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Optimization of Fuzzy Production Inventory Model with Repairable Defective Products Under Crisp or Fuzzy Production Quantity

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Abstract—In this paper, we introduce a fuzzy Economic Production Quantity (EPQ) model with defective products that can be repaired. In this model, we consider a fuzzy opportunity cost, trapezoidal fuzzy costs and quantities into the traditional production inventory model. We use Function Principle and Graded Mean Integration Representation Method to find optimal economic production quantity of the fuzzy production inventory model.

Keywords—Fuzzy production inventory model, Economic production quantity, Function principle, Graded mean integration representation method, Optimization, Imperfect production, Defective products

1. INTRODUCTION

The fuzzy set concept has been used to treat the classical inventory model recently. Park (1987) used fuzzy inventory cost in economic order quantity model. Chang (1999) discussed how to get the economic production quantity, when the quantity of demand is uncertain. Chen et al. (2000b) established a fuzzy economic production model to treat the inventory problem with all the parameters and variables which are fuzzy numbers. Hsieh (2002), Lee et al. (1998), Lin et al. (2000) also wrote some papers about fuzzy production model.

In the real world, imperfect products cannot be avoided in most production process. It is reasonable to discuss the models with imperfect production process. Recently, Salameh et al. (2000), Mohamed (2002), Lin et al. (2003), Chung et al. (2003), Lee (2005) have written papers about imperfect production process. From the previous researchers, we can find some papers discussed fuzzy costs, but they did not discuss imperfect production model, the other researchers discussed imperfect production processes but did not discuss fuzzy cost. Therefore a Fuzzy Economic Production Quantity model with imperfect products that can be repaired is a good topic for us to treat vague environment problems.

In viewing of production management, the cost includes explicit cost and implicit cost. This paper includes both implicit costs and explicit costs in the fuzzy economic production quantity model, and use Function Principle to calculate the fuzzy total production inventory cost (FTPIC). The Graded Mean Integration Representation Method is used to defuzzify the FTPIC.

This paper is organized as following: In section 2, the methodology is introduced. In section 3, two different production inventory models are discussed. In section 4, a numerical example is given to test the proposal model, and in section 5, we give a conclusion of our discussion.

2. METHODOLOGY

In this paper, we use Function Principle and Graded Mean Integration Representation method to find the optimal economic production quantity with a fuzzy inventory model. When the quantities are fuzzy numbers we need to use the Kuhn-Tucker conditions to solve the model. Therefore we introduce this three methodologies as following.

2.1 Graded mean integration representation method

Chen et al. (1998, 1999c, 2000b) introduced Graded Mean Integration Representation method based on the integral value of graded mean h-level of generalized fuzzy number for defuzzifing generalized fuzzy number. They also found this method is better than the methods of Adamo (1980), Campos et al. (1989), Yager (1981), Kaufmann et al. (1991), Chen (1998), Lee et al. (1998), Liou et al. (1992), Heilpern (1997). Now, we describe generalized fuzzy number as following.

Suppose $\tilde{A}$ is a generalized fuzzy number as shown in

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Figure 1. It is described as any fuzzy subset of the real line R, whose membership function μ_A satisfies the following conditions.

1. μ_A is a continuous mapping from R to the closed interval [0, 1],
2. $\mu_A = 0$, $-\infty < x \le a_1$,
3. $\mu_A = L(x)$ is strictly increasing on $[a_1, a_2]$,
4. $\mu_A = w_A$, $a_2 \le x \le a_3$,
5. $\mu_A = R(x)$ is strictly decreasing on $[a_3, a_4]$,
6. $\mu_A = 0$, $a_4 \le x < \infty$,

where $0 < w_A \le 1$, and a_1, a_2, a_3, and a_4 are real numbers.

Also this type of generalized fuzzy number be denoted as $\tilde{A} = (a_1, a_2, a_3, a_4;\ w_A)_{LR}$. When $w_A = 1$, it can be simplified as $\tilde{A} = (a_1, a_2, a_3, a_4)_{LR}$.

