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An Integrated View of Risk Management: Portfolio Allocation Incorporating Credit Risk

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Abstract—Markowitz (1952, 1959) first proposed a well-known mean-variance analysis for optimizing portfolio diversification that has been long served as a foundation of modern finance. The risk diversification was formulated by a quadratic optimization model. Unfortunately, the quadratic optimization had a computational difficulty in dealing with a large number of asset allocations. To enhance the computational capability, Konno and Yamazaki (1991) incorporated the concept of time into the mean-variance analysis and expressed the market risk by a variance of return in an absolute form. The risk diversification was formulated by goal programming and solved by linear programming. The computational issue is solved, indeed. However, both approaches consider only the market risk measured by a variance of return, but not paying attention to credit risk (e.g., bankruptcy). Furthermore, they do not pay attention to a fact that the market risk is expressed by Value-at-Risk (VaR). To overcome such methodological issues on risk management, this study explores how to incorporate both credit risk and market risk into a VaR model. The proposed approach is applied to asset allocation that is composed of stocks related to Japanese electric industry. The performance of the proposed approach is compared with the risk diversification model proposed by Konno and Yamazaki (1991). In the comparison, it is confirmed that the former performs at least as well as the latter in sluggish economy.

Keywords—Portfolio analysis, Credit risk, Market risk

1. INTRODUCTION

For fifty years, the argument that asset-specific risk must be diversified in optimal risky asset allocations has been well established. Markowitz (1952, 1959) first proposed a mean-variance analysis for optimizing portfolio diversification, assuming that it is reasonable for a risk-averse individual to maximize an expected profit or gain while restricting the variability of the expected return. His contribution depends upon an assumption that financial (market) risk is expressed by a variance of rate of return. The variance needs to be minimized under the condition that the expected return is larger than the minimum return required by each investor. Mathematically, the principle of risk diversification is formulated by a quadratic optimization model (which is referred to as a L_2 risk model). After his contribution was published, many research efforts have been dedicated to the extension of the mean-variance analysis. For instance, see Sharpe (1963, 1964) and Constantinides and Malliaris (1995) which compile approximately 100 previous important research efforts on portfolio theory.

Unfortunately, quadratic optimization has a computational disadvantage in the mean-variance portfolio selection problem when it has to deal with a large number of asset allocations. To enhance the computational capability, Konno and Yamazaki (1991, hereafter K-Y) have proposed an alternative to the quadratic formulation. They incorporated the concept of time into the analytical framework of the mean-variance analysis and then expressed the financial risk by the variance in an absolute form. Mathematically, their risk diversification is formulated by a goal programming model (so, it is a L_1 risk model) and solved by any linear programming algorithm. A consequence of their contribution is that investors can handle a large-scale risk diversification problem.

While acknowledging the importance and contribution of these previous efforts on the mean-variance analysis, we consider another important perspective regarding long-term financial risk diversification. That is, all financial institutions are carefully audited by various regulatory agencies. The Basel Accords, for example, stipulate financial practices that should be followed by banks in their risk management. Those requirements are now regulatory standards for supervisory authorities worldwide. The Basel Accords allows financial institutions to develop their internal models that are used to measure risk exposures for regulatory fulfillment. Value-at-Risk (VaR) has become the standard of such measurement in establishing a long-term financial strength. Here, VaR implies a fractile of a return distribution. Shareholders usually seriously care about an extreme loss that may drive them and/or their institutions

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into bankruptcy. For instance, a value of VaR, e.g., 90%, implies that a single potential disaster may occur once in every 10 days, hence requiring some action for restoration of financial health. Many institutions are required to monitor their VaR and keep an adequate amount of capital to prepare for a major financial loss. Limited liability protects most of modern corporations in a case when an institution faces bankruptcy. (See, for instance, Basak and Shapiro (2001), Campbell et al. (2001), Fusai and Luciano (2001), Jansen et al. (2000), Rockafeller and Uryasev (2000) for recent VaR developments. See also Duffie and Pan (1997) who have provided an overview on VaR.)

The type of risk hedge (VaR) is not new, dating back to the research effort of Kataoka (1963) who has formulated the portfolio optimization problem by a probability model. In his model, the concept of portfolio risk is mathematically expressed by likelihood that a portfolio return becomes lower than its allowable limit. This type of portfolio diversification was called "Safety First". (See Roy (1952) and Telser (1955) that first proposed the criterion. See also Arzac and Bawa (1977), Bawa (1975, 1978), Bawa and Lindenberg (1977) and Gotoh and Konno (2000) who have discussed the relationship between stochastic dominance and the concept of safety first.) As expressed in VaR, an underlying assumption of the approach is that investors are unable (or unwilling) to use the mean-variance analysis, but rather use a simpler decision rule that limits the chance of a bad outcome (or an expected shortfall). The risk-averse portfolio diversification has been long neglected under the shadow of Markowitz's mean-variance analysis. However, the above description on the relationship between Basel Accords and VaR clearly indicates a research need to explore how to incorporate VaR into our portfolio diversification.

