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Minimizing Deviations Models for Solving MADM Problems with Preference Information on Alternatives in Uncertain Linguistic Setting

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Abstract—In this paper, we investigate the multiple attribute decision making (MADM) in uncertain linguistic setting where the information about attribute weights is incompletely known and the attribute values are uncertain linguistic variables, and the decision maker (DM) has preferences on alternatives. We establish two optimization models, which minimize deviations between the overall attribute values of alternatives and the overall preference values. Based on these two models and a formula of possibility degree for the comparison between uncertain linguistic variables, we propose a method for MADM with preference information on alternatives in uncertain linguistic setting. The method can sufficiently meet the DM's requirements and can also be performed on computer easily. Finally, we apply the method to evaluate university faculty for tenure and promotion.

Keywords—Multiple attribute decision making (MADM), Uncertain linguistic variables, Model, Deviation, Preference

1. INTRODUCTION

Multiple attribute decision making (MADM) with preference information on alternatives is an interesting research topic having received a great deal of attention from researchers (Kruskal, 1964a, 1964b; Srinivasan and Shocker, 1973; Hwang and Yoon, 1981; Malakooti and Zhou, 1994; Xu, 2004a). Up to now, many methods have been proposed for dealing with the MADM problems with numerical preference information on alternatives, such as the multidimensional scaling method with ideal point (Kruskal, 1964a, 1964b), linear programming techniques for multidimensional analysis of preference (Srinivasan and Shocker, 1973), interactive simple additive weighting method (Hwang and Yoon, 1981), artificial neural network method (Malakooti and Zhou, 1994), nonlinear programming method based on fuzzy preference relation (Fan et al., 2002), interval numbers based optimization approach (Xu, 2004a). In some situations, however, the decision information takes the form of uncertain linguistic variables rather than numerical ones because of time pressure, lack of knowledge, and the decision maker (DM)'s limited attention and information processing capabilities (Xu, 2004b). All these methods are unsuitable for solving the MADM problem, in which the information about attribute weights is incompletely known and the attribute values are uncertain linguistic variables, and the DM has also uncertain linguistic preferences on alternatives. To overcome this limitation, in this paper, we shall propose a practical method for MADM with preference information on alternatives in uncertain linguistic setting. In order to do so, the remainder of this paper is structured as follows. In Section 2 we introduce some basic concepts, and several operational laws of uncertain linguistic variables. In Section 3 we give a representation of the problem. In Section 4 we shall establish two optimization models. Based on the models and a formula of possibility degree for the comparison between uncertain linguistic variables, we propose a method for ranking alternatives. In Section 5 we apply the method to evaluate university faculty for tenure and promotion, and in Section 6, some concluding remarks are pointed out.

2. PRELIMINARIES

In decision making with linguistic information, the DM generally provides his/her assessment information by mean of linguistic variables (Delgado et al., 1993; Herrera, 1995; Herrera et al., 1996; Torra, 1996; Bordogna et al., 1997; Zadeh and Kacprzyk, 1999; Herrera and Martínez, 2000; Herrera and Martínez, 2001; Xu and Da, 2003; Xu, 2004c, 2004d). For example, when evaluating the comfort or design of a car, labels like good, fair, poor can be used; when evaluating a car's speed linguistic labels like fast, very fast, slow can be used (Bordogna et al., 1997). Suppose that $S = \{s_i \mid i = -t, \ldots, t\}$ is a finite and totally ordered discrete label set. Any label, s_i, represents a possible value for a linguistic variable, and it requires that $s_i < s_j$ iff $i < j$. To preserve all the given information, we extend the

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discrete label set S to a continuous label set $\bar{S} = \{s_a \mid a \in [-q, q]\}$, where $q (q > t)$ is a sufficiently large positive integer. If $s_\alpha \in S$, then we call s_α an original label, otherwise, we call s_α a virtual label. In general, the DM uses the original linguistic labels to evaluate alternatives, and the virtual linguistic labels can only appear in calculation (Xu, 2005).

Let $\tilde{s} = [s_\alpha, s_\beta]$, where $s_\alpha, s_\beta \in \bar{S}$, s_α and s_β are the lower and the upper limits, respectively, we then call $\tilde{s}$ an uncertain linguistic variable (Xu, 2004a). Let $\tilde{S}$ be the set of all the uncertain linguistic variables.