By Graded Mean Integration Representation method, L^{-1} and R^{-1} are the inverse functions of L and R respectively, and the graded mean h-level value of generalized fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4;\ w_A)_{LR}$ is $h(L^{-1}(h)+R^{-1}(h))/2$ as Figure 1. Then the graded mean integration representation of $\tilde{A}$ is $P(\tilde{A})$ with grade w_A where

$$P(\tilde{A}) = \int_0^{w_A} h\left(\frac{L^{-1}(h) + R^{-1}(h)}{2}\right)dh \Bigg/ \int_0^{w_A} h\, dh, \tag{1}$$

with $0 < h \le w_A$ and $0 < w_A \le 1$.

Throughout this paper, we only use normal trapezoidal fuzzy number as the type of all fuzzy parameters in our proposed fuzzy production inventory models. Let $\tilde{B}$ be a trapezoidal fuzzy number, and be denoted as $\tilde{B} = (b_1, b_2, b_3, b_4)$. Then we can get the graded mean integration representation of $\tilde{B}$ by formula (1) as

$$\begin{aligned} \mathrm{P}(\tilde{B}) &= \int_0^1 h\left(\frac{b_1 + b_4 + (b_2 - b_1 - b_4 + b_3)h}{2}\right)dh \Bigg/ \int_1^0 hdh \\ &= \frac{b_1 + 2b_2 + 2b_3 + b_4}{6}. \end{aligned} \tag{2}$$

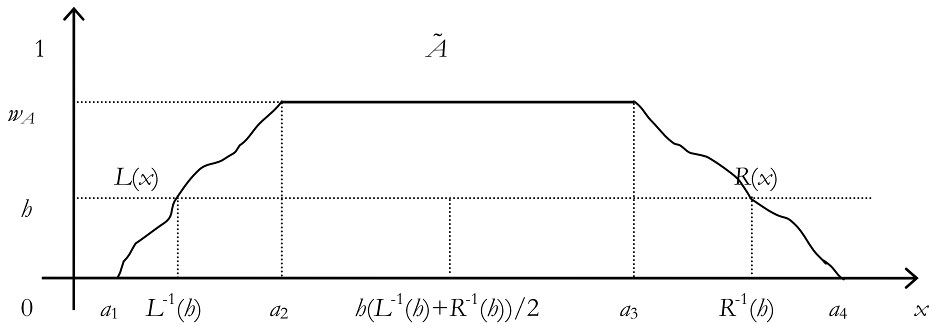

Figure 1. The graded mean h-level value of generalized fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4;\ w_A)_{LR}$.

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2.2 The fuzzy arithmetical operations under function principle

Function Principle is introduced by Chen (1985) to treat the fuzzy arithmetical operations with trapezoidal fuzzy numbers. We will use this principle as the operation of addition, multiplication, subtract, division of trapezoidal fuzzy numbers, because (1) Function Principle is easier to calculate than Extension Principle, (2) Function Principle will not change the shape of trapezoidal fuzzy number after the multiplication of two trapezoidal fuzzy numbers, but the multiplication of two trapezoidal fuzzy numbers will become drum like shape fuzzy number by using Extension Principle, (3) If we have to multiple more than four trapezoidal fuzzy numbers then the Extension Principle can not solve the operation, but Function Principle can easy to find the result by pointwise. Here we describe some fuzzy arithmetical operations under Function Principle as following.

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers. Then,

1. The addition of $\tilde{A}$ and $\tilde{B}$ is
   $\tilde{A} \oplus \tilde{B} = (a_1+b_1,\ a_2+b_2,\ a_3+b_3,\ a_4+b_4)$,