In addition to the VaR-based portfolio optimization, we need to add another important perspective associated with modern investment. Recently, investors can easily access information regarding market indices through Internet and other information devices. Financial indices provide information necessary to guide investors in assessing how well each firm manages its own risks. Bankruptcy is the most extreme of many corporate credit risks and the worst disaster. (See, for instance, Altman (1968, 1983, 1984), Frydman and Altman (1985), Wilcox (1973), Ohlson (1980), Burgstahler et al. (1989), Laitinen (1993), Hensen et al. (1996), and Westgaard and Wijst (2001).) It is possible now to predict the default probability of each portfolio component and allocation, using financial indices. Thus, investors must take an integrated view of the financial risk management process, considering simultaneously both market and credit risks. The integration of such two different risks requires an analytical tool that incorporates them over a long-run horizon. It is mathematically envisioned that such an analytical approach with a large-scale optimization capability may provide a powerful tool to integrate the risk management process. The task is the main research issue of this study.

The structure of this article is organized as follows: The next section (2) proposes a probability model for VaR. In Section 3, a default probability associated with each security allocation is incorporated into the proposed probability model. Mathematical properties of the probability model are explored in Section 4. Section 5 applies the proposed model for analyzing Japanese electric industry stocks. The performance of the proposed approach is compared with that of the L_1 risk model. A conclusion and future extensions are all summarized in the last section (6).

2. VAR AS MARKET RISK

2.1 Market risk

Deviation as market risk: As mentioned previously, Markowitz (1952, 1959) has proposed the mean-variance analysis for optimizing portfolio diversification in which market risk is defined by a variance of return. The portfolio variance needs to be minimized under the condition that an expected return is larger than the minimum return required by each investor.

Figure 1 depicts such a deviation of return. It is indeed acceptable that we need to minimize the deviation at the left hand side of Figure 1 where a return is equal to or lower than its mean. However, the minimization of variance also implies the reduction of variance (at the right hand side of Figure 1) where the return is equal to or larger than the mean. Such an analytical feature is often problematic in real portfolio analysis, since variance is a "two-sided" risk measure.

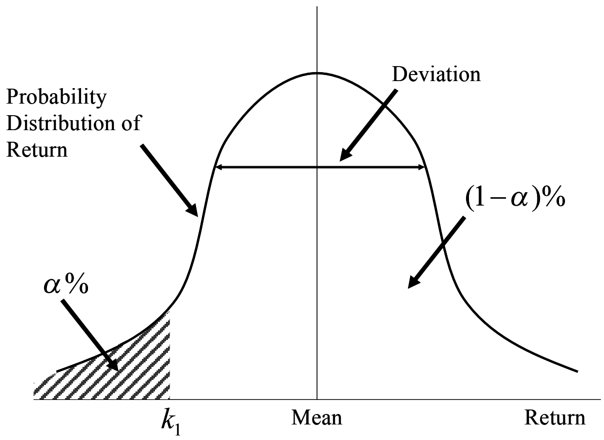

Figure 1. $(1-\alpha)$ VaR.

VaR as market risk: Meanwhile, as found in the Basel Accords, many financial institutions are recently concerned about the concept of VaR, where an investor attempts to avoid a major financial loss. In Figure 1, such a major loss, or an expected shortfall, implies a case where the return is equal to or less than a specific amount of tolerance limit (k_1). Here, the $(1-\alpha)\%$ VaR indicates a probability $(1-\alpha)\%$ which the return is equal to or more than k_1. The opposite case can be identified by the remaining probability ($\alpha\%$) that indicates a shortfall probability. For example, a value of VaR (e.g., 99%VaR) implies that a single major loss may occur on return once in every 100 annual periods. (The concept of VaR is not limited on the return, rather including much larger implications on financial decisions. In this study, the concept is used only on return, so that we can compare it with the conventional mean-variance analysis.)

Important features of VaR: Returning to the definition of market risk, the following comments are useful in understanding the VaR model explored in this study. First, the market risk discussed in this study is VaR, not the deviation of return used in the mean-variance analysis. The risk measure can be defined as a shortfall probability (α%) regarding an occurrence of a major financial loss where the return becomes equal to or less than the lower tolerance level of return (k_1). Second, a downside risk measure (i.e., VaR) is more attractive to many individuals than the variance. That is, if the value of VaR is set to be relatively high, then they need to pay attention to only a portfolio allocation that produces a possible occurrence of an extreme financial loss specified by VaR. A monitoring process is easily implemented under the VaR measurement. Consequently, VaR is now conceptually widely used by many financial institutions (Konno, 2001).

