modifications of the perturbed schedule to satisfy the operating constraints. From this, a DSS might be helpful for the application of these strategic models to real time operations.

5. CONCLUSIONS

We develop several network models to help carriers handle schedule perturbations resulting from the expected aircraft maintenance, for the operations of multiple fleets as well as non-stop and one-stop flights. These models minimize the schedule-perturbed time so that carriers can resume their normal services as soon as possible in order to maintain their levels of service. Besides, these models combine flight cancellations, the swap of aircraft types, flight delays, the modification of multi-stop flights, and the ferrying of idle aircraft in a combined and systematic framework to effectively adjust a schedule, so that a carrier can maintain its profitability. These strategic models are formulated as pure network flow problems or multi-commodity network flow problems. We used the network simplex method to solve the pure network flow problems and developed a Lagrangian relaxation-based algorithm to solve the multi-commodity network flow problems.

To test the models in practice, a case study regarding the international operations of a major Taiwan airline was performed on an HP735 workstation. Sixteen strategic models and the SSP approach were tested, with substantial problem sizes of up to 4,285 nodes and 14,288 arcs. Several C programs and an automatic data process were developed to apply these models using an HP735 workstation. The algorithms performed very well. In particular, models "B" and "Bd" were optimally solved in about one second of CPU time; other models converged to 0.1% of the error in at most 112 seconds of CPU time. All our models out performed the SSP approach and improved the traditional scheduling process significantly. All of these showed that our models could be useful for actual operations. Since the case study is only for demonstration at the current stage, the evaluation of impact on the application of this model to actual operations is left to future work.

Although a multi-stop flight is evaluated as to whether to cancel its segments in our research, how to combine the flight segments of different flights to form a multi-stop flight can be a direction of future research. How to combine several flights of smaller types to form a larger type flight, and systematic models for handling schedule perturbation caused by other types of expected events, can be performed in the future research as well. Finally, a computerized decision support system for users, useful for applying these strategic models in practice, could also be another direction of future research.

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