   where a_1, a_2, a_3, a_4, b_1, b_2, b_3, and b_4 are any real numbers.
2. The multiplication of $\tilde{A}$ and $\tilde{B}$ is
   $\tilde{A} \otimes \tilde{B} = (c_1, c_2, c_3, c_4)$,
   where $T = \{a_1b_1,\ a_1b_4,\ a_4b_1,\ a_4b_4\}$, $T_1 = \{a_2b_2,\ a_2b_3,\ a_3b_2,\ a_3b_3\}$, $c_1 = \min T$, $c_2 = \min T_1$, $c_3 = \max T_1$, $c_4 = \max T$.
   If a_1, a_2, a_3, a_4, b_1, b_2, b_3, and b_4 are all nonzero positive real numbers, then
   $\tilde{A} \otimes \tilde{B} = (a_1b_1, a_2b_2, a_3b_3, a_4b_4)$
3. $-\tilde{B} = (-b_4, -b_3, -b_2, -b_1)$, then the subtraction of $\tilde{A}$ and $\tilde{B}$ is
   $\tilde{A} \ominus \tilde{B} = (a_1-b_4, a_2-b_3, a_3-b_2, a_4-b_1)$,
   where a_1, a_2, a_3, a_4, b_1, b_2, b_3, and b_4 are any real numbers.
4. $1/\tilde{B} = \tilde{B}^{-1} = (1/b_4, 1/b_3, 1/b_2, 1/b_1)$, where b_1, b_2, b_3, and b_4 are all positive real numbers.
   If a_1, a_2, a_3, a_4, b_1, b_2, b_3, and b_4 are all nonzero positive real numbers, then the division of $\tilde{A}$ and $\tilde{B}$ is
   $\tilde{A} \oslash \tilde{B} = (a_1/b_4, a_2/b_3, a_3/b_2, a_4/b_1)$.

5. Let $\alpha \in$ R, then

$$\begin{cases}(i)\ \alpha \ge 0,\ \alpha \otimes \tilde{A} = (\alpha a_1,\ \alpha a_2,\ \alpha a_3, \alpha a_4), \\ (ii)\ \alpha < 0,\ \alpha \otimes \tilde{A} = (\alpha a_4,\ \alpha a_3,\ \alpha a_2, \alpha a_1).\end{cases}$$

We do not introduce a new addition symbol, as the sum under the Extension Principle is the same as Figure 2. For a mathematically minded reader, we observe that the Extension Principle is a form of convolution (Chen et al., 1996) while the Function Principle is akin to a pointwise multiplication as Figure 3.

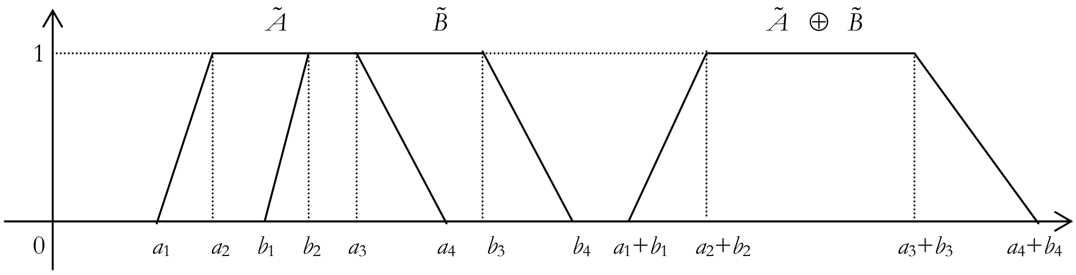
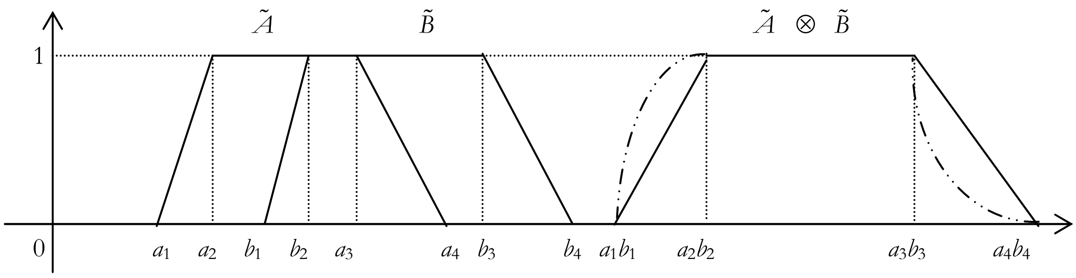


Figure 2. The fuzzy addition operation of function principle and extension principle.



Function Principle: ——— Extension Principle: —··—··

Figure 3. The comparing of fuzzy multiplication operation under function principle and extension principle.

2.3 The Kuhn-Tucker conditions

Taha (1997) discussed how to solve the optimum solution of nonlinear programming problem subject to inequality constraints by using the Kuhn-Tucker conditions. The development of the Kuhn-Tucker conditions is based on the LaGrangean method.

