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Discount Cash-Flow Analysis on Inventory Control under Various Supplier's Trade Credits

Jinn-Tsair Teng*

Department of Marketing and Management Sciences William Paterson University of New Jersey 1600 Valley Road Wayne, New Jersey 07470-2103, U.S.A.

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Abstract—In practices, the supplier may simultaneously offer the customer: (1) a permissible delay in payments to attract new customers and increase sales, and (2) a cash discount to motivate faster payment and reduce credit expenses. Since all cash outflows related to inventory control that occur at different points of time have different values, we use the discount cash-flow (or DCF) approach to establish the models, and obtain the optimal ordering policies to the problem. We find that the DCF approach is not only simple to understand but also easy to identify which alternative is less cost. In addition, we also characterize the optimal solution and provide the closed-form solution to the problem. Furthermore, we also compare the optimal order quantity under supplier credits with the classical economic order quantity.

Keywords—Inventory, Decision theory, Cash discount, Delay payment, Trade credit

1. INTRODUCTION

In the classical inventory economic order quantity (or EOQ) model, it was tacitly assumed that the customer must pay for the items as soon as the items are received. However, in practices or when the economy turns sour, the supplier frequently offers its customers a permissible delay in payments to attract new customers who consider it to be a type of price reduction. To motivate faster payment, stimulate more sales, or reduce credit expenses, the supplier also often provides its customers a cash discount. For examples, several years ago, US gas stations adopted a pricing policy that charged less money per gallon to the customer who paid by cash, instead of by a credit card. Likewise, a storeowner in many China towns around the world usually charges a customer 5% more if the customer pays by a credit card, instead of by cash. As a result, the customer must decide which alternative to take when the supplier provides not only a cash discount but also a permissible delay.

Goyal (1985) derived an EOQ model under the conditions of permissible delay in payments. Shah (1993) developed a stochastic inventory model when inventory items deteriorate and delay in payment is permissible. Aggarwal and Jaggi (1995) extended Goyal's model to allow for deteriorating items. Jamal et al. (1997) then further generalized the model to allow for shortages. Liao et al. (2000) developed an inventory model for stock-dependent consumption rate when a delay in payment is permissible. Lately, Arcelus et al. (2001) analyzed the pros and cons of price discount vs. trade credit. Chang and Dye (2000) proposed an inventory model for deteriorating items with partial backlogging and permissible delay. Teng (2002) amended Goyal's model by considering the fact that the unit price usually is higher than the unit cost. Chang et al. (2003) established an EOQ model for deteriorating items under supplier credits linked to order quantity. Recently, Ouyang et al. (2005) generalized Goyal's model (in which the retailer pays the supplier only the costs when items are sold) to obtain an optimal order policy for the retailer when the supplier offers not only a cash discount but also a permissible delay. Concurrently, Chang and Teng (2004) solved the same problem by assuming that the retailer pays the supplier the sales revenue when items are sold. Several interesting and relevant papers related to trade credits are Arcelus and Srinivasan (1993, 1995, 2001), Chung and Liao (2004), Davis and Gaither (1985), Huang (2003, 2004), Shah (1997), Teng et al. (2005) and others.

All of the above mentioned articles do not use the discount cash-flows (DCF) approach for the analysis of the optimal inventory policy when the supplier offers trade credits. In fact, inventories of all types, cycle stocks, safety stocks, etc. are required for the normal operation of a firm in the same way as it may require machinery, land or premises. Therefore, inventories are an integral and necessary part of a firm's total investment portfolio, and compete with other investment projects for a firm's limited sources of funds. From a financial standpoint, all cash outflows related to the inventory control that occur at different points of time have different values. As a result, it

* Corresponding author's email: TengJ@wpunj.edu

is necessary to consider the effects of the time value of money on the inventory system. Trippi and Lewin (1974) and Kim and Chung (1990) recognized the need to explore the inventory problems by using the present value concept or DFC approach. Chung (1989) then adopted the DCF approach to establish the closed-form solutions for the basic EOQ model with various trade credits. Several interesting and relevant papers related to trade credits by using the DCF approach are Chapman et al. (1984), Chung and Liao (2005), Daellenbach (1986), Jaggi and Aggarwal (1994), and Ward and Chapman (1987).

