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Horizontal Suppliers Coordination with Uncertain Suppliers Deliveries

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Abstract—In this study, two (or more) supply chains where the suppliers share information on their production, inventory and delivery status are considered. Due to uncertain supplier deliveries and lead time, horizontal supplier coordination is implemented to reduce lead time and deliveries uncertainty, and thus the safety inventory due to risk pooling effect. As a result, the total system profit increases. A mathematical model, illustrating a case study and sensitivity analysis are developed to show the significant of horizontal coordination.

Keywords—Horizontal suppliers, Coordination, Uncertain deliveries

1. INTRODUCTION

With the advancement of technology and varying customer's behavior, the lifecycle of new products has become shorter. This is especially true for mobile phone, digital camera, and notebook PC. In order to improve collaboration, management must focus on improving the cooperation between the suppliers so as to avoid uncertain in supply by each supplier. Coordinated ordering strategy for deterministic demand has been discussed from different viewpoints including the production management, the marketing chain coordination, and the economic theory. Oren et al. (1983), Tirole (1993) focused on price discrimination due to quantity discount on vertical channel integration. Most of the production management literatures focused on how to determine the integrated strategies for ordering and stocking (Jeuland and Shugan, 1983; Lal and Staelin, 1984; Kohli and Park, 1994; Goyal and Gupta, 1989). The literatures on marketing chain coordination focused on the coordination mechanism provided by quantity discount (Benton and Park, 1996). Buzzell and Ortmeyer (1995) pointed out the benefit of information sharing between the buyers and the suppliers. Weng (1995) and Fites (1996) showed that significant profit increase could be achieved through effective coordination of the supply chain. Weng (2003) demonstrated that under uncertain delivery time between the buyers and the suppliers. Weng (2004) further extended the model to consider quantity discount. The kind of supply chain discussed so far is known as the "vertical coordination".

In our study, two or more independent supply chains with their own buyers and suppliers are assumed. The buyer obtains the product from the supplier for reproduction or direct selling to the customers. The buyer has to consider the uncertainty customers' need and delivery time (lead time) from the supplier's perspective. Placing an optimal order before the selling period of the product is vital to all buyers. Two cases may occur; that is when the order quantity is higher than the demand, excess inventory will result in an obsolescence and higher inventory cost. When the order quantity is less than the demand, there will be shortages and lost of goodwill costs. Uncertain supplier lead-time is also a problem to the buyers. Our study develops a model with two or more suppliers through information sharing and coordinated supply chain (Please see Figure 1). This coordination, commonly known as the horizontal coordination, will allow the suppliers to adjust their lead-time and inventory control.

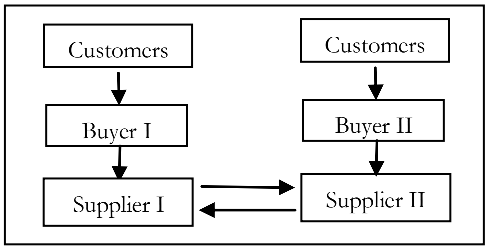

Figure 1. The framework of horizontal coordination of two supply chains.

2. ASSUMPTIONS AND NOTATIONS

Two supply chains (I and II) with the same product quality and cost of production. Each supply chain has one buyer and one supplier with fixed selling period. With the uncertainty in the customers' demand, the buyer will place its order independently of the supplier according to its optimal expected profit. However, there will be some unpredictable factors during the production and delivery processes. If the supplier accomplishes the order earlier than needed by the buyer, inventory cost will incur by the supplier. If the supplier has late order, lost sale will incur. Backordering involves

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crushed order cost and shortage penalty cost, and the buyer will be compensated by the supplier through a lower purchase price (Whang, 1992).

Assuming the buyer places its order independently, we discuss the different strategies adopted by the suppliers with and without coordination. The following notations from Weng's paper (2003) are adopted.

$i = 1$: the supply chain I
$i = 2$: the supply chain II
E_{bi}: buyer's maximum expected profit without coordination for ith supply chain
E_{si}: supplier's maximum expected profit without coordination for ith supply chain
E_{ST}: $=E_{s1} + E_{s2}$
E_i: maximum expected system profit without coordination for ith supply chain($E_i = E_{bi} + E_{si}$)
E_{Jsi}: supplier's maximum expected profit with coordination for ith supply chain
E_{JsT}: $=E_{Js1} + E_{Js2}$
E_{Ji}: maximum expected system profit with coordination for ith supply chain ($E_{Ji} = E_{bi} + E_{Jsi}$)
r_i: buyer's shortage cost per unit; represents costs of lost goodwill for ith supply chain
s_i: buyer's salvage value per unit for ith supply chain
Q_{bi}: buyer's order quantity for ith supply chain
x_i: random demand faced by the buyer for ith supply chain
y_i: supplier's order completion time ($y_i \le 0$ means delivery to the buyer will be on time; $y_i > 0$ means delivery to the buyer is delayed by y_i periods) for ith supply chain, i.e. lead-time
$g_{Y_i}(y_i)$: PDF of the supplier's order completion date for ith supply chain
t: supplier's per unit production cost
h: supplier's per unit holding cost per period (note: not an annual cost)
p_i: buyer's selling price per unit for ith supply chain
c_0: buyer's wholesale purchase price per unit when the supplier delivers on time
k: total inventory ratio

The demand and delivery time are assumed to be independent continuous random variables. Let $f_{X_i}(x_i | y_i, y_i \le 0)$ and $F_{X_i}(x_i | y_i, y_i \le 0)$ denote the PDF and CDF of demand for ith supply chain given that the order is delivered on time, and $f_{X_i}(x_i | y_i, y_i > 0)$, $F_{X_i}(x_i | y_i, y_i > 0)$ denote the PDF and CDF of demand given that the order is delivered late.

3. MODELING AND ANALYSIS

In this section, we formulate the expected profit models for the suppliers. The buyer's conditional expected profit function $EB(\cdot)$ for ith supply chain is given as follows:

$$EB(Q_{bi} | y_i, y_i \le 0) =$$
$$\int_0^{Q_{bi}} [(p_i - c_0)x_i - (c_0 - s_i)(Q_{bi} - x_i)] f_{X_i}(x_i | y_i, y_i \le 0) dx_i$$
$$+\int_{Q_{bi}}^{\infty} [(p_i - c_0)Q_{bi} - r_i(x_i - Q_{bi})] f_{X_i}(x_i | y_i, y_i \le 0) dx_i,$$
$$i = 1, 2, \tag{1}$$

where the first term represents the expected profit when demand, x_i, is less than the buyer's order quantity, Q_{bi}, and the second term represents the expected profit when demand, x_i, is greater than the buyer's order quantity, Q_{bi}. The buyer's maximum expected profit for ith supply chain, $E_{bi} = EB\left(Q^*_{bi} | y_i, y_i \le 0\right)$, where Q^*_{bi} represents the buyer's optimal order quantity. The" newsboy" problem (Hadley and Whitin, 1963) follows, $F_{X_i}(Q^*_{bi} | y_i, y_i \le 0) = (p_i - c_0 + r_i) / (p_i - s_i + r_i)$, under certain condition, $p_i > c_0 > s_i$, and $r_i \ge 0$. E_{bi} can be written as

$$E_{bi} = (p_i - c_0)Q^*_{bi} - r_i[E(x_i | y_i, y_i \le 0) - Q^*_{bi}]$$
$$-(p_i - s_i + r_i)\int_0^{Q^*_{bi}} F_{X_i}(x_i | y_i, y_i \le 0) dx_i,$$

where

$$E(x_i | y_i, y_i \le 0) = \int x_i f_{X_i}(x_i | y_i, y_i \le 0) dx_i$$

represents the buyer's expected demand and $F_{X_i}(x_i | y_i, y_i \le 0)$ denote the CDF of demand given that the supplier delivers on time. Since there is a potential lose on the buyer due to delivery delay from the supplier, we define "$c_i(y_i)$"to be the discounted wholesale price satisfying the contractual requirement $EB(Q^*_{bi} | y_i, y_i > 0) = E_{bi}$ (Weng, 2003):

$$c_i(y_i) = p_i - \left(\frac{1}{Q^*_{bi}}\right)[E_{bi} + (p_i - s_i + r_i)\int_0^{Q^*_{bi}} F_{X_i}(x_i | y_i,$$
$$y_i > 0) dx_i + r_i(E(x_i | y_i, y_i > 0) - Q^*_{bi})], \quad i = 1, 2, \tag{2}$$

where $E(x_i | y_i, y_i > 0) = \int x_i f_{X_i}(x_i | y_i, y_i > 0) dx_i$ represents the buyer's expected demand and $F_{X_i}(x_i | y_i, y_i > 0)$ denote the CDF of demand given that the order is delivered late. Then, maximum expected system profit without coordination for ith supply chain, E_i, will be

$$E_i = E_{bi} + E_{si}$$
$$= EB(Q^*_{bi} | y_i, y_i \le 0) + Q^*_{bi}\left[\int_0^{\infty}(c_i(y_i) - t) g_{Y_i}(y_i) dy_i\right.$$
$$\left.+\int_{-\infty}^{0}(c_0 - t + hy_i) g_{Y_i}(y_i) dy_i\right], \quad i = 1, 2, \tag{3}$$

where Q^*_{bi} denotes the buyer's optimal order quantity for it supply chain.