Consider any three uncertain linguistic variables $\tilde{s} = [s_\alpha, s_\beta]$, $\tilde{s}_1 = [s_{\alpha_1}, s_{\beta_1}]$, and $\tilde{s}_2 = [s_{\alpha_2}, s_{\beta_2}]$, then their operational laws are defined as follows (Xu, 2004b):

(i) $\tilde{s}_1 \oplus \tilde{s}_2 = [s_{\alpha_1}, s_{\beta_1}] \oplus [s_{\alpha_2}, s_{\beta_2}] = [s_{\alpha_1} \oplus s_{\alpha_2}, s_{\beta_1} \oplus s_{\beta_2}]$;

(ii) $\lambda \tilde{s} = \lambda [s_\alpha, s_\beta] = [\lambda s_\alpha, \lambda s_\beta] = [s_{\lambda\alpha}, s_{\lambda\beta}]$, where $\lambda \in [0, 1]$.

In what follows, we introduce a simple formula for comparing two uncertain linguistic variables $\tilde{s}_1 = [s_{\alpha_1}, s_{\beta_1}]$ and $\tilde{s}_2 = [s_{\alpha_2}, s_{\beta_2}]$, that is,

$$p(\tilde{s}_1 \geq \tilde{s}_2) = \frac{\min\{(\beta_1 + \beta_2) - (\alpha_1 + \alpha_2), \max(\beta_1 - \alpha_2, 0)\}}{(\beta_1 + \beta_2) - (\alpha_1 + \alpha_2)} \tag{1}$$

where $p(\tilde{s}_1 \geq \tilde{s}_2)$ is called the degree of possibility of $\tilde{s}_1 \geq \tilde{s}_2$. Especially, if both the linguistic variables $\tilde{s}_1$ and $\tilde{s}_2$ express precise information (that is, both $\tilde{s}_1$ and $\tilde{s}_2$ are reduced to linguistic variables with $s_{\alpha_1} = s_{\beta_1}$ and $s_{\alpha_2} = s_{\beta_2}$, in this case, $\alpha_1 = \beta_1$ and $\alpha_2 = \beta_2$, i.e., $\alpha_1 + \alpha_2 = \beta_1 + \beta_2$), then we define the degree of possibility of $\tilde{s}_1 > \tilde{s}_2$ as

$$p(\tilde{s}_1 > \tilde{s}_2) = \begin{cases} 1, & \text{if } \alpha_1 > \alpha_2 \\ 1/2, & \text{if } \alpha_1 = \alpha_2 \\ 0, & \text{if } \alpha_1 < \alpha_2 \end{cases} \tag{2}$$

Obviously, the possibility degree $p(\tilde{s}_1 \geq \tilde{s}_2)$ satisfies the following properties:

$$0 \leq p(\tilde{s}_1 \geq \tilde{s}_2) \leq 1, \quad p(\tilde{s}_1 \geq \tilde{s}_2) + p(\tilde{s}_2 \geq \tilde{s}_1) = 1,$$
$$p(\tilde{s}_1 \geq \tilde{s}_1) = p(\tilde{s}_2 \geq \tilde{s}_2) = 0.5$$

Below we define the distance between the linguistic variables $\tilde{s}_1 = [s_{\alpha_1}, s_{\beta_1}]$ and $\tilde{s}_2 = [s_{\alpha_2}, s_{\beta_2}]$ as:

$$D(\tilde{s}_1, \tilde{s}_2) = \frac{1}{2}\left[|\alpha_2 - \alpha_1| + |\beta_2 - \beta_1|\right] \tag{3}$$

3. REPRESENTATION OF THE PROBLEM

The MADM problem considered in this paper can be represented as follows:

Let $X = \{x_1, x_2, ..., x_n\}$ be a finite set of alternatives and $G = \{G_1, G_2, ..., G_m\}$ be a finite set of attributes, and $w = \{w_1, w_2, ..., w_m\} \in H$ be the weight vector of attributes, where $w_i \geq 0, \sum_{i=1}^{m} w_i = 1$, H is the set of the known weight information, which can be constructed by the following forms (Park and Kim, 1997; Kim and Ahn, 1999), for $i \neq j$:

(i) A weak ranking: $\{w_i \geq w_j\}$;

(ii) A strict ranking: $\{w_i - w_j \geq \alpha_i\}$;

(iii) A ranking with multiples: $\{w_i \geq \alpha_i w_j\}$;

(iv) An interval form: $\{\alpha_i \leq w_i \leq \alpha_i + \varepsilon_i\}$;

(v) A ranking of differences: $\{w_i - w_j \geq w_k - w_l\}$, for $j \neq k \neq l$,

where $\{\alpha_i\}$ and $\{\varepsilon_i\}$ are non-negative constants. Let $\tilde{A} = (\tilde{a}_{ij})_{m\times n}$ be the uncertain linguistic decision matrix, where $\tilde{a}_{ij} \in \tilde{S}$, which is an attribute value, given by the DM for the alternative $x_j \in X$ with respect to the attribute $G_i \in G$. The DM has also an overall preference for the alternative $x_j \in X$, and let the overall preference value be $\tilde{v}_j \in \tilde{S}$.

Based on the uncertain linguistic decision matrix $\tilde{A} = (\tilde{a}_{ij})_{m\times n}$, the overall attribute value of the alternative x_j can be expressed as

$$\tilde{z}_j(w) = w_1\tilde{a}_{1j} \oplus w_2\tilde{a}_{2j} \oplus \cdots \oplus w_m\tilde{a}_{mj}, \quad j = 1, 2, ..., n \tag{4}$$

Obviously, the greater the value $\tilde{z}_j(w)$, the better the alternative x_j will be.

In the situation where the information about attribute weights is completely known, we can rank all the alternatives by Eq. (4). To do so, the following steps are involved:

(Algorithm I)

Step 1. By Eq. (4), we calculate the overall attribute values $\tilde{z}_j(w)$ $(j = 1, 2, ..., n)$.

Step 2. Compare each $\tilde{z}_j(w)$ with all the $\tilde{z}_j(w)$ (j = 1, 2, …, n) by using (1). For simplicity, let $p_{ij} = p(\tilde{z}_i(w) \geq \tilde{z}_j(w))$, then we develop a complementary matrix (Chiclana et al., 2001; Xu and Da, 2002, 2005; Xu, 2004e) as $P = (p_{ij})_{n\times n}$, where $p_{ij} \geq 0$, $p_{ij} + p_{ji} = 1$, $p_{ii} = 1/2$, i, j = 1, 2, …, n.

Summing all elements in each line of the matrix *P*, we have

$$p_i = \sum_{j=1}^{n} p_{ij},\ i = 1,\ 2,\ ...,\ n, \tag{5}$$

Step 3. Rank the overall attribute values $\tilde{z}_j(w)$ (j = 1, 2, …, n) in descending order in accordance with the values of p_i ($i = 1,\ 2,\ ...,\ n$).

Step 4. Rank all the alternatives x_j ($j = 1,\ 2,\ ...,\ n$) and select the most desirable one(s) in accordance with the overall attribute values $\tilde{z}_j(w)$ ($j = 1,\ 2,\ ...,\ n$).

Step 5. End.

However, in this paper, the information about the attribute weights in the problem considered is incompletely known. Thus, we need to determine the attribute weights in advance. In the next section, we shall develop two models to determine the attribute weights.

4. A METHOD FOR RANKING ALTERNATIVES

In the real life, there always exist some differences between the overall attribute values $\tilde{z}_j(w)$ ($j = 1,\ 2,\ ...,\ n$) and the corresponding overall preference value $\tilde{v}_j$ ($j = 1,\ 2,\ ...,\ n$) given by the DM for the alternative x_j ($j = 1,\ 2,\ ...,\ n$). We introduce the following deviation between $\tilde{z}_j(w)$ and $\tilde{v}_j$:

$$D_j(w) = \sum_{i=1}^{m} D(\tilde{a}_{ij}, \tilde{v}_j) w_i,\ j = 1,\ 2,\ ...,\ n \tag{6}$$

To determine the attribute weights, we shall minimize the sum of all deviations between the overall attribute values and the corresponding overall preference values for alternatives. Therefore, by Eq. (3) and (6), we can establish the following linear programming model:

$$\text{(M-1)}\quad \min D(w) = \sum_{j=1}^{n} D_j(w) = \sum_{i=1}^{m}\sum_{j=1}^{n} D(\tilde{a}_{ij}, \tilde{v}_j) w_i$$

$$s.t.\ \begin{cases} w \in H, \\ w_i \geq 0,\ \sum_{i=1}^{m} w_i = 1. \end{cases}$$

By solving the model (M-1), we can get the optimal solution $w^* = (w_1^*, w_2^*, ..., w_m^*)^T$.