As stated above, the DCF approach permits a proper alternative analysis of the inventory cost under the effect of the time value of money. Consequently, in this paper, we apply the DCF approach to establish an EOQ model when the supplier provides not only a cash discount but also a permissible delay. By comparing the DCF approach and the method used in Ouyang et al. (2005), we find that the DCF method is simpler to understand and easier to apply because it uses less number of variables, equations and cases. We then study the necessary and sufficient conditions for finding the optimal solution to the problem, and provide an explicitly closed-form solution to find the optimal replenishment interval and order quantity. Furthermore, we compare and characterize the optimal replenishment interval and the optimal present value of all future cost under each alternative. Finally, we provide several numerical examples for illustration the theoretical results.

The rest of the paper is organized as follows. In Section 2, we describe the assumptions and notation used throughout this study. In Section 3, we develop the mathematical models by DFC approach for both alternatives (i.e., both a cash discount and a permissible delay). In Section 4, the necessary and sufficient conditions are derived, an approximately closed-form solution to the optimal replenishment interval is developed, and several important theorems are established to characterize the optimal solution. We then compare the optimal order quantity under a cash discount and/or a permissible delay in payment with the classical economic order quantity (in which the supplier must be paid for the items as soon as the purchaser receives them) in Section 5. In Section 6, we provide two numerical examples to illustrate the results. Finally, we draw the conclusions and the future research in Section 7.

2. ASSUMPTIONS AND NOTATION

The following assumptions are similar to those in Goyal (1985).

(1) The demand for the item is constant with time.
(2) Shortages are not allowed.
(3) Replenishment is instantaneous.
(4) Time horizon is infinite.
(5) If the supplier provides a cash discount, then the outstanding balance must be paid at time M_1. Otherwise, the full payment is paid at time M_2. Notice that if $M_1 = 0$, then the cash discount is available only the customer (or the retailer) pays cash on delivery.

In addition, the following notation is used throughout this paper.

- D = the demand rate per year.
- h = the out-of-packet holding cost (including the insurance, investment, and obsolescence costs) as a proportion of the value of inventory per unit time.
- C_p = the unit purchasing cost.
- C_0 = the ordering cost per order.
- r = the opportunity cost (i.e., the continuous discounting rate) per unit time.
- Q = the order quantity.
- d = the cash discount rate, $0 < d < 1$.
- M_1 = the fixed period of cash discount in settling account.
- M_2 = the fixed period of permissible delay in settling account, with $M_2 > M_1$.
- T = the replenishment time interval.
- $PV(T)$ = the present value of cash outflows for the first replenishment cycle.

3. DISCOUNT CASH-FLOW MODELS

In this section, we first present the DCF approach for the EOQ model when the supplier offers a cash discount. We then discuss the other situation in which the supplier provides a fixed credit period (i.e., a delay payment).

3.1 Cash-discount model

At the beginning of each replenishment cycle, there will be cash outflows of the ordering cost, C_0. The customer pays the full purchase cost (i.e., $C_p(1-d)DT_1$, where T_1 stands for the replenishment interval for Case 1) on the last day of the credit period, M_1. Hence, the present value of the purchase cost is $C_p(1-d)DT_1e^{-rM_1}$. Since the out-of-pocket inventory holding cost is assumed to be proportional to the value of the item (i.e., $(1-d)C_p$), the out-of-pocket inventory holding cost per unit time at t is $hC_p(1-d)D(T_1-t)$. Then the present value of the out-of-pocket holding cost at the continuous discounting rate r is $hC_p(1-d)D(T_1-t)e^{-rt}$. Consequently, the present value of cash outflows for the first cycles is

$$\begin{aligned} PV_1(T_1) &= C_0 + C_p(1-d)DT_1e^{-rM_1} \\ &\quad + hC_p(1-d)D\int_0^{T_1}(T_1-t)e^{-rt}dt \\ &= C_0 + C_p(1-d)DT_1e^{-rM_1} \\ &\quad + hC_p(1-d)D\frac{1}{r}\{T_1 + \frac{1}{r}(e^{-rT_1}-1)\} \end{aligned} \tag{1}$$

Then the present value of all future cash outflows is

$PV_1(\infty)$

$$= \sum_{n=0}^{\infty} PV_1(T_1)e^{-nrT_1}$$
$$= PV_1(T_1)\sum_{n=0}^{\infty} e^{-nrT_1}$$
$$= PV_1(T_1)/(1-e^{-rT_1})$$
$$= \frac{\left[C_0 r + (re^{-rM_1} + h)C_p(1-d)DT_1\right]}{\left[r(1-e^{-rT_1})\right]} - \frac{hC_p(1-d)D}{r^2} \quad (2)$$