We then consider how the suppliers in supply chain I and II apply horizontal coordination. If the suppliers from the two supply chains share their information for the horizontal coordination and control their production and stocking, then the lead time for supplier I is $Z_1 = k(Y_1+Y_2)$, $0 \le k \le 1$, and the lead time for supplier II is $Z_2 = (1 - k)(Y_1 + Y_2)$, where k is total inventory ratio. The maximum expected system profit with coordination for ith supply chain, E_{Ji}, is

$$E_{Ji} = E_{bi} + E_{Jsi}$$
$$= EB(Q^*_{bi} \mid y_i, y_i \le 0) + Q^*_{bi}[\int_0^{\infty}(c_i(z_i) - t)g_{Z_i}(z_i)dz_i$$
$$+\int_{-\infty}^{0}(c_0 - t + hz_i)g_{Z_i}(z_i)dz_i], i = 1, 2. \quad (4)$$

We then investigate the effects of coordination by an illustrative case study.

4. AN ILLUSTRATIVE CASE STUDY

In this section a practical probability distribution is used to explain the results of the previous section. The demand for ith supply chain is uniformly distributed over the range 0 and b_i, where b_i represents the number of time periods in the planning season. We define a period such that the expected demand is one unit per period. We assume that the order is completed by the supplier i at time y_i. When the order is delivered on time ($y_i \le 0$), during the planning period $[0, b_i]$, the largest uniform distribution demand x_i can be as large as b_i. Thus, the CDF of the buyer's demand is

$$F_{X_i}\left(x_i \mid y_i, y_i \le 0\right) = x_i / b_i, \quad i = 1, 2. \quad (5)$$

When there is late delivery ($y_i > 0$), then the selling period is reduced to $[y_i, b_i]$, the largest demand x_i is $b_i - y_i$. Thus

$$F_{X_i}\left(x_i \mid y_i, y_i > 0\right) = x_i / (b_i - y_i), \quad i = 1, 2. \quad (6)$$

Recall that the target delivery date for the supplier i is the start of the selling season of ith supply chain, at time 0. Let $\alpha_i \in [0,1]$ represents the degree of uncertainty in the lead-time of supplier i, the actual order completion date for supplier i will be uniformly distributed over the interval $(-\alpha_i b_i, \alpha_i b_i)$. If the suppliers from the two supply chains share their information for the horizontal coordination and control their production and stocking, then the lead time for supplier I is $Z_1 = k(Y_1+Y_2)$, $0 \le k \le 1$, and the lead time for supplier II is $Z_2 = (1-k)(Y_1+Y_2)$, where k is total inventory ratio. The PDF of Z_1 is

$$g_{Z_1}(z_1) = \begin{cases} \dfrac{\frac{z_1}{k} + \alpha_1 b_1 + \alpha_2 b_2}{4k\alpha_1 b_1 \alpha_2 b_2}, & k(-\alpha_1 b_1 - \alpha_2 b_2) \le z_1 < k(\alpha_1 b_1 - \alpha_2 b_2), \\ \dfrac{1}{2k\alpha_2 b_2}, & k(\alpha_1 b_1 - \alpha_2 b_2) \le z_1 < k(\alpha_2 b_2 - \alpha_1 b_1), \\ \dfrac{-\frac{z_1}{k} + \alpha_1 b_1 + \alpha_2 b_2}{4k\alpha_1 b_1 \alpha_2 b_2}, & k(\alpha_2 b_2 - \alpha_1 b_1) \le z_1 \le k(\alpha_1 b_1 + \alpha_2 b_2), \end{cases} \quad (7)$$

The PDF of Z_2 is

$$g_{Z_2}(z_2) = \begin{cases} \dfrac{\frac{z_2}{(1-k)} + \alpha_1 b_1 + \alpha_2 b_2}{4(1-k)\alpha_1 b_1 \alpha_2 b_2}, & (1-k)(-\alpha_1 b_1 - \alpha_2 b_2) \le z_2 < (1-k)(\alpha_1 b_1 - \alpha_2 b_2), \\ \dfrac{1}{2(1-k)\alpha_2 b_2}, & (1-k)(\alpha_1 b_1 - \alpha_2 b_2) \le z_2 < (1-k)(\alpha_2 b_2 - \alpha_1 b_1), \\ \dfrac{\frac{-z_2}{(1-k)} + \alpha_1 b_1 + \alpha_2 b_2}{4(1-k)\alpha_1 b_1 \alpha_2 b_2}, & (1-k)(\alpha_2 b_2 - \alpha_1 b_1) \le z_2 \le (1-k)(\alpha_1 b_1 + \alpha_2 b_2), \end{cases} \quad (8)$$

Without coordination, assuming fixed c_0, and t, one has

$$E_i = E_{bi} + E_{si}$$
$$= E_{bi} + Q^*_{bi}(\frac{p_i}{2} - \frac{1}{Q^*_{bi}}\{\frac{E_{bi}}{2} + \frac{p_i - s_i + r_i}{2\alpha_i b_i}\frac{Q^{*\,2}_{bi}}{2}[-\ln(1-\alpha_i)]$$
$$+ r_i[\frac{b_i(2-\alpha_i)}{8} - \frac{Q^*_{bi}}{2}]\} - t + \frac{c_0}{2} - \frac{\alpha_i b_i}{4}h), \quad i = 1, 2, \quad (9)$$

Where Q^*_{bi} satisfies
$F_{X_i}(Q^*_{bi} \mid y_i, y_i \le 0) = (p_i - c_0 + r_i)/(p_i - s_i + r_i)$.

With coordination, one has (please see appendix A for detail)

$$E_{J1} = E_{b1} + E_{Js1}$$
$$= E_{b1} + Q^*_{b1}\left[\int_0^{\infty}(c_1(z_1) - t)g_{Z_1}(z_1)dz_1 + \int_{-\infty}^{0}(c_0 - t + hz_1)g_{Z_1}(z_1)dz_1\right],$$

$$E_{J2} = E_{b2} + E_{Js2}$$
$$= E_{b2} + Q^*_{b2}\left[\int_0^{\infty}(c_2(z_2) - t)g_{Z_2}(z_2)dz_2 + \int_{-\infty}^{0}(c_0 - t + hz_2)g_{Z_2}(z_2)dz_2\right].$$

Two scenarios of the facility are considered. **Case(i)**: When $p_1 = p_2 = p$, $s_1 = s_2 = s$, $r_1 = r_2 = r$, $\alpha_1 = \alpha_2 = \alpha$, $b_1 = b_2 = b$, $b_1 = b_2 = b$, and $k = 1/2$, the basic condition of the facility is the same for the buyers and suppliers from two supply chains. Maximum expected system profit with coordination for ith supply chain is

$$E_{Ji} = E_{bi} + E_{Jsi}$$
$$= E_{bi} + Q^*_{bi}\{\frac{p}{2} - \frac{1}{Q^*_{bi}}[\frac{E_{bi}}{2} + \frac{Q^{*\,2}_{bi}}{2}(p - s + r)$$
$$(\frac{-\alpha\ln(1-\alpha) + \alpha + \ln(1-\alpha)}{\alpha^2 b}) + r(\frac{b(3-\alpha)}{12} - \frac{Q^*_{bi}}{2})]$$
$$-t + \frac{c_0}{2} - \frac{\alpha b}{6}h\}, i = 1, 2. \quad (10)$$

From Theorem 2, one can ensure maximum expected system profit with coordination will be better than without coordination.

