

## Optimal Selection of Arrival and Service Rates in Tandem Queues

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**Abstract**—We consider  $n$  M/M/1 queues in series. At queue one the arrival and service rates are chosen in pair from a finite set whenever there are arrivals or service completions at any queue. Customers arriving to queue  $k$  ( $k = 1, 2, \dots, n - 1$ ) must go on to queue  $k + 1$  after finishing service at server associated queue  $k$ . Customers arriving to queue  $n$  leave the system after finishing service at the last server. Arrival and service rates are fixed at queues 2 to  $n$ . The objective is to minimize the expected discounted cost of the system over finite and infinite horizons. We show that there is a monotone hysteretic optimal policy in which the arrival and service rates are decreasing and increasing, respectively, in the queue length. In order to establish the result, we formulate the optimal control problem with an equivalent Linear Programming. We believe that many optimal control queueing problems, in which the dynamic programming formulation fails, can be treated successfully via Linear Programming techniques.

**Keywords**—Queueing networks, Markov decision processes, Stochastic linear programming, Sample-path arguments

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