2.2 Methodological issues on market risk (VaR) and credit risk

It is indeed true that VaR has conceptually opened up a new avenue for a monitoring process of financial and banking institutions. However, when the risk measure is applied to real portfolio allocation, we must consider the following methodological and practical difficulties, all of which need to be overcome in this study:

Computational issue: The VaR measurement is mathematically formulated by a probability model. It is not impossible, but very difficult, to solve the probability model directly. So, the computational issue becomes very serious when we deal with a large scale fund allocation. Hence, there is an open question regarding how to incorporate the VaR measurement computationally into portfolio analysis. Thus, the computational feasibility is critical in the implementation of VaR for asset allocation.

Credit risk: In this study, the market risk measured by VaR which is usually closely linked to another type of risk, referred to as "credit risk". In particular, many investors pay attention to corporate bankruptcy as the worst disaster in their decisions on portfolio allocation. It is easily imagined that VaR practically involves a default probability of each portfolio component. Hence, the default probability is defined as the credit risk in this research. However, the close linkage does not imply a perfect relationship between the two risk measures. Hence, we need to develop an analytical approach which can incorporate simultaneously the two risk measures in portfolio allocation. It is expected that the resulting combined portfolio scheme may provide investors with more return at a given risk level than the fund allocation based upon the market risk only.

General view on market risk and credit risk: First, many portfolio managers are interested in not only a default probability, but also credit rating of portfolio. Hence, the measurement of the default probability needs to be assessed with the corporate rating of non-default firms. Second, as mentioned in Thomas Wilson (1997a, b), the credit risk needs to be examined by many uncontrollable factors such as the current status of economy, a country risk and an industry-specific risk. Finally, there is a certain level of correlation between market risk and credit risk. In other words, there is some level of dependency between the rate of return and the default probability. However, it is true that we can conceptually describe the existence of such correlation between market risk and credit risk, but it is very difficult (not impossible) to numerically assess the correlation.

2.3 Formulation

To incorporate the two types of risk into portfolio analysis, this study proposes a VaR portfolio model that has the following assumptions and these related notations:

Assumptions and notations: First, it is assumed that an investor makes his/her decision on fund allocation at the current period, accessing only previous information regarding the rate of return and the default probability. Both are measured in "T" annual periods (t = 1, …, T), where the initial and last annual periods are expressed by "1" and "T", respectively. Second, a total amount of investment needs to be allocated to n assets (j = 1, …, n). Third, a set of information to be prescribed is as follows: w_1 and w_2 are weights representing for relative importance between market and credit risks ($w_1 + w_2 = 1$). α is a shortfall probability of VaR where an investor accepts an occurrence of a major financial loss with a chance of α%. I is a total amount of portfolio investment to be allocated. u_j is the upper bound of investment on the j-th asset. R_{tmj} is the rate of return invested to the j-th asset, whose performance is measured in each month (m = 1, …, b) of the t-th annual period (t), where "b" stands for the last observed month. P_{tj} is the probability in the state of non-default. Finally, unknown decision variables to be determined by the proposed approach are as follows: k_1 is an unknown decision variable that indicates the lower tolerance limit on a total portfolio return. k_2 is also an unknown decision variable indicating the lower tolerance limit on a total amount of fund allocations weighted by the non-default probabilities. An unknown decision variable (x_j), as part of I, indicates the amount to be invested to the j-th asset.

The problem of optimal risky portfolio allocations in the absence of a riskless asset is formulated as follows:

$$
\begin{aligned}
&\max.\ w_1 k_1 + w_2 k_2 \\
&s.t.\quad Prob\left(\sum_{j=1}^{n} \mathrm{R}_{tmj} x_j \le k_1\right) \le \alpha \qquad t = 1,\ ...,\ T \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \&\ m = 1,\ ...,\ b \\
&\qquad \sum_{j=1}^{n} P_{tj} x_j - k_2 \ \ge 0 \qquad\qquad t = 1, ..., T \qquad\qquad (1) \\
&\qquad \sum_{j=1}^{n} x_j = I \\
&\qquad 0 \le x_j \le u_j \qquad\qquad\qquad j = 1,\ ...,\ n \\
&\qquad k_1 \ge 0,\ k_2 \ge 0,\ and\ x_j \ge 0 \qquad j = 1,\ ...,\ n
\end{aligned}
$$

In the first set of equations, given the shortfall probability (α), the probability (*Prob*) that a total portfolio return, which is equal to or less than k_1, becomes equal to or less than the value of α. The second set of constraints indicates that the total portfolio allocation weighted by non-default probabilities is equal to or lager than k_2. The two tolerance limits are maximized in the objective of (1) in a manner that the two are weighted by w_1 and w_2, respectively.