Theorem 1. For a uniform demand and lead time distribution, if $p_1 = p_2 = p$, $s_1 = s_2 = s$, $r_1 = r_2 = r$, $\alpha_1 = \alpha_2 = \alpha$, $h_1 = h_2 = h$, $b_1 = b_2 = b$, and $k = 1/2$, then, the inventory holding cost per unit saved for supplier i without coordination is $H_i = \dfrac{\alpha b}{12} h$, $i = 1, 2$.
(Please see appendix B)

Lemma 1. For differentiable $f(y)$ and strictly decreasing with positive values on $[0, a]$,

$$g_1(y) = \frac{1}{2a}, y \in [0, a], \quad g_2(y) = \frac{-1}{a^2}(y - a), y \in [0, a]$$

$$\int_0^a f(y) g_1(y) dy \le \int_0^a f(y) g_2(y) dy \tag{11}$$

(Please see appendix C)

Lemma 2. For differentiable $f(y)$ and strictly increasing with positive values on $[-a, 0]$,

$$g_1(y) = \frac{1}{2a}, y \in [-a, 0], \quad g_2(y) = \frac{1}{a^2}(y + a), y \in [-a, 0]$$

$$\int_{-a}^0 f(y) g_1(y) dy \le \int_{-a}^0 f(y) g_2(y) dy. \tag{12}$$

(Please see appendix D)

Lemma 3.

$$c_i(y_i) = p - \frac{1}{Q^*_{bi}}[E_{bi} + (p - s + r)\frac{{Q^*_{bi}}^2}{2}\frac{1}{b - y_i} + r(\frac{b - y_i}{2} - Q^*_{bi})], i = 1, 2$$

is strictly decreasing on $[0, \alpha b]$.
(Please see appendix E)

Theorem 2. For uniform demand and lead time distribution, If $p_1 = p_2 = p$, $s_1 = s_2 = s$, $r_1 = r_2 = r$, $\alpha_1 = \alpha_2 = \alpha$, $h_1 = h_2 = h$, $b_1 = b_2 = b$, and $k = 1/2$, then

$$E_{Ji} \ge E_i, \quad i = 1, 2 .$$

(Please see appendix F)

Case (ii): When the parameters of the two supply chains are not exactly the same, then Q^*_{bi} and E_{bi} will be different, the expected total profit of the two supply chains with coordination is $E_{JT}(k) = E_{J1}(k) = E_{J1}(k)$. The optimal k^* can be derived when the conditions $\dfrac{dE_{JT}}{dk}(k^*) = 0$, $\dfrac{d^2 E_{JT}}{dk^2}(k^*) < 0$ are also satisfied.

5. NUMERICAL STUDY

The above model is demonstrated by the following examples and sensitivity analysis. In order to exploit the effect of the horizontal coordination further under uncertain demand and the lead-time, different parameters values are assumed. In Theorem 2, we proved that for the uniform distributed demand and lead-time, the maximum expected system profit with coordination is better than without coordination. In table 1, we showed the various parameter value used in the numerical studies. With identical parameters for the two supply chains, we found that when a_0 is close to t, $c_i(y_i)$ might be negative. This will not be practical. If α is higher (e.g. α = 0.5 or 0.7), the supplier's expected profit would often be negative and the suppliers would choose not to produce. When the parameters are different for the two suppliers with a significant discrepancy for their basic condition (e.g. $\alpha_1 b_1, \alpha_2 b_2$), the k* could be more than 1 or less than 0. This means that the horizontal coordination can still be achieved. When $0 \le k \le 1$, the total profit increases.
Since the buyer's optimal ordering quantity Q^*_{bi} is fixed, the buyer's profit E_{bi} remains the same. Figures 5-13 show, the percentage of profit increases with and without coordination. The amount in Figures 5-8 was calculated as $\left[(E_{Jsi} - E_{si}) / E_{si}\right] \cdot 100\%$ (Case (i)). The columns 4 and 6 in Figures 9-13 were calculated as $\left[(E_{Jsi} - E_{si}) / E_{si}\right]$ · 100%, also, column 8 was $\left[(E_{JsT} - E_{sT}) / E_{sT}\right] \cdot 100\%$ (Case (ii)).
Case (i) We next describe the numerical results with coordination by (a) two suppliers (b) three suppliers, with identical conditions.
(a) The basic conditions of the buyers and supplier are exactly the same for two supply chains with the ordering quantity from the buyers fixed. That is $p_1 = p_2 = p$, $s_1 = s_2 = s$, $r_1 = r_2 = r$, $\alpha_1 = \alpha_2 = \alpha$, $h_1 = h_2 = h$, $b_1 = b_2 = b$.

Example 1. Given p = 100, α = 0.1, b = 1000, s = 4.5, t = 15, r = 7.5, h = 0.001, a_0 = 45, k = 0.5, then Q^*_{bi} = 607 is derived, and E_{bi} = 1.521×10^4, E_{si} = 1.777×10^4, E_{Jsi} = 1.792×10^4, $E_{jsi} - E_{si}$ = 149, i = 1, 2. Supplier's expected profit increases by 0.84% after horizontal coordination.

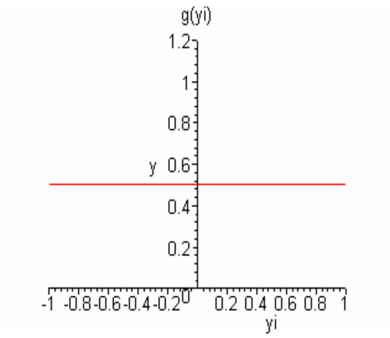


Figure 2. Graph with
PDF of Yi, Yi i, i, d~U(−1, 1)

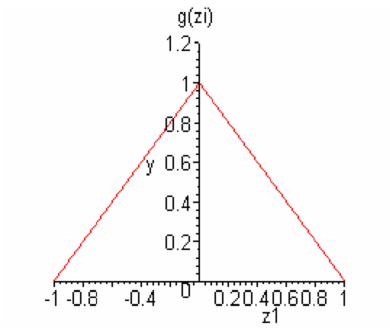


Figure 3. Graph with
PDF of $Zi = (Y_1+Y_2)/2$

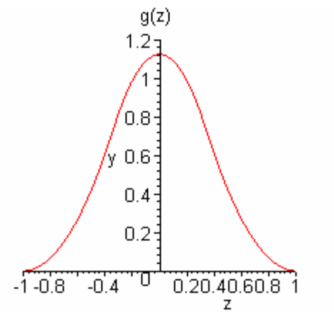


Figure 4. Graph with
PDF of $Z=(Y_1+Y_2+Y_3)/3$

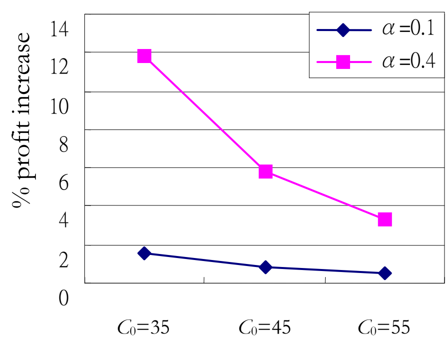
Table 1. Parameter values

Parameter	Value
Delivery time variability factor（α）	0.1, 0.4
Unit cost to buyer（c_o）	35, 45, 55
Unit salvage value（s）	0.1t, 0.3t, 0.5t
Unit shortage cost（r）	0.5t, t, 2t
Unit production cost（t）	5, 15, 25
Unit revenue to buyer（p）	100
Period holding cost factor（h）	0.001
Upper bound on demand（b）	1000