If the information about attribute weights is completely unknown, and Eq. (6) is replaced with the following deviation function:

$$\bar{D}_j(w) = \sum_{i=1}^{m} \bar{D}(\tilde{a}_{ij}, \tilde{v}_j) w_i^2,\ j = 1,\ 2,\ ...,\ n \tag{7}$$

where $\bar{D}(\tilde{s}_1,\ \tilde{s}_2) = \frac{1}{2}\left[(\alpha_2 - \alpha_1)^2 + (\beta_2 - \beta_1)^2\right]$, , for any $\tilde{s}_1 = [s_{\alpha_1},\ s_{\beta_1}]$ and $\tilde{s}_2 = [s_{\alpha_2},\ s_{\beta_2}] \in \tilde{S}$. Then, we can establish the following optimization model:

$$\text{(M-2)}\quad \min \bar{D}(w) = \sum_{j=1}^{n} \bar{D}_j(w) = \sum_{i=1}^{m}\sum_{j=1}^{n} \bar{D}(\tilde{a}_{ij}, \tilde{v}_j) w_i^2$$

$$s.t.\ w_i \geq 0,\ \sum_{i=1}^{m} w_i = 1.$$

To solve this model, we construct the Lagrange function

$$L(w, \lambda) = \bar{D}(w) + 2\lambda\left(\sum_{i=1}^{m} w_i - 1\right) \tag{8}$$

where λ is the Lagrange multiplier.

Differentiating Eq. (8) with respect to $w_i (i = 1,\ 2,\ ...,\ m)$ and λ, and setting these partial derivatives equal to zero, the following set of equations is obtained:

$$\begin{cases} \dfrac{\partial L(w,\lambda)}{\partial w_i} = 2\sum_{j=1}^{n} \bar{D}\left(\tilde{a}_{ij}, \tilde{v}_j\right) w_i + 2\lambda = 0 \\ \dfrac{\partial L(w,\lambda)}{\partial \lambda} = \sum_{i=1}^{m} w_i - 1 = 0 \end{cases} \tag{9}$$

By solving Eq. (9), we get the optimization solution

$$w_i^* = \left(\sum_{i=1}^{m}\left(\sum_{j=1}^{n} \bar{D}\left(\tilde{a}_{ij}, \tilde{v}_j\right)\right)^{-1}\right)^{-1}\left(\sum_{j=1}^{n} \tilde{D}\left(\tilde{a}_{ij}, \tilde{v}_j\right)\right)^{-1}, \tag{10}$$

$i = 1,\ 2,\ ...,\ m$

Obviously, $w_i^* \geq 0$, for all i.

From the above analysis, we know that both Eqs. (6) and (7) reflect the deviation between the overall attribute value $\tilde{z}_j(w)$ and the corresponding preference value $\tilde{v}_j$ given by the DM for the alternative x_j. For convenience of calculation, we utilize Eq. (6) to establish a simple linear programming model (M-1), and utilize Eq. (7) to establish a nonlinear programming model (M-2). It is worth pointing out that by solving the nonlinear programming model (M-2), we can obtain the simple formula (10), by which an exact solution of the weight vector of attributes can be obtained.

Based on the above results, in the following, we shall propose a method for MADM with preference information on alternatives under uncertain linguistic environment.

Step 1. Let $X = \{x_1, x_2, ..., x_n\}$ and $G = \{G_1, G_2, ..., G_m\}$ be the finite sets of alternatives and attributes, respectively, and $w = (w_1,\ w_2,\ ...,\ w_m) \in H$ be the weight vector of attributes, where $w_i \geq 0$,

$\sum_{i=1}^{m} w_i = 1$, H is the set of the known weight information, which can be constructed by the forms (i)~(v). Let $\tilde{A} = (\tilde{a}_{ij})_{m\times n}$ $(\tilde{a}_{ij} \in \tilde{S})$ be the uncertain linguistic decision matrix, and the DM provides also an overall preference value $\tilde{v}_j \in \tilde{S}$ for the alternative $x_j \in X$.