Suppose that the problem is given by

Minimize $y = f(x)$

Subject to $g_i(x) \ge 0$, i=1, 2, ⋯, m.

The nonnegative constraints $x \ge 0$, if any, are included in the m constraints.

The inequality constraints may be converted into equations by using nonnegative surplus variables. Let's S_i^2 be the surplus quantity added to the ith constraint $g_i(x) \ge 0$. Let $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_m)$, $g(x) = (g_1(x), g_2(x), \ldots, g_m(x))'$ and $S^2 = (S_1^2, S_2^2, \ldots, S_m^2)'$. Then the LaGrangean functions are given by

$L(x, s, \lambda) = f(x) - \lambda[g(x) - S^2]$.

Given the constraints $g_i(x) \ge 0$.

Taking the partial derivatives of L with respect to x, s, and λ, we obtain

$$\frac{\partial L}{\partial X} = \nabla f(x) - \lambda \nabla g(x) = 0,$$

$$\frac{\partial L}{\partial S_i} = 2\lambda_i S_i = 0,\ i = 1, 2, \ldots, m.$$

$$\frac{\partial L}{\partial \lambda_i} = -g_i(x) + S_i^2 = 0,\ i = 1, 2, \ldots, m.$$

From the second and third sets of equations it shows that

$\lambda_i g_i(x) = 0$, $i = 1, 2, \cdots, m$.

The Kuhn-Tucker conditions need x and λ to be a stationary point of the minimization problem which can be summarized as following:

$$\begin{cases}\lambda \le 0, \\ \nabla f(x) - \lambda \nabla g(x) = 0, \\ \lambda_i g_i(x) = 0,\ i = 1,\ 2,\ \ldots, m, \\ g_i(x) \ge 0,\ i = 1,\ 2,\ \ldots, m.\end{cases} \tag{3}$$

3. FUZZY PRODUCTION INVENTORY MODELS

We thereby discuss two cases of imperfect productions that can be repaired, one case with fuzzy costs but crisp quantities, the other case with fuzzy costs and fuzzy quantities.

Throughout this paper, we use the following variables in order to simplify the treatment of the fuzzy production inventory model:

3.1 Nomenclature

$\tilde{H}$: fuzzy daily storage cost per unit,
$\tilde{K}$: fuzzy setup cost,
c : the unit production cost,
$\tilde{E}$: fuzzy cost incurred by repairing a defective item,
$\tilde{I}$: fuzzy opportunity cost percentage,
p : the probability that the production process can go 'out-of-control',
a_p : the investment is required to reduce the' out-of-control' probability with p,
R: daily demand,
B: daily production,
D: total demand over the planning time period [0, T],
Q: crisp production quantity.

3.2 Case with crisp production quantity

The manufacturer produces and sells products in the time OG, SG_1, …, etc., and he only sells products in time GS, G_1S_1, …, etc, as shows in Figure 4.

Inventory quantity

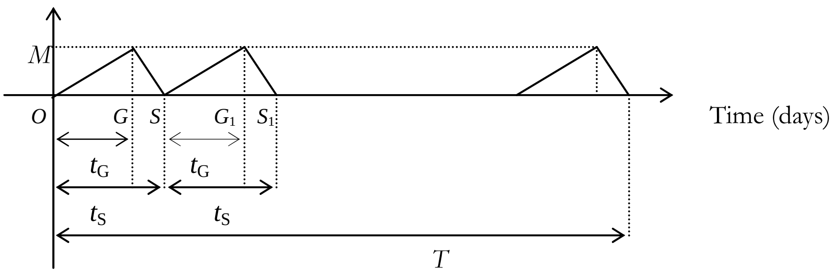


Figure 4. Inventory control and the production process.

Furthermore, $t_G = Q/B$ is the length of the product run in days, $Rt_G = RQ/B$ is the sale quantity for the product run, $M = Q-RQ/B = Q(1-R/B)$ is the inventory quantity at the end of the product run, where R should be less than B, $M_S = M/2 = Q(1-R/B)/2$ is the average inventory quantity on the time periods with length $t_s = TQ/D$, because the number of product runs in the plan period is given by D/Q.