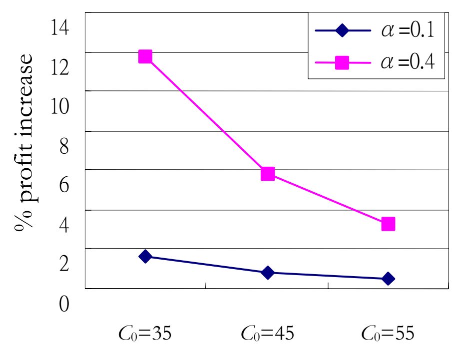
Sensitivity analysis

The changes in parameters in example 1 for fixed *b* and *p* are discussed as follows:

$s = 1.5$, $r = 7.5$, $t = 15$				
C_0	α	Q^*_{bi}	$E_{Jsi} - E_{si}$	% profit increase
35	0.1	684	204	1.6 %
45	0.1	590	144	0.83 %
55	0.1	495	93	0.4 %
35	0.4	684	1240	11.8 %
45	0.4	590	892	5.8 %
55	0.4	495	595	3.2 %



$s = 4.5$, $r = 7.5$, $t = 15$				
C_0	α	Q^*_{bi}	$E_{Jsi} - E_{si}$	% profit increase
35	0.1	704	211	1.6 %
45	0.1	607	149	0.84 %
55	0.1	510	97	0.48 %
35	0.4	704	1280	11.8 %
45	0.4	607	922	5.8 %
55	0.4	510	616	3.3 %



s = 7.5, r = 7.5, t = 15				
C_0	α	Q^*_{bi}	$E_{Jsi} - E_{si}$	% profit increase
35	0.1	725	218	1.6 %
45	0.1	625	155	0.84 %
55	0.1	525	101	0.48 %
35	0.4	725	1322	11.8 %
45	0.4	625	953	5.8 %
55	0.4	525	638	3.3 %

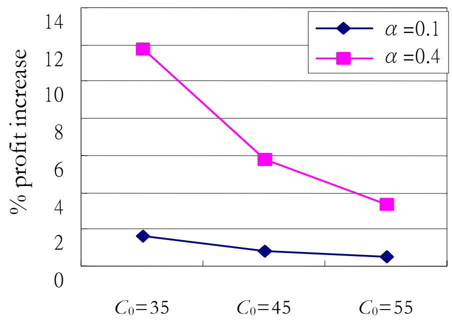
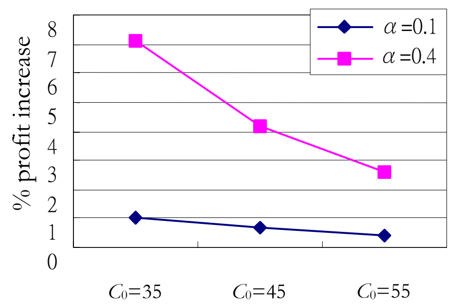
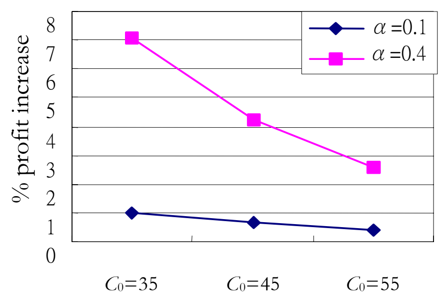


Figure 5. The effect of salvage value s on the profit increase due to coordination.

s = 2.5, r = 2.5, t = 5				
C_0	α	Q^*_{bi}	$E_{Jsi} - E_{si}$	% profit increase
35	0.1	675	206	1 %
45	0.1	575	147	0.65 %
55	0.1	475	98	0.42 %
35	0.4	675	1214	7.1 %
45	0.4	575	872	4.2 %
55	0.4	475	585	2.6 %



s = 2.5, r = 5, t = 5				
C_0	α	Q^*_{bi}	$E_{Jsi} - E_{si}$	% profit increase
35	0.1	683	206	1 %
45	0.1	585	146	0.63 %
55	0.1	488	96	0.39 %
35	0.4	683	1233	7.1 %
45	0.4	585	887	4.2 %
55	0.4	488	593	2.6 %



s = 2.5, r = 10, t = 5				
C_0	α	Q^*_{bi}	$E_{Jsi} - E_{si}$	% profit increase
35	0.1	698	206	1 %
45	0.1	605	145	0.61 %
55	0.1	512	93	0.36 %
35	0.4	698	1273	7.2 %
45	0.4	605	917	4.2 %
55	0.4	512	612	2.5 %

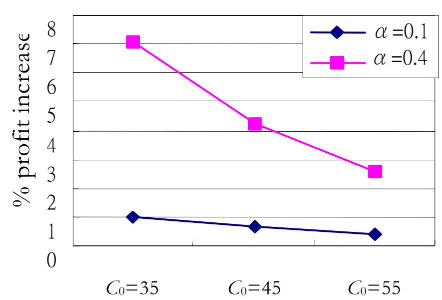
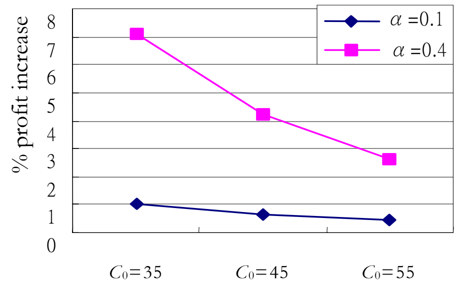
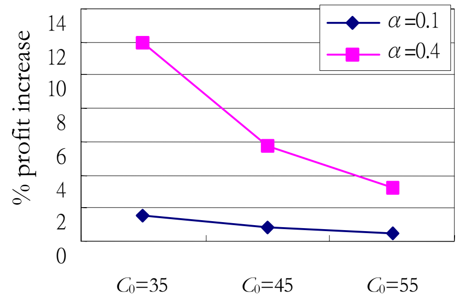


Figure 6. The effect of goodwill cost r on the profit increase due to coordination.

s = 1.5, r = 5, t = 5				
C_0	α	Q^*_{bi}	$E_{Jsi} - E_{si}$	% profit increase
35	0.1	676	204	1 %
45	0.1	580	145	0.63 %
55	0.1	483	95	0.39 %
35	0.4	676	1221	7.1 %
45	0.4	580	877	4.2 %
55	0.4	483	586	2.6 %



s = 7.5, r = 25, t = 25				
C_0	α	Q^*_{bi}	$E_{Jsi} - E_{si}$	% profit increase
35	0.1	724	211	1.5 %
45	0.1	633	148	0.79 %
55	0.1	543	93	0.43 %
35	0.4	724	1342	12 %
45	0.4	633	971	5.8 %
55	0.4	543	650	3.2 %



s = 7.5, r = 25, t = 25				
C_0	α	Q^*_{bi}	$E_{Jsi} - E_{si}$	% profit increase
35	0.1	766	221	3.1 %
45	0.1	681	153	1.2 %
55	0.1	596	94	0.53 %
35	0.4	766	1475	36 %
45	0.4	681	1080	9.8 %
55	0.4	596	732	4.5 %

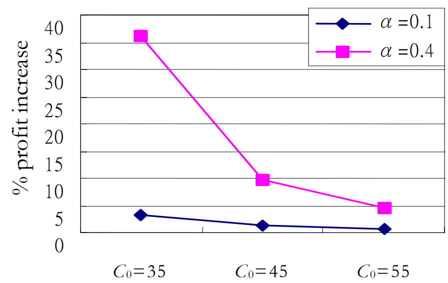


Figure 7. The effect of production cost t on the profit increase due to coordination.

s = 4.5, r = 7.5, t = 15						
			two suppliers		three suppliers	
C_0	α	Q^*_{bi}	$E_{Jsi} - E_{si}$	% profit increase	$E_{Jsi} - E_{si}$	% profit increase
35	0.1	704	211	1.6 %	289	2.1 %
45	0.1	607	149	0.84 %	204	1.1 %
55	0.1	510	97	0.48 %	132	0.65 %
35	0.4	704	1280	11.8 %	1726	15.9 %
45	0.4	607	922	5.8 %	1243	7.8 %
55	0.4	510	616	3.3 %	830	4.4 %

Figure 8. The comparison of profit with coordination for two suppliers vs. three suppliers.