Step 2. If the information about the attribute weights is partly known, then we solve the model (M-1) to obtain the attribute weights. If the information about the attribute weights is completely unknown, then we solve the formula (10) to determine the attribute weights.

Step 3. Utilize the algorithm I to rank alternatives and get the most desirable one(s).

Step 4. End.

5. ILLUSTRATIVE EXAMPLE

In this section, a MADM of evaluating university faculty for tenure and promotion (adapted from Bryson and Mobolurin, 1995) is used to illustrate the proposed method.

A practical use of the proposed method involves the evaluation of university faculty for tenure and promotion. The criteria (attributes) used at some universities are G_1: teaching, G_2: research, and G_3: service. Five faculty candidates (alternatives) x_j $(j = 1, 2, 3, 4, 5)$ are to be evaluated using the label set

S = {s_{-4} = extremely poor, s_{-3} = very poor, s_{-2} = poor, s_{-1} = slightly poor, s_0 = fair, s_1 = slightly good, s_2 = good, s_3 = very good, s_4 = extremely good}

by the DM under these three attributes, as listed in Tables 1.

Table 1. Uncertain linguistic decision matrix $\tilde{A}$

G_i	x_1	x_2	x_3	x_4	x_5
G_1	$[s_{-2}, s_0]$	$[s_2, s_3]$	$[s_{-1}, s_1]$	$[s_3, s_4]$	$[s_1, s_3]$
G_2	$[s_2, s_4]$	$[s_3, s_4]$	$[s_{-3}, s_{-1}]$	$[s_1, s_3]$	$[s_2, s_3]$
G_3	$[s_0, s_1]$	$[s_2, s_4]$	$[s_1, s_3]$	$[s_{-2}, s_0]$	$[s_0, s_2]$

Suppose that the DM provides his/her overall preference values for the alternatives x_j $(j = 1, 2, 3, 4, 5)$ as follows:

$\tilde{v}_1 = [s_0, s_2]$, $\tilde{v}_2 = [s_2, s_3]$, $\tilde{v}_3 = [s_0, s_1]$, $\tilde{v}_4 = [s_1, s_2]$, $\tilde{v}_5 = [s_1, s_3]$,

(i) If the known weight information is as follows:

$$H = \{0.40 \le w_1 \le 0.42,\ 0.26 \le w_2 \le 0.30,\ w_3 \ge w_2,\ w_1 - w_3 \ge 0.10,\ w_1 - w_2 \ge w_3 - w_1\}$$

then by the model (M-1), we establish the following linear programming model:

$$\begin{aligned}
&\min D(w) = 4.5w_1 + 6w_2 + 6w_3 \\
&s.t. \\
&0.40 \le w_1 \le 0.42, \\
&0.26 \le w_2 \le 0.30, \\
&w_3 \ge w_2,\ w_1 - w_3 \ge 0.10, \\
&w_1 - w_2 \ge w_3 - w_1,\quad w_i \ge 0,\ i = 1,2,3, \\
&w_1 + w_2 + w_3 = 1
\end{aligned}$$

Solving this model, we get the weight vector of attributes: $w^* = (0.42, 0.26, 0.32)$

By Eq. (4), we obtain the overall attribute values $\tilde{z}_j(w^*)$ (j = 1, 2, 3, 4, 5) of alternatives x_j(j = 1, 2, 3, 4, 5):

$\tilde{z}_1(w^*) = [s_{-0.32}, s_{1.36}]$, $\tilde{z}_2(w^*) = [s_{2.26}, s_{3.58}]$,

$\tilde{z}_3(w^*) = [s_{-0.88}, s_{1.12}]$, $\tilde{z}_4(w^*) = [s_{0.88}, s_{2.46}]$,