3.2 Fixed-credit model

In fact, if the supplier offers a permissible delay in payments, then it is a special case of the above cash-discount model with $d = 0$ and $M_1 = M_2$. Therefore, we know from the cash-discount model above that the present value of cash outflows for the fixed-credit model in first cycles as

$$PV_2(T_2)$$
$$= C_0 + C_p DT_2 e^{-rM_2} + hC_p D\int_0^{T_2}(T_2 - t)e^{-rt}\,dt \quad (3)$$
$$= C_0 + C_p DT_2 e^{-rM_2} + \frac{hC_p D[T_2 + (e^{-rT_2} - 1)/r]}{r}$$

where T_2 stands for the replenishment interval of the fixed-credit model. As a result, the present value of all future cash outflow is

$$PV_2(\infty) = \sum_{n=0}^{\infty} PV_2(T_2)e^{-nrT_2} = PV_2(T_2)\sum_{n=0}^{\infty} e^{-nrT_2}$$
$$= \frac{\left[C_0 r + (re^{-rM_2} + h)C_p DT_2\right]}{\left[r(1-e^{-rT_2})\right]} - \frac{hC_p D}{r^2} \quad (4)$$

Now, the decision problem for the customer is to find which present value of all future cash outflow (i.e., either (2) or (4)) is smaller than the other.

4. THEORETICAL RESULTS

Since the cash-discount model is a general case of the fixed-credit model, we establish the theoretical results for the cash-discount model first, and then obtain the results for the fixed-credit model by substituting $d = 0$ and $M_2 = M_1$ into the cash-discount model. By taking the first derivatives of $PV_1(\infty)$ in (2) with respect to T_1, we have

$$\frac{dPV_1(\infty)}{dT_1} = \frac{(re^{-rM_1} + h)C_p(1-d)D}{r(1-e^{-rT_1})}$$
$$- \frac{\left[C_0 r + (re^{-rM_1} + h)\,C_p(1-d)DT_1\right]r^2 e^{-rT_1}}{r^2(1-e^{-rT_1})^2} \quad (5)$$

Letting $dPV_1(\infty)/dT_1 = 0$, and rearranging terms, we obtain

$$\{(re^{-rM_1} + h)\,C_p(1-d)D$$
$$+[C_0 r + (re^{-rM_1} + h)\,C_p(1-d)DT_1]\,r\}e^{-rT_1}$$
$$= (re^{-rM_1} + h)\,C_p(1-d)D \quad (6)$$

From (5) and (6), we can obtain the following result.

Theorem 1.
(a) The solution to (6) not only exists but also is unique.
(b) There exists a unique optimal solution $T_1^* \in (0, \infty)$ that minimizes $PV_1(\infty)$.

Proof. See APPENDIX A for the detail proof.

Next, we derive the explicitly closed-form solution of T_1^*. Utilizing the fact that

$e^{rT} \approx 1 + rT + (rT)^2/2$, as rT is small,

and (6), we obtain

$$e^{rT_1} = 1 + r\{T_1 + C_0 r/[(re^{-rM_1} + h)\,C_p(1-d)D]\}$$
$$\approx 1 + rT_1 + (rT_1)^2/2 \quad (7)$$

Consequently, we have the optimal replenishment cycle time

$$T_1^* \approx \sqrt{2C_0/[(re^{-rM_1} + h)\,C_p(1-d)D]} \quad (8)$$

and the optimal order quantity

$$Q_1^* = DT_1^* \approx \sqrt{2C_0 D/[(re^{-rM_1} + h)\,C_p(1-d)]} \quad (9)$$

Substituting (8) into (2), we obtain the optimal present value of all future cash outflows as

$$PV_1^*(\infty) \approx$$
$$[C_0 r + \sqrt{2C_p C_0 D(1-d)(re^{-rM_1} + h)}]/[r(1-e^{-rT_1})]$$
$$-hC_p(1-d)D/r^2 \quad (10)$$

We then substitute $d = 0$ and $M_2 = M_1$ into (8)–(10), and obtain the following results.
The optimal replenishment cycle time is approximately equal to

$$T_2^* \approx \sqrt{2C_0/[(re^{-rM_2} + h)\,C_p D]} \quad (11)$$

The optimal order quantity is approximately equal to

$$Q_2^* = DT_2^* \approx \sqrt{2C_0 D/[(re^{-rM_2} + h)\,C_p]}\,. \quad (12)$$

Likewise, the optimal present value of all future cash outflows is

$$PV_2^*(\infty) \approx [C_0 r + \sqrt{2C_p C_0 D(re^{-rM_2} + h)}] / [r(1-e^{-rT_2})] - hC_p D / r^2 \tag{13}$$

By comparing the optimal replenishment intervals and order quantities in both cases, we have the following theoretical result.