(b) The basic conditions of the buyers and suppliers are the same for three supply chains with the ordering quantity from the buyers fixed (i.e. $p_i = p$, $r_i = r, \ldots$, i = 1, 2, 3). The lead-time is $Z = \frac{1}{3}(y_1 + y_2 + y_3)$, the PDF is

$$g_Z(z) = \begin{cases} \frac{9}{4a^3}\left(\frac{3}{4}z^2 + \frac{3}{2}az + \frac{3}{4}a^2\right), & -a \le z < -\frac{a}{3} \\ \frac{9}{4a^3}\left(\frac{-3}{2}z^2 + \frac{1}{2}a^2\right), & -\frac{a}{3} \le z < \frac{a}{3} \\ \frac{9}{4a^3}\left(\frac{3}{4}z^2 - \frac{3}{2}az + \frac{3}{4}a^2\right), & \frac{a}{3} \le z \le a. \end{cases} \tag{13}$$

where $a = \alpha b$. The following table compares the profit of coordination for two suppliers and three suppliers. In Figure 5, with holding goodwill cost r and production cost t, we found the change in salvage value s has no significant influence to the percentage of increase in supplier's profit with coordination. This is because the change in s is limited to be less than t. It is apparent that the profit difference by this change is less than a_0 and α. When a_0 increases, $E_{Jsi} - E_{si}$ drops, the percentage of profit increase also drops, which means the higher the supplier's unit profit the poorer the coordination effect. As a comparison, when α increases, there will be a better $E_{Jsi} - E_{si}$ value and a higher percentage of profit increase. That means better effect of coordination is achieved when the supply delivery is less efficient. In Figure 6, when s and t are fixed, the goodwill cost r has no significant influence to the percentage of profit increase in supplier's profit with coordination. In Figure 7, the effect of production cost, t, on the profit increase due to coordination is shown. Since the percentage profit increase is higher than Figures 5 and 6 we can conclude that the production cost, t, is more critical than s and r in profit improvement. In Figures 5-7, when a_0 increases, the percentage profit increase elevates; this means that the higher the supplier's profit the worse the effect of coordination. Figure 8 shows the expected profit of coordination between two suppliers and three suppliers. It is obvious that the expected profit increases as the number of suppliers increases. This is because more suppliers in the coordination reduce the probability of delivery uncertainty. The condition can be appreciated from the shape of PDF (in Figures 2-4) that centralize as the number of coordinated suppliers increases, and this reduces the holding and goodwill costs (e.g. if $\alpha_i b_i = 1$, that is $Y_i \overset{i.i.d}{\sim} U(-1, 1)$, $Z_i = (Y_1+Y_2)/2$, $Z = (Y_1+Y_2+Y_3)/3$ the expectations of Y_i, Z_i, Z all equal to 0, the variances are 1/3,1/6,1/9 respectively).

Case (ii) If the parameters for the two supply chains are different, the optimal ordering quantity of the buyer will change. If we coordinate horizontally (choose an appropriate k value), the expected total profit for the two suppliers can increase. One party will compensate for the loss of its partner; both suppliers share the benefit of coordination.

Example 2. $p_1 = 100$, $s_1 = 4.5$, $r_1 = 7.5$, $b_1 = 1000$, $p_2 = 110$, $s_2 = 4.5$, $r_2 = 15$, $b_2 = 900$, $\alpha_1 = 0.1$, $\alpha_2 = 0.2$, $b = 0.001$, $C_0 = 45$, $t = 15$.

With coordination, the supplier I takes k ($0 < k < 1$) part of the total inventory and the supplier II takes $(1 - k)$ part. By the optimal conditions, $\frac{dE_{JsT}}{dk}(k^*) = 0$ and $\frac{d^2E_{JsT}}{dk^2}(k^*) < 0$, we derive the optimal $k^* = 0.81$ for the best total profit. The maximum expected profit for supplier I is $E_{Js1} = 17466$ and the maximum expected profit for supplier II is $E_{Js2} = 17732$, with the maximum expected total profit for both suppliers is $E_{JsT} = E_{Js1} + E_{Js2} = 35198$.The maximum expected total profit without coordination is $E_{sT} = E_{s1} + E_{s2} = 34627$. The profit gain after the coordination is 571 (1.6%); the profit of supplier I drops by 1.7% and the profit supplier II increases by 5.2%. There should be a contract for supplier II to compensate the loss of supplier I so that both parties will benefit from the coordination.

Sensitivity analysis

Sensitivity analysis with different parameter changes in example 2 are carried out as follows:

p_1=100,s_1=4.5,r_1=7.5,b_1=1000,p_2=110,s_2=4.5,r_2=15,b_2=900, b=0.001,C_0=45,t=15							
(a_1,a_2)	K^*	$E_{Js1} - E_{s1}$	% profit increase	$E_{Js2} - E_{s2}$	% profit increase	$E_{JsT} - E_{sT}$	% total profit increase
(0.1,0.15)	0.87	−197	−1.1 %	660	3.8 %	463	1.3 %
(0.1,0.2)	0.81	−308	−1.7 %	879	5.2 %	571	1.6 %
(0.1,0.25)	0.77	−571	−3.2 %	1255	7.6 %	683	2 %
(0.12,0.15)	0.84	−126	−0.7 %	626	3.6 %	500	1.4 %
(0.12,0.2)	0.79	−229	−1.3 %	848	5 %	619	1.8 %
(0.12,0.25)	0.76	−357	−2 %	1103	6.7 %	746	2.2 %

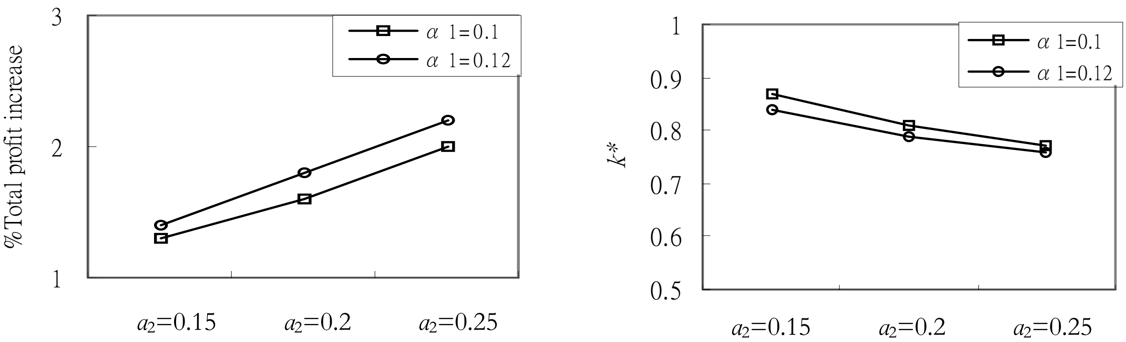


Figure 9. The effect of α_i on the total profit increase due to coordination.

p_1=100, s_1=4.5, r_1=7.5, b_1=1000, p_2=110, s_2=4.5, r_2=15, b_2=800, h=0.001, C_0=45, t=15							
(a_1,a_2)	K^*	$E_{Js1}-E_{s1}$	% profit increase	$E_{Js2}-E_{s2}$	%profit increase	$E_{JsT}-E_{sT}$	% total profit increase
(0.1,0.15)	0.9	−167	−0.9 %	604	4 %	437	1.3 %
(0.1,0.2)	0.84	−263	−1.5 %	804	5.4 %	541	1.7 %
(0.1,0.25)	0.8	−375	−2.1 %	1029	7 %	654	2 %
(0.12,0.15)	0.87	−105	−0.6 %	573	3.8 %	468	1.4 %
(0.12,0.2)	0.82	−190	−1.1 %	776	5.2 %	585	1.8 %
(0.12,0.25)	0.79	−302	−1.7 %	1008	6.9 %	706	2.2 %