The following three comments are important in understanding (1): First, R_{tmj} is a random variable, because the rate of return fluctuates in every moment. In solving (1), we need an exact number to specify a change of R_{tmj}. For such a purpose, an observed value (r_{tmj}) on rate of return is used as its real substitute for R_{tmj}. The monthly average rate of return (r_{tmj}) is incorporated into the proposed approach. Here, the subscript (m) stands for a month. Of course, the month can be replaced by a day or another time period. A rationale concerning why we use the monthly average exists in "Bank for International Settlement" listed in http://www.bis.org/publ/bcbs24.pdf. The statement suggests that VaR needs to be measured within 250 days (one year). Therefore, it is natural expectation that this type of measurement should be less than an annual period. Second, P_{tj} is also a random variable to represent an occurrence of a non-default in the j-th asset in the t-th annual period. It is true that the bankruptcy may occur anytime. However, the statistics of corporate bankruptcy in Japan are usually documented in an annual report. Hence, the default probability is measured annually and incorporated into (1) in this study. Furthermore, we need many financial indices regarding each asset in order to estimate P_{tj}. A logit model is used for such a purpose. Third, as mentioned previously, both k_1 and k_2 are unknown decision variables. Here, k_1 indicates the lower bound of an expected portfolio return. Meanwhile, k_2 indicates the lower bound of the portfolio allocation weighted with non-default probabilities. The two tolerance limits are determined by (1).

2.4 Reformulation

It is impossible to compute the first set of equations in (1) so that we need to change it into a deterministic equivalence. This research utilizes a chance constraint programming approach for such reformulation. (See Brockett et al. (1992), Charnes and Cooper (1959, 1963) and Li (1995) for their descriptions on how to reformulate a probability model to a deterministic equivalence.)

As mentioned previously, R_{tmj} needs to be approximated by its realization (r_{tmj}). Returning to (1), the first group of equations can be reformulated by

$$Prob\left(\sum_{j=1}^{n} r_{tmj} x_j \le k_1\right) \le \alpha, \quad \text{for all } m \text{ and } j. \tag{2}$$

It is assumed that rate of return is expressed by

$$r_{tmj} = \bar{r}_{t\cdot j} + s_{t\cdot j}\eta \tag{3}$$

where $\bar{r}_{t\cdot j} = \dfrac{\sum_{m=1}^{b} r_{tmj}}{b}$ and $s_{t\cdot j} = \left(\dfrac{1}{b-1}\cdot\sum_{m=1}^{b}(r_{tmj}-\bar{r}_{t\cdot j})^2\right)^{\frac{1}{2}}$.

The two indicate the annual average and standard deviation of the rate of return if $b = 12$, respectively.

It is also assumed that a random variable (η) follows the standard normal distribution N(0, 1). The incorporation of (3) into (2) produces

$$Prob\left\{\sum_{j=1}^{n}\left(\bar{r}_{t\cdot j} + s_{t\cdot j}\eta\right)x_j \le k_1\right\} \le \alpha \quad \text{for all } t. \tag{4}$$

The equation can be further reformulated as follows:

$$Prob\left\{\eta\sum_{j=1}^{n} s_{t\cdot j}x_j \le k_1 - \sum_{j=1}^{n}\bar{r}_{t\cdot j}x_j\right\} \le \alpha \quad \text{for all } t. \tag{5}$$

Eq. (5) is equivalent to

$$Prob\left\{\eta \le \left(k_1 - \sum_{j=1}^{n}\bar{r}_{t\cdot j}x_j\right)\Big/\left(\sum_{j=1}^{n} s_{t\cdot j}x_j\right)\right\} \le \alpha \quad \text{for all } t. \tag{6}$$

Eq. (6) can be expressed by the following formulation:

$$F\left(\left(k_1 - \sum_{j=1}^{n}\bar{r}_{t\cdot j}x_j\right)\Big/\left(\sum_{j=1}^{n} s_{t\cdot j}x_j\right)\right) \le \alpha \quad \text{for all } t \tag{7}$$

where $F(\bullet)$ indicates the cumulative distribution function of the random variable (η).