Now, the fuzzy total cost $\tilde{F}$ per cycle time T with imperfect productions is defined as an approximation as,

"Production cost + fuzzy investment cost required for fixed process + fuzzy setup cost + fuzzy storage cost + fuzzy repaired cost for defectives".

That is, $\tilde{F}=c\times D\oplus\tilde{I}\otimes a_p\otimes T\oplus\tilde{K}\otimes(D/Q)\oplus(\tilde{I}\otimes c\oplus\tilde{H})\otimes(Q(1-R/B)/2)\otimes T\oplus D\otimes p\otimes\tilde{E}$ where $\oplus$, $\otimes$ are the fuzzy arithmetical operations under Function Principle.

Here, we suppose $\tilde{H} = (h_1, h_2, h_3, h_4)$, $\tilde{K} = (k_1, k_2, k_3, k_4)$, $\tilde{I} = (i_1, i_2, i_3, i_4)$ and $\tilde{E} = (e_1, e_2, e_3, e_4)$ are nonnegative trapezoidal fuzzy numbers. Then we solve the optimal fuzzy total cost of formula (4) as the following steps.

Firstly, we get the fuzzy total production inventory cost $\tilde{F}$ by formula (4) as,

$$\tilde{F} = [D(c+e_1p)+a_pTi_1+Dk_1/Q+T/2(1-R/B)(ci_1+h_1)Q,$$
$$D(c+e_2p)+a_pTi_2+Dk_2/Q+ T/2(1-R/B)(ci_2+h_2)Q,$$
$$D(c+e_3p)+a_pTi_3+Dk_3/Q + T/2\,(1-R/B)(ci_3+h_3)Q,$$
$$D(c+e_4p)+a_pTi_4+Dk_4/Q+ T/2\,(1-R/B)(ci_4+h_4)Q].$$

Secondly, we defuzzify the fuzzy total production inventory cost using graded mean integration representation method. The result is,

$$P(\tilde{F})=\{D[2c+p(e_1+e_4)]+a_pT(i_1+i_4)+D(k_1+k_4)/Q+T/2(1-R/B)((i_1+i_4)c+h_1+h_4)Q+2[D(2c+p(e_2+e_3))+a_pT(i_2+i_3)+D(k_2+k_3)/Q+T/2(1-R/B)((i_2+i_3)c+h_2+h_3)Q]\}/6.$$

Thirdly, we can get the optimal production quantity Q^* when $P(\tilde{F})$ is a minimization. In order to find the minimization of $P(\tilde{F})$, the derivative of $P(\tilde{F})$ with Q is,

$$dP(\tilde{F})/d\,Q = \{-D(k_1+k_4)/Q^2+T/2(1\text{-}R/B)((i_1+i_4)c+h_1+h_4)2[-D(k_2+k_3)/Q^2+T/2(1-R/B)((i_2+i_3)c+h_2+h_3)]\}/6.$$

Let $dP(\tilde{F})/d\,Q = 0$, it becomes,

$$-D(k_1+2k_2+2k_3+k_4)/Q^2+T/2(1-R/B)[(i_1+2i_2+2i_3+i_4)c+h_1+2h_2+2h_3+h_4]=0.$$

Since

$$d^2P(\tilde{F})/d\,Q^2 = [D\,(k_1+2\,k_2+\,2k_3+\,k_4)/Q^3\,]/3>0.$$

Hence, we find the optimal production quantity Q^* by the above equation as,

$$Q^{*}=\sqrt{\frac{2D(k_1+2k_2+2k_3+k_4)}{T(1-R/B)[(i_1+2i_2+2i_3+i_4)c+h_1+2h_2+2h_3+h_4]}} \quad (5)$$

When costs are real numbers, they are $h_1 = h_2 = h_3 = h_4 = H$, $k_1 = k_2 = k_3 = k_4 = K$, $i_1 = i_2 = i_3 = i_4 = I$, $e_1 = e_2 = e_3 = e_4 = E$. Formula (5) can be revised as $Q^* = [2KD/(T(1-R/B)(cI+H))]^{1/2}$.