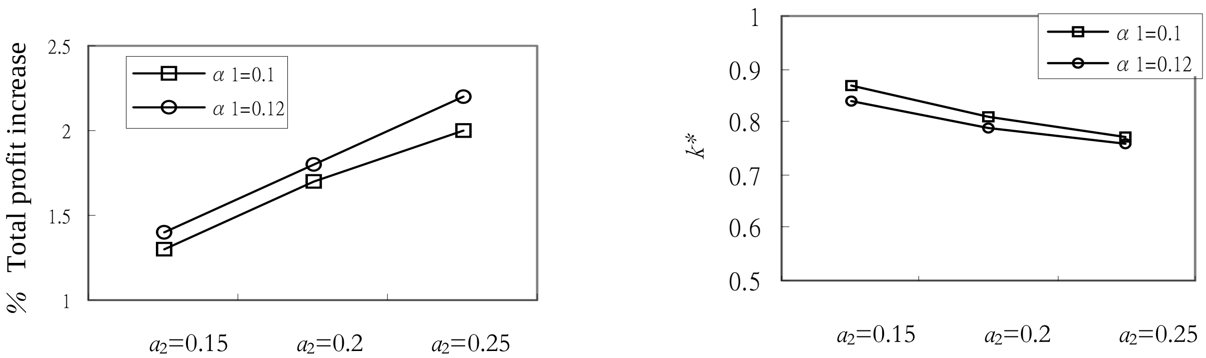


Figure 10. The effect of the upper bound on demand b_i on the total profit increase due to coordination.

p_1=100, s_1=4.5, r_1=7.5, b_1=1000, p_2=90,s2=4.5, r_2=15, b_2=900, h=0.001, C_0=45, t=15							
(a_1,a_2)	K^*	$E_{Js1}-E_{s1}$	% profit increase	$E_{Js2}-E_{s2}$	% profit increase	$E_{JsT}-E_{sT}$	% total profit increase
(0.1,0.15)	0.1	366	2.1 %	−44	−0.3 %	321	0.96 %
(0.1,0.2)	0.19	276	1.5 %	70	0.45 %	346	1 %
(0.1,0.25)	0.25	180	1 %	195	1.3 %	375	1.1 %
(0.12,0.15)	0.14	428	2.4 %	−58	−0.36 %	370	1.1 %
(0.12,0.2)	0.22	337	1.9 %	66	0.4 %	403	1.2 %
(0.12,0.25)	0.27	300	1.7 %	136	0.9 %	436	1.3 %

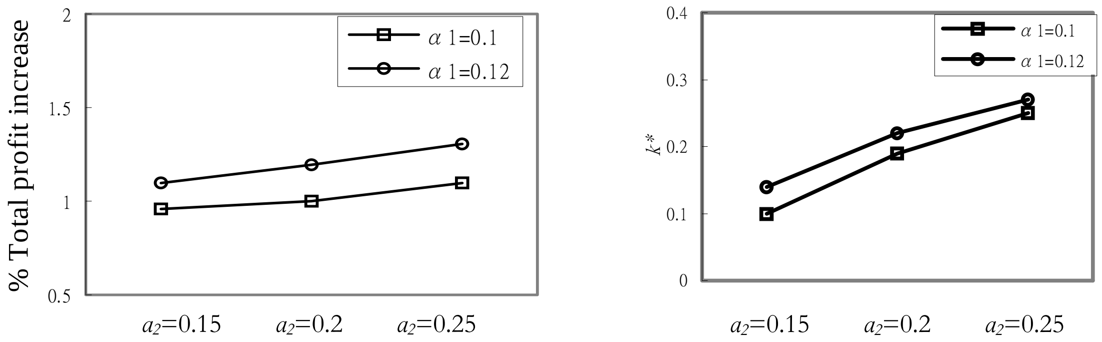


Figure 11. The effect of unit revenue to buyer p_i on the total profit increase due to coordination.

p_1=100,s_1=4.5,r_1=7.5,b_1=1000, p_2=110,s_2=7.5,r_2=15,b_2=900, h=0.001,C_0=45,t=15							
(a_1,a_2)	K^*	$E_{Js1}-E_{s1}$	% profit increase	$E_{Js2}-E_{s2}$	% profit increase	$E_{JsT}-E_{sT}$	% total profit increase
(0.1,0.15)	0.92	−238	−1.3 %	724	4.1 %	486	1.4 %
(0.1,0.2)	0.85	−351	−2 %	952	5.5 %	601	1.7 %
(0.1,0.25)	0.8	−475	−2.7 %	1203	7.1 %	728	2.1 %
(0.12,0.15)	0.89	−171	−0.96 %	693	3.9 %	522	1.5 %
(0.12,0.2)	0.83	−274	−1.5 %	922	5.3 %	648	1.9 %
(0.12,0.25)	0.79	−399	−2.3 %	1181	7 %	782	2.3 %

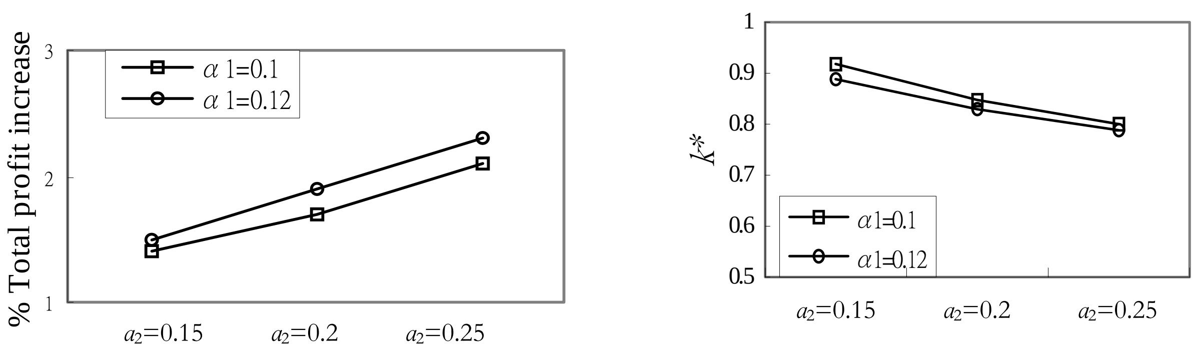


Figure 12. The effect of unit salvage value s_i on the total profit increase due to coordination.

p_1=100,s_1=4.5,r_1=7.5,b_1=1000, p_2=110,s_2=4.5,r_2=7.5,b_2=900, h=0.001,C_0=45,t=15							
(a_1,a_2)	K^*	$E_{Js1}-E_{s1}$	% profit increase	$E_{Js2}-E_{s2}$	% profit increase	$E_{JsT}-E_{sT}$	% total profit increase
(0.1,0.15)	0.89	−213	−1.2 %	676	4.1 %	463	1.3 %
(0.1,0.2)	0.82	−319	−1.8 %	883	5.4 %	564	1.7 %
(0.1,0.25)	0.77	−435	−2.4 %	1108	7 %	673	2 %
(0.12,0.15)	0.86	−144	−0.8 %	644	3.9 %	500	1.5 %
(0.12,0.2)	0.8	−240	−1.4 %	852	5.2 %	612	1.8 %
(0.12,0.25)	0.76	−357	−2 %	1085	6.8 %	728	2.2 %

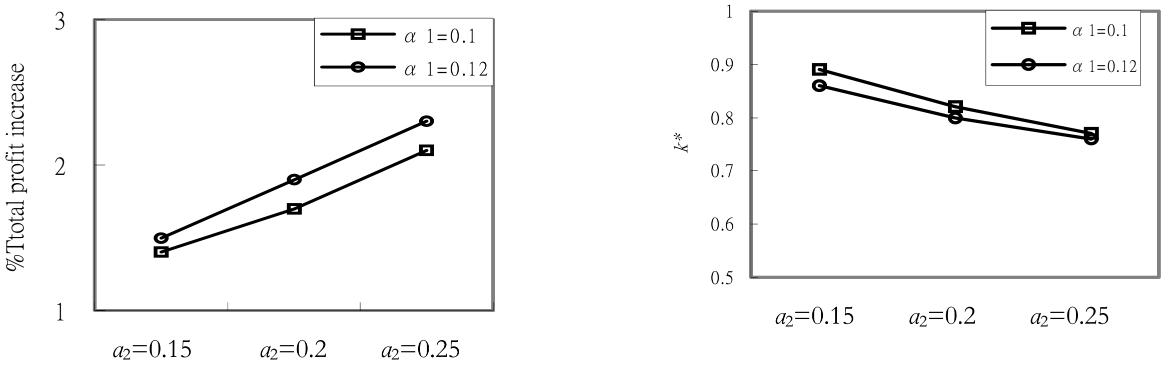


Figure 13. The effect of unit shortage cost r_i on the total profit increase due to coordination.