Here, let $F^{-1}(\bullet)$ be the inverse function of the cumulative distribution function, then (7) becomes

$$\left(k_1 - \sum_{j=1}^{n}\bar{r}_{t\cdot j}x_j\right)\Big/\left(\sum_{j=1}^{n} s_{t\cdot j}x_j\right) \le F^{-1}(\alpha) \quad \text{for all } t \tag{8}$$

or

$$k_1 - \sum_{j=1}^{n}\bar{r}_{t\cdot j}x_j \le F^{-1}(\alpha)\cdot\sum_{j=1}^{n} s_{t\cdot j}x_j \quad \text{for all } t. \tag{9}$$

Consequently, Eq. (1) is equivalent to the following linear programming model:

$$\begin{aligned} &\max.\ w_1k_1 + w_2k_2 \\ &s.t.\quad \sum_{j=1}^{n}\{\bar{r}_{t\cdot j} + F^{-1}(\alpha)s_{t\cdot j}\}x_j \ge k_1 \qquad t = 1, ..., T \\ &\qquad\ \ \sum_{j=1}^{n} P_{tj}x_j \ge k_2 \qquad\qquad\qquad\qquad t = 1, ..., T \end{aligned}$$

$$\sum_{j=1}^{n} x_j = I$$
$$0 \le x_j \le u_j \qquad (10)$$
$$k_1 \ge 0,\ k_2 \ge 0,\ and\ x_j \ge 0 \qquad j = 1, ..., n$$

The following two comments are useful in understanding the proposed reformulation from (1) to (10): First, this study uses the standard normal distribution for η in (3). The use of the normal distribution is due to the popularity and computational easiness of the distribution. As long as the cumulative distribution function of a distribution are available to us, we can use it for the distribution of η. Of course, we know that there is another VaR reformulation approach. For example, Kalin and Zagst (1999) have provided an approach that weakens the normality assumption in obtaining the VaR measure. Their approach requests only a condition that the return distribution is asymmetric. Second, a difference between their approach (Kalin and Zagst, 1999) and our approach is that the former maximizes an expected portfolio return, while the latter maximizes both the lower bound of the expected portfolio return and the lower bound of the portfolio allocation weighted by non-default probabilities. Furthermore, their approach expresses VaR from probability conditions. Meanwhile, we transform it as the chance-constraint programming problem. Consequently, the portfolio allocation problem examined in our approach can be solved as linear programming. Consequently, the proposed approach can maintain a computational feasibility in dealing with a large scale portfolio problem.

3. ESTIMATION OF DEFAULT/NON-DEFAULT PROBABILITIES

To describe how to estimate the non-default probabilities (P_{tj}) in (10), we need to specify a data matrix that is structured as follows:

$$Y_{itj} = \begin{bmatrix} y_{1t_1} & y_{1t_2} & \cdots & y_{1t_n} \\ y_{2t_1} & y_{2t_2} & \cdots & y_{2t_n} \\ \vdots & \vdots & \ddots & \vdots \\ y_{qt_1} & y_{qt_2} & \cdots & y_{qt_n} \end{bmatrix} \quad t = 1, ..., T \qquad (11)$$

where there are q indices (i = 1, …, q) that characterize the financial status of the j-th asset (j = 1, …, n) to be invested. The default (π_{tj} = 0) or non-default (π_{tj} = 1) of each asset is expressed by a binary response. In this study, the default/non-default status is predicted by the following linear probability model:

$$c_{tj} = \beta_0 + \sum_{i=1}^{q} \beta_i y_{itj} + \varepsilon_{tj} \quad \text{for all } t \text{ and } j \qquad (12)$$

where c_{tj} is an unobserved variable whose magnitude is determined in such a manner that π_{tj} = 1 if c_{tj} > 0 and π_{tj} = 0 if $c_{tj} \le 0$. All parameter estimates are denoted by β_i (i = 0, 1, …, q). An observational error is ε_{tj}.

It is assumed in (12) that the status of default may occur independently on any asset and any annual period. (See Wilson (1997c, d) for a detailed description on how to compute the default probabilities over an observed period, using the logit model discussed in this section.)

To identify the non-default probability (P_{tj}), we need to examine the sign of c_{tj} by

$$P_{tj} = Prob(\pi_{tj} = 1) = Prob(c_{tj} > 0)$$

and

$$1 - P_{tj} = Prob(\pi_{tj} = 0) = Prob(c_{tj} \le 0)$$

Here, 1 − P_{tj} indicates the default probability. Using (13), each non-default probability is expressed by

$$\begin{aligned} P_{tj} &= Prob(\pi_{tj} = 1) = Prob(c_{tj} > 0) \\ &= Prob(\beta_0 + \sum_{i=1}^{q} \beta_i y_{itj} + \varepsilon_{tj} > 0) \\ &= Prob\{\varepsilon_{tj} > -(\beta_0 + \sum_{i=1}^{q} \beta_i y_{itj})\} \end{aligned} \qquad (13)$$