Then, the fuzzy economic production quantity model will be reduced to the traditional economic production quantity model.

3.3 Case with fuzzy production quantity

In this case, the quantities are fuzzy numbers so we have the fuzzy quantities notations instead. Where, $\tilde{R}$: fuzzy daily demand, $\tilde{B}$: fuzzy daily production, $\tilde{D}$: fuzzy total demand over the planning time period $[0, T]$, $\tilde{Q}$: fuzzy production quantity.

The manufacturer's inventory control and the production process are as shown in Figure 4. Furthermore, t_G, the length of the product run in days, is approximated to $\tilde{Q}\oslash\tilde{B}$. $\tilde{R}t_G$, the sale quantity for the product run, is approximated to $\tilde{R}\otimes\tilde{Q}\oslash\tilde{B}$. $\tilde{M}$, the fuzzy inventory quantity at the end of the product run, is approximated to $\tilde{Q}\Theta\tilde{R}\otimes\tilde{Q}\oslash\tilde{B}=\tilde{Q}$ $(1\Theta\tilde{R}\oslash\tilde{B})$, where $\tilde{R}$ should be absolute less than $\tilde{B}$. $\tilde{M}_S$ $(=\tilde{M}\oslash 2)$, the average inventory quantity on the time periods with length t_S is approximated to $T\otimes\tilde{Q}\oslash\tilde{D}$, is approximated to $=\tilde{Q}\otimes(1\Theta\tilde{R}\oslash\tilde{B})\oslash 2$ and the number of product runs in the plan period is given by $\tilde{D}\oslash\tilde{Q}$.

Now, the fuzzy total cost $\tilde{F}$ per cycle time for the EPQ model with imperfect production process and with defective productions is defined as an approximation as,

"Fuzzy production cost + fuzzy investment cost required for fixed process + fuzzy setup cost + fuzzy storage cost + fuzzy repaired cost for defectives".

That is,

$$\tilde{F}=c\otimes\tilde{D}\oplus\tilde{I}\otimes a_p\otimes T\oplus\tilde{K}\otimes\tilde{D}\oslash\tilde{Q}\oplus(\tilde{I}\otimes c\oplus\tilde{H})\otimes\tilde{Q}\otimes(1\Theta\tilde{R}\oslash\tilde{B})\oslash 2\otimes T\oplus\tilde{D}\otimes p\otimes\tilde{E} \quad (6)$$

where $\oplus$, Θ, $\otimes$ and $\oslash$ are the fuzzy arithmetical operations under Function Principle.

Here, we suppose $\tilde{H}=(h_1, h_2, h_3, h_4)$, $\tilde{K}=(k_1, k_2, k_3, k_4)$, $\tilde{I}=(i_1, i_2, i_3, i_4)$, $\tilde{E}=(e_1, e_2, e_3, e_4)$, $\tilde{R}=(r_1, r_2, r_3, r_4)$, $\tilde{B}=(b_1, b_2, b_3, b_4)$, $\tilde{D}=(d_1, d_2, d_3, d_4)$, and $\tilde{Q}=(q_1, q_2, q_3, q_4)$ are nonnegative trapezoidal fuzzy numbers. Then we solve the optimal fuzzy total cost of formula (6) as the following steps.

Firstly, we get the fuzzy total production inventory cost $\tilde{F}$ by formula (6) as,

$$\begin{aligned}\tilde{F}=[&cd_1+d_1e_1p+a_pTi_1+d_1k_1/q_4+T/2(ci_1+h_1)(1-r_4/b_1)q_1,\\ &cd_2+d_2e_2p+a_pTi_2+d_2k_2/q_3+T/2(ci_2+h_2)(1-r_3/b_2)q_2,\\ &cd_3+d_3e_3p+a_pTi_3+d_3k_3/q_2+T/2(ci_3+h_3)(1-r_2/b_3)q_3,\\ &cd_4+d_4e_4p+a_pTi_4+d_4k_4/q_1+T/2(ci_4+h_4)(1-r_1/b_4)\,q_4].\end{aligned}$$