Theorem 2.
(a) If $(re^{-rM_1} + h)(1-d) < (re^{-rM_2} + h)$, then $T_1^* > T_2^*$, and $Q_1^* > Q_2^*$.
(b) If $(re^{-rM_1} + h)(1-d) = (re^{-rM_2} + h)$, then $T_1^* = T_2^*$, and $Q_1^* = Q_2^*$.
(c) If $(re^{-rM_1} + h)(1-d) > (re^{-rM_2} + h)$, then $T_1^* < T_2^*$, and $Q_1^* < Q_2^*$.

Proof. It immediately follows from (8) and (11) that

if and only if $T_1^* > T_2^*$,
then $(re^{-rM_1} + h)(1-d) < (re^{-rM_2} + h)$. (14)

Similarly, from (9) and (12), we can easily obtain that

if and only if $Q_1^* > Q_2^*$,
then $(re^{-rM_1} + h)(1-d) < (re^{-rM_2} + h)$. (15)

This completes the proof.

Theorem 2 provides the supplier the necessary and sufficient information to know which alternative will encourage more sales. A simple economic interpretation of Theorem 2 is as follows. The customer always loves to buy the same product at less expensive costs. Consequently, if the sum of the opportunity cost (i.e., the discounted discount rate $= re^{-rM_1}$) and the holding cost (i.e., h) under the cash-discount model (i.e., $(re^{-rM_1} + h)(1-d)$) is cheaper than that under the fixed-credit model (i.e., $re^{-rM_2} + h$), then it is obvious that the customer will buy more quantity under the cash-discount model, and vice versa.

Similarly, by comparing the optimal present values of all future cash outflows in (10) and (13), we obtain that

if and only if

$$\Delta = \frac{-[C_0 r + \sqrt{2C_p C_0 D(re^{-rM_2} + h)}]}{[r(1-e^{-rT_2})]} + \frac{hC_p dD}{r^2} + \frac{[C_0 r + \sqrt{2C_p C_0 D(1-d)(re^{-rM_1} + h)}]}{[r(1-e^{-rT_1})]} < 0,$$

then $PV_1^*(\infty) < PV_2^*(\infty)$. (16)

From Theorem 2 and (16), we can easily obtain the following result.

Theorem 3.
If $(re^{-rM_1} + h)(1-d) \geq (re^{-rM_2} + h)$, then $PV_1^*(\infty) > PV_2^*(\infty)$, $T_1^* \leq T_2^*$, and $Q_1^* \leq Q_2^*$.

Proof. If $(re^{-rM_1} + h)(1-d) \geq (re^{-rM_2} + h)$, then

$$C_0 r + \sqrt{2C_p C_0 D(re^{-rM_2} + h)} \leq C_0 r + \sqrt{2C_p C_0 D(1-d)(re^{-rM_1} + h)} \tag{17}$$

By Theorem 2, we know that $T_1^* \leq T_2^*$, which in turn implies that $(1-e^{-rT_2^*}) \geq (1-e^{-rT_1^*})$. Consequently, we have $\Delta \geq hC_p dD / r^2 > 0$. We then know from (16) that taking the permissible delay is less expensive than taking the cash discount (i.e., $PV_1^*(\infty) > PV_2^*(\infty)$). The proof is completed.

Theorem 3 tells us that if $(re^{-rM_1} + h)(1-d) \geq (re^{-rM_2} + h)$ then the customer prefers to take the permissible delay to buy more quantity (i.e., $Q_1^* \leq Q_2^*$) but pay less present value of all future cash outflows (i.e., $PV_1^*(\infty) > PV_2^*(\infty)$).