It is assumed that ε_{tj} follows a logit model whose cumulative distribution function is

$$F(z) = \int_{-\infty}^{z} \left(e^u\right) / \left((1+e^u)^2\right) du = (1) / \left(1 + e^{-z}\right).$$

Then, (13) may be changed as follows:

$$\begin{aligned} P_{tj} &= Prob\{\varepsilon_{tj} > -(\beta_0 + \sum_{i=1}^{q} \beta_i y_{itj})\} \\ &= 1 - \frac{1}{1 + EXP(\beta_0 + \sum_{i=1}^{q} \beta_i y_{itj})} \\ &= \frac{EXP(\beta_0 + \sum_{i=1}^{q} \beta_i y_{itj})}{1 + EXP(\beta_0 + \sum_{i=1}^{q} \beta_i y_{itj})} \end{aligned} \qquad (14)$$

Eq. (14) implies

$$\beta_0 + \sum_{i=1}^{q} \beta_i y_{itj} = \ln(P_{tj} / (1 - P_{tj})) \text{ for all } t. \qquad (15)$$

The above equations suggest that the non-default probabilities (P_{tj}) can be determined immediately from the parameter estimates of the logit model. (See Madalla (1983), Hausman and McFadden (1984).)

Returning to (10), let $\hat{P}_{tj}$ be the non-default probabilities estimated by the logit model. Then, the VaR asset allocation problem can be formulated as follows:

$$\begin{aligned}
\max.\ & w_1 k_1 + w_2 k_2 \\
s.t.\ & \sum_{j=1}^{n} \{\bar{r}_{t\cdot j} + F^{-1}(\alpha) s_{t\cdot j}\} x_j \geq k_1 \quad t = 1, ..., T \\
& \sum_{j=1}^{n} \hat{P}_{tj} x_j \geq k_2 \quad t = 1, ..., T \\
& \sum_{j=1}^{n} x_j = I \\
& 0 \leq x_j \leq u_j \quad j = 1, ..., n \\
& k_1 \geq 0,\ k_2 \geq 0,\ and\ x_j \geq 0 \quad j = 1, ..., n
\end{aligned} \tag{16}$$

Finally, to implement the asset allocation model (16), an investor needs to specify the two weights (w_1 and w_2). At this stage, we must admit that there is no perfect approach to identify the best weight combination. Such is still an open question to be explored in future. In this study, we propose a use of sensitivity analysis under different combinations of the two weights. (This issue is discussed in Section 5.)

4. COMPARISON WITH L_1 RISK MODEL

Konno and Yamazaki (1991) have proposed the following L_1 risk model as an alternative to enhance the computational capability of Markowitz's quadratic formulation:

$$\begin{aligned}
\min.\ & \sum_{t=1}^{T} \left| \sum_{j=1}^{n} a_{tj} x_j \right| \Big/ T \\
s.t.\ & \sum_{j=1}^{n} \bar{r}_{\cdot j} x_j \geq hI, \\
& \sum_{j=1}^{n} x_j = I, \\
& 0 \leq x_j \leq u_j \quad j = 1, ..., n
\end{aligned} \tag{17}$$

where $a_{tj} = r_{tj} - \bar{r}_{\cdot j}$ and $\bar{r}_{\cdot j} = \sum_{t=1}^{T} r_{tj} / T$. The prescribed h is a minimum rate of portfolio return required by an investor(s). A major benefit of the L_1 risk model is that it can be an alternative to the L_2 model since both produce very similar portfolio allocations (Konno and Yamazaki, 1991). As formulated by Cooper et al. (1997), Problem (17) is equivalent to the following L_1 risk model:

$$\begin{aligned}
\min.\ & \sum_{t=1}^{T} (v_t + w_t) / T \\
s.t.\ & \sum_{j=1}^{n} a_{tj} x_j - v_t + w_t = 0 \quad t = 1, ..., T \\
& \sum_{j=1}^{n} \bar{r}_{\cdot j} x_j \geq hI \\
& \sum_{j=1}^{n} x_j = I \\
& 0 \leq x_j \leq u_j \quad j = 1, ..., n \\
& v_t \geq 0\ \ and\ w_t \geq 0 \quad t = 1, ..., T
\end{aligned} \tag{18}$$

Here, $v_t = \sum_{j=1}^{n} a_{tj} x_j \geq 0$ and $-w_t = \sum_{j=1}^{n} a_{tj} x_j \leq 0$. Eq. (18) incorporates the two variables into (17) in order to transform the variance of return ($\sum_{j=1}^{n} a_{tj} x_j$) into an equivalent linear programming problem. (The two variables must satisfy the bilinear conditions ($v_t w_t = 0$ for each t = 1, ..., T) that are needed to preserve equivalence with the minimization of an absolute value functional in (17). Fortunately, (18) can avoid such a need to incorporate the bilinear conditions into its formulation (Charnes and Cooper, 1977).)