$\tilde{z}_5(w^*) = [s_{0.94}, s_{2.68}]$

Comparing each $\tilde{z}_j(w)$ with all the $\tilde{z}_j(w)$ (j = 1, 2, 3, 4, 5) by using Eq. (1), we develop a complementary matrix as

$$P = \begin{bmatrix} 0.5 & 0 & 0.6087 & 0.1472 & 0.1228 \\ 1 & 0.5 & 1 & 0.9310 & 0.8627 \\ 0.3913 & 0 & 0.5 & 0.0670 & 0.0481 \\ 0.8528 & 0.0690 & 0.9330 & 0.5 & 0.4578 \\ 0.8772 & 0.1373 & 0.9519 & 0.5422 & 0.5 \end{bmatrix}$$

Summing all elements in each line of the matrix P, we have p_1 = 1.3787, p_2 = 4.2937, p_3 = 1.0064, p_4 = 2.8126, p_5 = 3.0086 then, we rank the overall attribute values $\tilde{z}_j(w)$ $(j = 1, 2, 3, 4, 5)$ in descending order in accordance with the values of p_i (i = 1, 2, 3, 4, 5): $\tilde{z}_2(w) > \tilde{z}_5(w) > \tilde{z}_4(w) > \tilde{z}_1(w) > \tilde{z}_3(w)$ and thus the ranking of all alternatives x_j $(j = 1, 2, ..., n)$ is $x_2 \succ x_5 \succ x_4 \succ x_1 \succ x_3$. Therefore, the most desirable alternative is x_2.

(ii) If the information about the attribute weights is completely unknown, then by Eq. (10), we have w^* = (0.385, 0.271, 0.344)

By Eq. (4), we obtain the overall attribute values $\tilde{z}_j(w^*)$ $(j = 1,2,3,4,5)$ of alternatives x_j (j = 1, 2, 3, 4, 5):

$\tilde{z}_1(w^*) = [s_{-0.228}, s_{1.428}]$, $\tilde{z}_2(w^*) = [s_{2.271}, s_{3.615}]$,

$\tilde{z}_3(w^*) = [s_{-0.854}, s_{1.146}]$, $\tilde{z}_4(w^*) = [s_{0.738}, s_{2.353}]$,

$\tilde{z}_5(w^*) = [s_{0.927}, s_{2.656}]$

Comparing each $\tilde{z}_j(w)$ with all the $\tilde{z}_j(w)$ (j = 1, 2, 3, 4, 5) by using Eq. (1), we develop a complementary matrix as

$$P = \begin{bmatrix} 0.5 & 0 & 0.624 & 0.211 & 0.148 \\ 1 & 0.5 & 1 & 0.972 & 0.865 \\ 0.376 & 0 & 0.5 & 0.113 & 0.059 \\ 0.789 & 0.028 & 0.887 & 0.5 & 0.426 \\ 0.852 & 0.125 & 0.941 & 0.74 & 0.5 \end{bmatrix}$$

Summing all elements in each line of the matrix P, we have $p_1 = 1.483$, $p_2 = 4.347$, $p_3 = 1.048$, $p_4 = 2.630$, $p_5 = 2.992$ then, we rank the overall attribute values $\tilde{z}_j(w)$ (j = 1, 2, 3, 4, 5) in descending order in accordance with the values of p_i $(i = 1, 2, 3, 4, 5)$: $\tilde{z}_2(w) > \tilde{z}_5(w) > \tilde{z}_4(w) > \tilde{z}_1(w) > \tilde{z}_3(w)$ and thus the ranking of all alternatives $x_j\,(j = 1,\ 2,\ ...,\ n)$ is $x_2 \succ x_5 \succ x_4 \succ x_1 \succ x_3$. Therefore, the most desirable alternative is x_2.

6. CONCLUDING REMARKS

At present, many methods have been proposed for dealing with the MADM problems with numerical preference information on alternatives. However, the increasing complexity of the socio-economic environment or time pressure, lack of knowledge or data, and his/her limited expertise related to the problem domain usually make a DM provides his/her decision information within uncertain linguistic variables. In this paper, we have investigated the MADM with preference information on alternatives in uncertain linguistic setting. We have established two optimization models, which minimize deviations between the overall attribute values of alternatives and the overall preference values, and proposed a method for ranking alternatives. The method can sufficiently meet the DM's requirements and can also be performed on computer easily. A practical application has also been given to illustrate the proposed method.

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