5. COMPARISONS

In this section, we compare the above two models (i.e., the cash-discount model and the fixed-credit model) with the classical economic order quantity (i.e., EOQ) model, in which the supplier must be paid for the items as soon as the customer receives them. As a result, the classical EOQ model is also a special case of the cash-discount model with $M_1 = d = 0$. Consequently, we have the following results. The optimal replenishment cycle time for the classical EOQ model is approximately equal to

$$T_3^* \approx \sqrt{2C_0 / [(r+h)\, C_p D]}\,. \tag{18}$$

The optimal order quantity for the classical EOQ model is approximately equal to

$$Q_3^* = DT_3^* \approx \sqrt{2C_0 D / [(r+h)\, C_p]} \tag{19}$$

Likewise, the optimal present value of all future cash flows for EOQ model is

$$PV_3^*(\infty) \approx \frac{[C_0 r + \sqrt{2C_p C_0 D(r+h)}]}{[r(1-e^{-rT_3})]} - \frac{hC_p D}{r^2}\,. \tag{20}$$

By comparing the optimal replenishment intervals, order quantities, and present values of all future cash flows among these three different models, we have the following theoretical result.

Theorem 4.
T_1^* and $T_2^* > T_3^*$, Q_1^* and $Q_2^* > Q_3^*$, and $PV_1^*(\infty)$ and $PV_2^*(\infty) < PV_3^*(\infty)$.

Proof. Since $(r+h)\,C_p D$ is larger than both $(re^{-rM_1}+h)\,C_p(1-d)D$ and $(re^{-rM_2}+h)\,C_p D$, we know from (8), (9), (11), (12), (18), and (19) that not only T_1^* and $T_2^* > T_3^*$, but also Q_1^* and $Q_2^* > Q_3^*$. By using the fact that T_1^* and $T_2^* > T_3^*$, and comparing (10), (13) and (20), we can easily obtain that $PV_1^*(\infty)$ and $PV_2^*(\infty) < PV_3^*(\infty)$. This completes the proof.

The result in Theorem 4 reveals that the supplier's trade credit (either a cash discount or a payment delay) makes the customer not only to buy more quantity but also to pay less cost than the classical EOQ model, in which the customer must pay cash on delivery.

6. NUMERICAL EXAMPLE

To illustrate and verify the above theoretical results, we provide a couple of numerical examples here.

Example 1. Suppose that the demand rate per year D=1000 units, C_0 = 30, h = 0.2, r = 0.06, d = 0.02, and C_p = 20 in appropriate units, M_1 = 15 days = 15/365 years, M_2 = 30 days = 30/365 years. Applying (8)-(13) and (18)-(20), we have the following results, which are shown in Table 1.

Table 1. Numerical results of Example 1

	Cash-discount Model	Fixed-credit Model	Classical EOQ Model
T^*	0.108539	0.107478	0.107417
Q^*	108.539	107.478	107.417
$PV^*(\infty)$	335095.5048	341017.7681	342662.8428

In this case, $(re^{-rM_1}+h)(1-d)$ = 0.25465 < $(re^{-rM_2}+h)$ = 0.25970, the computational results is coincident with Theorem 2, 3 and 4. Further, the sensitivity analysis on C_0 and the difference for each two models are shown in table 2.

Table 2. Numerical results for different ordering costs of Example 1

C_0	30	25	20	15	10
T_1^*	0.1085	0.0991	0.0886	0.0767	0.0627
Q_1^*	108.5386	99.0818	88.6214	76.7484	62.6648
$PV_1^*(\infty)$	335095.50	334289.42	333398.16	332386.98	331188.16
T_2^*	0.1075	0.0981	0.0878	0.0760	0.0621
Q_2^*	107.4783	98.1138	87.7556	75.9986	62.0526
$PV_2^*(\infty)$	341017.77	340203.76	339303.74	338282.62	337072.00
T_3^*	0.1074	0.0981	0.0877	0.0760	0.0620
Q_3^*	107.4172	98.0581	87.7058	75.9555	62.0174
$PV_3^*(\infty)$	342662.84	341848.38	340947.84	339926.14	338714.84
$PV_2^*(\infty) - PV_1^*(\infty)$	5922.2633	5914.3434	5905.5832	5895.6398	5883.8452
$PV_3^*(\infty) - PV_1^*(\infty)$	7567.3380	7558.9576	7549.6879	7539.1664	7526.6859
$PV_3^*(\infty) - PV_2^*(\infty)$	1645.0747	1644.6142	1644.1048	1643.5266	1642.8407

The computation results reveal that a lower value of ordering cost C_0 results in lower values for the optimal replenishment cycle time T^*, the optimal order quantity Q^*, and the optimal present value of all future cash outflows $PV^*(\infty)$, and vice versa.