In comparing (18) with (16), we need to consider the following differences: First, in (18), the market risk is expressed by a L_1-variance of the rate of return over an observed time period. The formulation is originally a deterministic nonlinear programming problem. On the other hand, the market risk is specified by a shortfall probability in (16). The formulation of (16) is originally a probability model. A common feature of both (16) and (18) is that those are reformulated into linear programming equivalent models. As a result of such reformulations, we can deal with large-scale portfolio problems. Second, (18) is formulated in such a manner that the variance (market risk) is minimized under the condition that the expected return is larger than the minimum return required by each investor. Meanwhile, given α (a shortfall probability), (16) maximizes both the lower tolerance limit of an expected total portfolio return measured at VaR and that of an expected total portfolio allocation weighted by non-default probabilities. Thus, the former has a single criterion, but the latter has double criteria (market and credit risks). Third, (16) assumes $r_{tmj} = \bar{r}_{t\cdot j} + s_{t\cdot j}\eta$, while (18) does not have such an assumption. In the latter model, the rate of return is measured in an observed period (t); while the former model needs not only the observed period (t) but also its sub-period (m). The return distribution in VaR is formulated by a probability model. Hence, the monthly-based sampling (m) is needed to reformulate the probability model to its deterministic equivalence.

5. AN EMPIRICAL STUDY

5.1 A flow chart for application

Figure 2 depicts a flow chart that visually describes an application process of (16). The process is separated into the following sub-processes: (a) A simple regression is first applied to a data set related to credit risk. This initial step is designed to reduce the number of financial indices. (b) A multiple regression is applied to find which financial indices are important in terms of non-default probabilities. (c) A logit model is applied to determine the non-default probabilities, using the selected financial indices. (d) An average rate of return on each asset and its related standard deviation are computed from a data set on portfolio allocation. (e) Finally, (16) is applied to determine an optimal asset allocation.

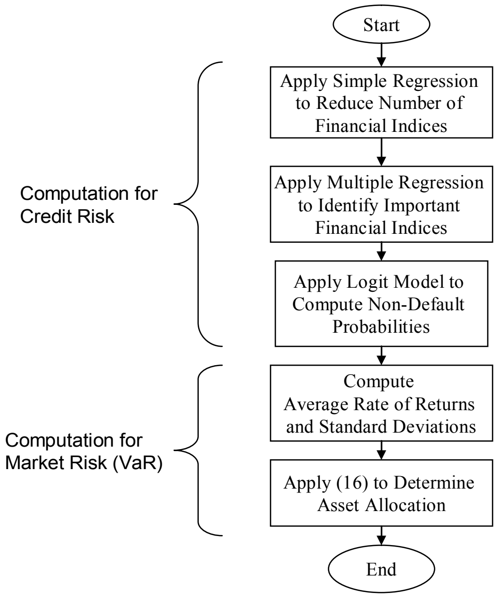

Figure 2. A flow chart for computation.

5.2 Japanese electric industry

Table 1 documents a list of 124 firms from the Japanese electric industry. All the 124 firms are listed in Tokyo Stock Exchange. To estimate the non-default probabilities, using Eq. (14), we have selected 53 firms from the 124 samples, because the firms have their rating scores. Then, we added 10 non-default firms, as listed at the next row to the bottom of Table 1. The ten non-default firms are middle-sized and well known in Japan. Therefore, those are additionally included in the whole sample set, even though those are not listed in Tokyo Stock Exchange. Furthermore, 10 default firms, listed at the bottom of Table 1, are also used for the estimation of the non-default probabilities. All the firms have experienced bankruptcy from 1991 to 2000. Each default firm was bankrupted in each specified annual period. So, it can be considered that those default probabilities distribute independently. Consequently, 73 (= 53 + 10 + 10) firms consist of a whole sample for the estimation of non-default probabilities. The 73 sample firms have their rating scores from Rating and Investment Information (August 31, 2001) Inc. that is a Japanese rating organization corresponding to Moody's and S&P in the United States. We cannot find any rating scores on the remaining other 71 (= 124 − 53) companies, even thought they are listed at Tokyo Stock Exchange. Consequently, these 71 firms are excluded from our estimation of the non-default probabilities, but are used for portfolio analysis by the proposed model.