In case (ii) the parameters for the two supply chains are different, so different combinations of parameters can yield different total inventory ratio k^*. We use the parameters in Figure 9 as the standard and compare different percentage of profit increase effect with changes in parameters from Figures 10 to13. In Figure 9, $k^* > 0.5$, the expected profit for supplier I drops while the expected profit for supplier II increases. When α_2 value increases, k^* becomes smaller and the percentage of profit increase for supplier I becomes lower. When α_1 increases, k^* becomes smaller and the percentage profit increase for supplier I becomes higher. When α_i (i = 1, 2) increases, the percentage total profit increases; this means that supplier with less efficient delivery will obtain a larger benefit as a result of coordination. In Figure 10, when b_2 is changed from 900 to 800, k^* becomes larger than the one in Figure 9 and the percentage of profit increase for supplier I and II is higher too. In Figure 11, when p_2 changes from 110 to 90, k^* changes from "> 0.5" to "< 0.5" ; this implies the profit gain for supplier I is more important. In Figure 12 , when s_2 increases from 4.5 to 7.5, k^* becomes larger ($k^* > 0.5$) which means the importance to the profit gain of supplier II increases. In Figure 13, when r_2 decreases from 15 to 7.5, k^* is higher than that in Figure 9. The importance of lower r_2 to profit gain of supplier II also increases, and the effect of horizontal coordination will be better.

6. CONCLUSION

Two (or more) supply chains where the suppliers share information of their production, inventory and delivery status are considered. Due to uncertain supplier deliveries and lead time, horizontal supplier coordination is implemented to reduce lead time and deliveries uncertainty, and thus reduce the safety inventory due to risk pooling effect. With this type of cooperation, both suppliers benefit in the long run. We have made two contributions in this study. Firstly we design a compensation mechanism such that both partners benefit. The second is to develop a mean to monitor the sharing of inventory information and manipulating the stocks jointly in both partners. Besides horizontal coordination among the suppliers, if the suppliers also agrees to coordinate vertically with the buyer (Weng, 2003), total profits can be improved further.

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REFERENCES

1. Benton, W. and Park, S. (1996). A classification of literature on determining the lot size under quantity discounts. *European Journal of Operational Research*, 92: 219-238.
2. Buzzell, R. and Ortmeyer, G. (1995). Channel partnerships streamline distribution. *Sloan Management Review*, 36: 85-95.
3. Fites, D. (1996). Make your dealers your partners. *Harvard Business Review*, 74: 84-95.
4. Goyal, S. and Gupta, Y. (1989). Integrated inventory models, The buyer–vendor coordination. *European Journal of Operational Research*, 41: 261-269.
5. Hadley, G. and Whitin, T. (1963). *Analysis of Inventory Systems*. Prentice-Hall, Englewood Cliffs, NJ.
6. Jeuland, A. and Shugan, S. (1983). Managing channel profits. *Marketing Science*, 2: 239-272.
7. Kohli, R. and Park, H. (1994). Coordinating buyer–seller transactions across multiple products. *Management Science*, 40: 1145-1150.
8. Lal, R. and Staelin, R. (1984). An approach for developing an optimal discount pricing policy. *Management Science*, 30: 1524-1539.
9. Oren, S., Smith, S., and Wilson, R. (1983). Competitive nonlinear tariffs. *Journal of Economic Theory*, 29: 49-71.
10. Rardin, R.L. (1998). *Optimization in Operation Research*. Prentice Hall, New Jersey.
11. Tirole, J. (1993). *The Theory of Industrial Organization*. The MIT Press, Cambridge, MA.

12. Weng, Z.K. (1995). Channel coordination and quantity discounts. *Management Science*, 41: 1509-1522.
13. Weng, Z.K. (2003). Coordinated ordering Decisions for Short Lifecycle Products with uncertainty in delivery time and demand. *European Journal of Operational Research*, 151: 12-24.
14. Weng, Z.K. (2004). Coordinating order quantities between the manufacturer and the buyer: a generalized newsvendor model. *European Journal of Operational Research*, 156: 148-161.
15. Whang, S. (1992). Contracting for software development. *Management Science*, 38: 307-324.

Appendix A

$$
\begin{aligned}
E_{J1} &= E_{b1} + E_{Js1} \\
&= E_{b1} + Q_{b1}^{*}\Big[\int_0^{\infty}(c_1(z_1)-t)g_{Z_1}(z_1)dz_1 + \int_{-\infty}^{0}(c_0 - t + hz_1)g_{Z_1}(z_1)dz_1\Big] \\
&= E_{b1} + Q_{b1}^{*}\Big[\frac{1}{2k\alpha_2 b_2}\{p_1 k(\alpha_2 b_2 - \alpha_1 b_1) - \frac{1}{Q_{b1}^{*}}[E_{b1}k(\alpha_2 b_2 - \alpha_1 b_1) + (p_1 - s_1 + r_1)\frac{Q_{b1}^{*\,2}}{2} \\
&\times \ln\frac{b_1}{k(\alpha_2 b_2 - \alpha_1 b_1) - b_1} + r_1\Big(\frac{k(\alpha_2 b_2 - \alpha_1 b_1)[2b_1 - k(\alpha_2 b_2 - \alpha_1 b_1)]}{4} \\
&- Q_{b1}^{*}\, k(\alpha_2 b_2 - \alpha_1 b_1)\Big)]\} + \frac{\alpha_1 b_1 + \alpha_2 b_2}{4k\alpha_1 b_1 \alpha_2 b_2}\{2p_1 k\alpha_1 b_1 - \frac{1}{Q_{b1}^{*}}[2E_{b1}k\alpha_1 b_1 + (p_1 - s_1 + r_1) \\
&\times \frac{Q_{b1}^{*\,2}}{2}\ln\frac{k(\alpha_2 b_2 - \alpha_1 b_1) - b_1}{k(\alpha_2 b_2 + \alpha_1 b_1) - b_1} + r_1(k\alpha_1 b_1(b_1 - k\alpha_2 b_2) - 2Q_{b1}^{*}k\alpha_1 b_1)]\} \\
&- \frac{1}{4k^2\alpha_1 b_1 \alpha_2 b_2}\{2p_1 k^2\alpha_2 b_2 \alpha_1 b_1 - \frac{1}{Q_{b1}^{*}}[2E_{b1}k^2\alpha_2 b_2 \alpha_1 b_1 + (p_1 - s_1 + r_1)\frac{Q_{b1}^{*\,2}}{2} \\
&\times\Big(-2k\alpha_1 b_1 - b_1\ln\frac{k(\alpha_2 b_2 + \alpha_1 b_1) - b_1}{k(\alpha_2 b_2 - \alpha_1 b_1) - b_1}\Big) + r_1\Big[\frac{\alpha_1 b_1(3b_1 k^2\alpha_2 b_2 - k^3(3\alpha_2^2 b_2^2 + \alpha_1^2 b_1^2))}{3} \\
&- 2Q_{b1}^{*}k^2\alpha_2 b_2 \alpha_1 b_1\Big]]\} - \frac{t}{2} + \frac{c_0 - t}{2} - k\Big(\frac{\alpha_1^2 b_1^2 + 3\alpha_2^2 b_2^2}{12\alpha_2 b_2}\Big)h\Big].
\end{aligned} \tag{14}
$$