7. CONCLUSIONS

In this paper, I adopt the DCF approach to establish the optimal order policies when the supplier offers a cash discount and a payment delay at the same time. In Ouyang et al. (2005), they used 4 cases and 31 equations to solve the problem. By contrast, the proposed DCF present-value method solves the same problem with 0 cases and 20 equations. Judging from the analysis in Section 3, one can easily conclude that the DCF method is simpler to understand and easier to apply than the future-value method by Ouyang et al. (2005). I then use Taylor's series approximation to obtain the explicit closed-form solution of the optimal replenishment interval. Furthermore, I derive a couple of theoretical results, which provide us: (1) a simply way to identify which alternative will encourage the customer to buy more as shown in Theorem 2, and (2) how large the cash discount must be to ensure more sales as shown in Theorem 3. I then compare the optimal economic order quantities among the cash discount, the permissible delay in payments, and the cash on delivery (i.e., the classical economic order quantity), and find that the customer will order more quantity than the classical EOQ model whenever the supplier offers any trade credit. Finally, I provide a numerical example to illustrate and verify the results.

The proposed model can be extended in several ways. For instance, we may extend the model for deteriorating items. Also, we could consider the demand as a function of selling price as well as varying time. Finally, we could generalize the model to allow for shortages, quantity discounts, and others.

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APPENDIX A

To prove (a), we set the left-hand side of (6) as

$$F(T_1)=\{(re^{-rM_1}+h)\,C_p(1-d)D+[C_0r+(re^{-rM_1}+h)\,C_p(1-d)DT_1]\,r\}e^{-rT_1} \tag{A1}$$

We then have

$$F(0)=(re^{-rM_1}+h)\,C_p(1-d)D+C_0r^2 > (re^{-rM_1}+h)\,C_p(1-d)D\text{, and } F(\infty)=0. \tag{A2}$$

Taking the first derivative of $F(T_1)$, we get

$$F'(T_1)$$
$$= (re^{-rM_1} + h)\, C_p(1-d)D\, r\, e^{-rT_1} - r\{(re^{-rM_1} + h)\, C_p(1-d)D + [C_0 r + (re^{-rM_1} + h)\, C_p(1-d)DT_1]\, r\}e^{-rT_1}$$
$$= -r^2[C_0 r + (re^{-rM_1} + h)\, C_p(1-d)DT_1]\, e^{-rT_1} < 0, \tag{A3}$$

which in turn implies that $F(T_1)$ is a strictly decreasing and continuous function. Consequently, there exists a unique solution $T_1^* \in (0,\infty)$ such that $F(T_1^*) = (re^{-rM_1} + h)\quad C_p(1-d)D$. This completes the proof of (a).

To prove (b), we simply check the second-order condition at T_1^*. Taking the second derivative of $PV_1(\infty)$ with respect to T_1, we obtain

$$\frac{d^2 PV_1(\infty)}{dT_1^2}$$
$$= -\frac{(re^{-rM_1} + h)C_p(1-d)Dr^2 e^{-rT_1}}{r^2(1-e^{-rT_1})^2} - \frac{(re^{-rM_1} + h)C_p(1-d)De^{-rT_1}}{(1-e^{-rT_1})^2}$$
$$+ \frac{[C_0 r + (re^{-rM_1} + h)\, C_p(1-d)DT_1]re^{-rT_1}}{(1-e^{-rT_1})^2} + \frac{2r[C_0 r + (re^{-rM_1} + h)\, C_p(1-d)DT_1]\, e^{-2rT_1}}{(1-e^{-rT_1})^3}$$
$$= -\left\{\frac{(re^{-rM_1} + h)C_p(1-d)D}{r(1-e^{-rT_1})^2} - \frac{[C_0 r + (re^{-rM_1} + h)C_p(1-d)DT_1]\, e^{-rT_1}}{(1-e^{-rT_1})^3}\right\}(2re^{-rT_1})$$
$$+ \frac{[C_0 r + (re^{-rM_1} + h)\, C_p(1-d)DT_1]\, r\, e^{-rT_1}}{(1-e^{-rT_1})^2} \text{ (by using (6))}$$
$$= \frac{[C_0 r + (re^{-rM_1} + h)\, C_p(1-d)DT_1]\, r\, e^{-rT_1}}{(1-e^{-rT_1})^2} > 0, \tag{A4}$$

which proves that the unique solution $T_1^* \in (0,\infty)$ minimizes $PV_1(\infty)$.