Secondly, we defuzzify the fuzzy total production inventory cost using graded mean integration representation method. The result is,

$$\begin{aligned}P(\tilde{F})=\{&[cd_1+d_1e_1p+a_pTi_1+d_1k_1/q_4+T/2(ci_1+h_1)(1-r_4/b_1)q_1+cd_4+d_4e_4p+a_pTi_4+d_4k_4/q_1+T/2(ci_4+h_4)(1-r_1/b_4)q_4]+2[cd_2+d_2e_2p+\\ &a_pTi_2+d_2k_2/q_3+T/2(ci_2+h_2)(1-r_3/b_2)q_2+cd_3+d_3e_3p+a_pTi_3+d_3k_3/q_2+T/2(ci_3+h_3)(1-r_2/b_3)q_3]\}/6. \quad (7)\end{aligned}$$

with $0 < q_1 \le q_2 \le q_3 \le q_4$.

It will not change the meaning of formula (7), if we replace inequality conditions $0 < q_1 \le q_2 \le q_3 \le q_4$ into the following inequality constrains.

$q_2 - q_1 \ge 0$, $q_3 - q_2 \ge 0$, $q_4 - q_3 \ge 0$, and $q_1 > 0$.

Thirdly, the Kuhn-Tucker condition is used to find the solution of q_1, q_2, q_3, and q_4 to minimize $P(\tilde{F})$ in formula (7), subject to $q_2 - q_1 \ge 0$, $q_3 - q_2 \ge 0$, $q_4 - q_3 \ge 0$, and $q_1 > 0$. The Kuhn-Tucker conditions are thus given as formula (3).

$\lambda \le 0$,

$\nabla f(P(\tilde{F})) - \lambda_i \nabla g(Q) = 0$,

$\lambda_i g_i(Q) = 0$,

$g_i(Q) \ge 0$,

These conditions simplify to the following,

$$\lambda_1, \lambda_2, \lambda_3, \lambda_4 \le 0, \quad (8\text{-}1)$$

$$-d_4k_4/q_1^2 + T/2(ci_1+h_1)(1-r_4/b_1)+\lambda_1-\lambda_4 = 0, \quad (8\text{-}2)$$

$$2[-d_3k_3/q_2^2 + T/2(ci_2+h_2)(1-r_3/b_2)]-\lambda_1+\lambda_2 = 0, \quad (8\text{-}3)$$

$$2[-d_2k_2/q_3^2 + T/2(ci_3+h_3)(1-r_2/b_3)]-\lambda_2+\lambda_3 = 0, \quad (8\text{-}4)$$

$$-d_1k_1/q_4^2 + T/2(ci_4+h_4)(1-r_1/b_4)-\lambda_3 = 0, \quad (8\text{-}5)$$

$$\lambda_1(q_2-q_1) = 0, \quad (8\text{-}6)$$

$$\lambda_2(q_3-q_2) = 0, \quad (8\text{-}7)$$

$$\lambda_3(q_4-q_3) = 0, \quad (8\text{-}8)$$

$$\lambda_4 q_1 = 0, \quad (8\text{-}9)$$

$$q_2 - q_1 \ge 0, \quad (8\text{-}10)$$

$$q_3 - q_2 \ge 0, \quad (8\text{-}11)$$

$$q_4 - q_3 \geq 0, \quad (8\text{-}12)$$

$$q_1 > 0. \quad (8\text{-}13)$$

Because $q_1 > 0$, and $\lambda_4 q_1 = 0$, then $\lambda_4 = 0$. If $\lambda_1 = \lambda_2 = \lambda_3 = 0$, then $q_4 < q_3 < q_2 < q_1$, it does not satisfy the constrains $0 < q_1 \leq q_2 \leq q_3 \leq q_4$. Therefore $q_2 = q_1$, $q_3 = q_2$, and $q_4 = q_3$, that is $q_1 = q_2 = q_3 = q_4 = Q$. Hence, from formula (8-2), (8-3), (8-4), and (8-5) we find the optimal production quantity Q^* by the above equation as,

$$Q_* = \sqrt{\frac{2(k_1d_1 + 2k_2d_2 + 2k_3d_3 + k_4d_4)}{T\left[(ci_1 + h_1)(1 - \frac{r_4}{b_1}) + 2(ci_2 + h_2)(1 - \frac{r_3}{b_2}) + 2(ci_3 + h_3)(1 - \frac{r_2}{b_3}) + (ci_4 + h_4)(1 - \frac{r_1}{b_4})\right]}} \quad (9)$$