$$
\begin{aligned}
E_{J2} &= E_{b2} + E_{Js2} \\
&= E_{b2} + Q_{b2}^{*}\Big[\int_0^{\infty}(c_2(z_2)-t)g_{Z_2}(z_2)dz_2 + \int_{-\infty}^{0}(c_0 - t + hz_2)g_{Z_2}(z_2)dz_2\Big] \\
&= E_{b2} + Q_{b2}^{*}\Big[\frac{1}{2(1-k)\alpha_2 b_2}\{p_2(1-k)(\alpha_2 b_2 - \alpha_1 b_1) - \frac{1}{Q_{b2}^{*}}[E_{b2}(1-k)(\alpha_2 b_2 - \alpha_1 b_1) + (p_2 - s_2 + r_2) \\
&\times \frac{Q_{b2}^{*\,2}}{2}\ln\frac{b_2}{(1-k)(\alpha_2 b_2 - \alpha_1 b_1) - b_2} + r_2\Big(\frac{(1-k)(\alpha_2 b_2 - \alpha_1 b_1)[2b_2 - (1-k)(\alpha_2 b_2 - \alpha_1 b_1)]}{4} \\
&- Q_{b2}^{*}(1-k)(\alpha_2 b_2 - \alpha_1 b_1)\Big)]\} + \frac{\alpha_1 b_1 + \alpha_2 b_2}{4(1-k)\alpha_1 b_1 \alpha_2 b_2}\{2p_2(1-k)\alpha_1 b_1 - \frac{1}{Q_{b2}^{*}}[2E_{b2}(1-k)\alpha_1 b_1 \\
&+ (p_2 - s_2 + r_2)\frac{Q_{b2}^{*\,2}}{2}\ln\frac{(1-k)(\alpha_2 b_2 - \alpha_1 b_1) - b_2}{(1-k)(\alpha_2 b_2 + \alpha_1 b_1) - b_2} + r_2((1-k)\alpha_1 b_1(b_2 - (1-k)\alpha_2 b_2) \\
&- 2Q_{b2}^{*}(1-k)\alpha_1 b_1)]\} - \frac{1}{4(1-k)^2\alpha_1 b_1 \alpha_2 b_2}\{2p_2(1-k)^2\alpha_2 b_2 \alpha_1 b_1 - \frac{1}{Q_{b2}^{*}} \\
&\times[2E_{b2}(1-k)^2\alpha_2 b_2 \alpha_1 b_1 + (p_2 - s_2 + r_2)\frac{Q_{b2}^{*\,2}}{2}\Big(-2(1-k)\alpha_1 b_1 - b_2\ln\frac{(1-k)(\alpha_2 b_2 + \alpha_1 b_1) - b_2}{(1-k)(\alpha_2 b_2 - \alpha_1 b_1) - b_2}\Big) \\
&+ r_2\Big[\frac{\alpha_1 b_1(3b_2(1-k)^2\alpha_2 b_2 - (1-k)^3(3\alpha_2^2 b_2^2 + \alpha_1^2 b_1^2))}{3} \\
&- 2Q_{b2}^{*}(1-k)^2\alpha_2 b_2 \alpha_1 b_1\Big]]\} - \frac{t}{2} + \frac{c_0 - t}{2} - (1-k)\Big(\frac{\alpha_1^2 b_1^2 + 3\alpha_2^2 b_2^2}{12\alpha_2 b_2}\Big)h\Big].
\end{aligned} \tag{15}
$$

Appendix B
Proof of Theorem 1:

$$H_i = \int_{-\infty}^{0} hz_i g_{Z_i}(z_i)dz_i - \int_{-\infty}^{0} hy_i g_{Y_i}(y_i)dy_i,\ i = 1,2, \tag{16}$$

where

$$g_{Z_i}(z_i) = \begin{cases} \dfrac{z_i}{\alpha^2 b^2} + \dfrac{1}{\alpha b}, & -\alpha b \le z_i < 0, \\ \dfrac{-z_i}{\alpha^2 b^2} + \dfrac{1}{\alpha b}, & 0 \le z_i \le \alpha b, \end{cases}$$

and

$$g_{Yi}(y_i) = \frac{1}{2\alpha b}, -\alpha b \le y_i \le \alpha b.$$

Therefore, one has

$$H_i = \int_{-\infty}^{0} hy\left(\frac{y+\alpha b}{\alpha^2 b^2} - \frac{1}{2\alpha b}\right)dy = \frac{\alpha b}{12} h.$$

Appendix C

Proof of Lemma 1:

$$\begin{aligned} & \int_0^a f(y)g_2(y)dy - \int_0^a f(y)g_1(y)dy \\ = \ & \int_0^a f(y)\left(\frac{-1}{a^2}y + \frac{1}{2a}\right)dy \\ = \ & \frac{1}{a^2}\left[\int_0^a F(y)dy - \int_0^a \frac{1}{2}F(a)dy\right] (\text{where } \mathrm{F(y)} = \int_0^y f(t)dt, 0 \le y \le a) \\ = \ & \frac{1}{a^2}\left[\int_0^a F(y)dy - \int_0^a \frac{F(a)}{a} ydy\right] \\ = \ & \frac{1}{a^2}\left[\int_0^a F(y) - \frac{F(a)}{a} ydy\right]. \end{aligned} \tag{17}$$

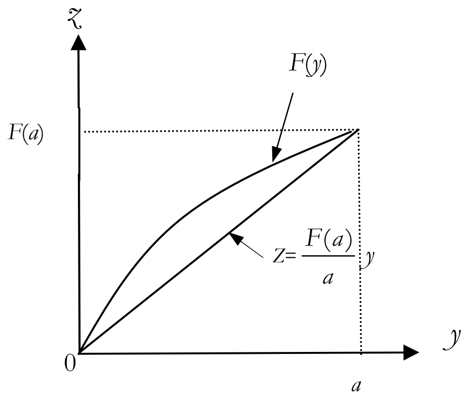

Figure 14. Shape of $F(y)$

Since $F''(y) = f'(y)$, the strictly decreasing property of $f(y)$ implies that $F''(y) < 0$.
Concavity is shown on [0, a] (see Figure .14).
Since $F(y) - \frac{F(a)}{a} y \ge 0$ (Rardin, 1998), the proof is completed.

Appendix D

Proof of Lemma 2:
The proof is analogous to Lemma 1.

Proof of Lemma 3:

Since $F_{X_i}\left(x_i \,|\, y_i, y_i \le 0\right) = {x_i}/{b}$, and $F_{X_i}(Q^*_{bi} \,|\, y_i, y_i \le 0) = (p - c_0 + r)/(p - s + r)$, i.e.
$Q^*_{bi} / b = (p - c_0 + r) / (p - s + r)$, or $Q^*_{bi} = b(p - c_0 + r) / (p - s + r)$, then

$$\begin{aligned} c_i'(y_i) &= \frac{-(p-s+r)Q^*_{bi}}{2(b-y_i)^2} + \frac{r}{2Q^*_{bi}} \\ &= \frac{-b(p-c_0+r)}{2(b-y_i)^2} + \frac{r}{2b}\frac{p-s+r}{p-c_0+r} \end{aligned}$$

$$\begin{aligned} &\le \frac{-b(p-c_0+r)}{2b^2}+\frac{r}{2b} \qquad (Note:\ b \ge b-y_i,\ c_0 \ge s,\ p \ge c_0) \\ &= \frac{-(p-c_0)}{2b} < 0, \quad y_i \in (0, \alpha b). \end{aligned} \tag{18}$$

It implies that $c_i(y_i)$ is strictly decreasing.

Appendix F

Proof of Theorem 2:

$$\begin{aligned} &E_{Ji}-E_i \\ &\quad = Q_{bi}^*\{\int_0^{\alpha b}[c_i(y)-t]g_2(y)dy - \int_0^{\alpha b}[c_i(y)-t]g_1(y)dy \quad + \int_{-\alpha b}^{0}(c_0-t+hy)g_2(y)dy \\ &\quad - \int_{-\alpha b}^{0}(c_0-t+hy)g_1(y)dy\}, i=1,2, \end{aligned} \tag{19}$$

where

$$g_1(y)=\frac{1}{2\alpha b}, \quad y \in [-\alpha b, \alpha b]$$

$$g_2(y)=\begin{cases} \dfrac{y}{\alpha^2 b^2}+\dfrac{1}{\alpha b}, & -\alpha b \le y < 0, \\ \dfrac{-y}{\alpha^2 b^2}+\dfrac{1}{\alpha b}, & 0 \le y \le \alpha b. \end{cases}$$

Since $c_i(y)-t$ is strictly decreasing in $[0, \alpha b]$ and c_0-t+hy is strictly increasing in $[-\alpha b, 0]$, $a=\alpha b$, and apply Lemma 1 and Lemma 2, we have $E_{Ji}-E_i \ge 0$.