In (9), when $r_1 = r_2 = r_3 = r_4 = R$, $b_1 = b_2 = b_3 = b_4 = B$, and $d_1 = d_2 = d_3 = d_4 = D$, then formula (9) becomes

$$Q_* = \sqrt{\frac{2D(k_1 + 2k_2 + 2k_3 + k_4)}{T(1 - R/B)\left[(i_1 + 2i_2 + 2i_3 + i_4)c + h_1 + 2h_2 + 2h_3 + h_4\right]}}.$$

It shows that equation (9) becomes equation (5).

When demand, production quantity and costs are all real numbers, that is $h_1 = h_2 = h_3 = h_4 = H$, $k_1 = k_2 = k_3 = k_4 = K$, $i_1 = i_2 = i_3 = i_4 = I$, $r_1 = r_2 = r_3 = r_4 = R$, $b_1 = b_2 = b_3 = b_4 = B$, and $d_1 = d_2 = d_3 = d_4 = D$, then formula (9) can be revised as $Q^* = [2KD/T(cI+H)(1-R/B)]^{1/2}$.

Then the fuzzy economic production quantity model will be reduced to the traditional economic production quantity model.

4. EXAMPLE

ABC manufacturing company produces commercial television units in batch. The firm estimated: the fuzzy daily storage cost ($\tilde{H}$) per unit is about NT$1, the fuzzy setup cost ($\tilde{K}$) is about NT$100000, the unit production cost (c) is NT$5000, the fuzzy cost incurred by repairing a defective item ($\tilde{E}$) is greater or less than NT$1000, the fuzzy opportunity cost percentage ($\tilde{I}$) is about 0.2%, the probability that the production process can go 'out-of-control '(p) is 0.01, the investment (a_p) required to reduce the 'out-of-control' probability with p is NT$100000, the fuzzy daily demand ($\tilde{R}$) is about 25 units, the fuzzy daily production ($\tilde{B}$) is about 30 units, the fuzzy total demand over the planning time period [0, 365] is $\tilde{D}$ (=365 $\tilde{R}$). How many television units should ABC manufacturing company produce in each batch?

Solving:

Here we use a general rule to transfer the linguistic data, "greater or less than X" and "about X",into trapezoidal fuzzy numbers as

"greater or less than X" = (0.9X, 0.95X, 1.05X, 1.1X), and "about X" = (0.95X, X, X, 1.05X).

By the above rule, the fuzzy parameters in this example can be transferred as follows: $\tilde{H}$ = (0.95, 1, 1, 1.05), $\tilde{K}$ = (95000, 100000, 100000, 105000), c=5000, $\tilde{E}$ = (900, 950, 1050, 1100), $\tilde{I}$ = (0.0019, 0.002, 0.002, 0.0021), p=0.01, a_p=100000, $\tilde{R}$ = (23.75, 25, 25, 26.25), $\tilde{B}$ = (28.5, 30, 30, 31.5), $\tilde{D}$ = (8668.75, 9125, 9125, 9581.25).

Replace the above fuzzy parameters values into formula (9), we find the optimal fuzzy production quantity $\tilde{Q}$ = (1652.13, 1652.13, 1652.13, 1652.13) ≈ (1653, 1653, 1653, 1653).

Then, the minimization fuzzy total production inventory cost is

$\tilde{F}$ = (44238333.4, 46889780, 46898904.98, 49554025.63).

The result shows that ABC manufacturing company is better to produce 1653 units per batch.

5. CONCLUSION

In real world, defective products cannot be avoided in some imperfective production processes. Therefore it is nature and reasonable to discuss the model with defective products. In this paper, we discuss the situation of defective products that can be repaired. We solve two production inventory model, one case is with fuzzy costs but crisp quantities, the other case is with fuzzy costs and fuzzy quantities. These models are applicable when inventory continuously flows or builds up over a period of time, after an order has been placed and units are produced and sold simultaneously. Under this circumstance, the length of the planning time period is measured in days.

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