Data sources on the financial performances of these firms are (a) Yahoo Finance (http://chart.yahoo.co.jp/d) and (b) Nikkei Finance Data on CD−ROM. The observed time period is from 1995 to 2000. This study applies the proposed model to the data sets from 1995 to 1999 as training samples. The remaining data in 2000 is used as a validation sample set to compare the proposed approach with the conventional L_1 risk model (K-Y, 1991).

5.3 Measurement of non-default probabilities

Financial indices used for our measurement of non-default probabilities include three major components: (a) Financial Stability, (b) Profitability and Efficiency and (c) Size. Here, the financial stability is measured by the eight financial indices: Interest Coverage, Interest-bearing Debt Ratio, Capital Asset Ratio, Ratio of Capital to Fixed Assets, Fixed Ratio, Debt Ratio, Current Ratio, and Quick Ratio. The profitability and efficiency are measured by seven financial indices: Operating Income/Net Sales, Ordinary Income/Net Sales, Return On Equity, Return On Asset, Fixed Assets Turnover, Total Capital Turnover, and Account Receivable Turnover. The size is measured by the three indices: Cash flows, Net Sales, and Total Assets. Thus, the performances of all the Japanese electric firms listed in Table 1 are measured by these eighteen financial indices.

In the estimation of non-default probabilities, it is desirable to reduce the number of financial indices from eighteen to a level at which we can more easily predict an occurrence of bankruptcy in the Japanese electric industry. To reduce the number of these financial indices, this research applies a simple linear regression model (with a single dependent variable) to 73 firms (all of which have their ranking scores) and then applies a multiple regression model to them. In the two regression models, the dependent variable of these firms is scored by their ranks. That is, y = 5 is given to firms belonging to AAA (AAA+, AAA, and AAA−). Similarly, 4: AA (AA+, AA, and AA−), 3: A (A+, A, A−), 2: BBB (BBB+, BBB, BBB−), 1: BB (BB+, BB, BB−) and 0: default firms. (Note that firms rated below BB− belong to a non-default status, but they are not recommended for any investment. Therefore, those firms are excluded from this study.)

In the simple regression model ($y = \beta_0 + \beta_1 x$), y corresponds to the above ranking score and x corresponds to each financial index. Examining whether each parameter estimate is significant based upon the t-test, this study identifies whether each financial index is important in terms of predicting these corporate ranks. Table 2 documents resulting parameter estimates of the single regression model. Six financial factors (Capital Asset Ratio, Ratio of Capital to Fixed Assets, Return on Equity, Total Capital Turnover, Account Receivable Turnover, and Cash Flows) are not significant at the 5% level of the t-test and these indices are omitted from our proceeding study, because the financial index does not statistically explain a change in the ranking sore. Here, "statistically" implies that the decision for elimination is correct in a confidence level of 95% but it is incorrect at the level of 5%. Next, we apply the linear regression model with multiple variables to the data set without the six financial indices. As listed in the right hand side of Table 2, the three indices (Current Ratio, Fixed Assets Turnover, and Net Sales) are significant at the t-test.

Table 1. A list of Japanese electric firms

	No.	Firms	Rating
Non-default firms for Portfolio & Default Analysis	(1)	IBIDEN	A–
	(2)	MINEBEA	A–
	(3)	HITACHI	AA
	(4)	TOSHIBA	AA–
	(5)	MITSUBISHI ELECTRIC	A–
	(6)	FUJI ELECTRIC	A–
	(7)	YASKAWA ELECTRIC	BBB
	(8)	SHINKO ELECTRIC	
	(9)	MEIDENSHA	
	(10)	ORIGIN ELECTRIC	
	(11)	HITACHI KOKI	A–
	(12)	MATSUSHITA SEIKO	
	(13)	TOSHIBA TEC	BBB+
	(14)	SHIBAURA MECHATRONICS	
	(15)	MABUCHI MOTOR	
	(16)	TAKAOKA ELECTRIC MFG.	
	(17)	DAIHEN	
	(18)	NISSIN ELECTRIC	
	(19)	OSAKI ELECTRIC	BBB
	(20)	OMRON	A+
	(21)	NITTO ELECTRIC WORKS	
	(22)	IDEC IZUMI	
	(23)	NEC	AA–
	(24)	FUJITSU	AA–
	(25)	OKI ELECTRIC INDUSTRY	BBB
	(26)	IWATSU ELECTRIC	
	(27)	NITSUKO	
	(28)	DENKI KOGYO	
	(29)	SANKEN ELECTRIC	A–
	(30)	TOYO COMMUNICATION EQUIPMENT	BBB
	(31)	TAMURA ELECTRIC WORKS	
	(32)	FUJITSU DENSO	
	(33)	NIPPON SIGNAL	
	(34)	KYOSAN ELECTRIC MFG.	
	(35)	NOHMI BOSAI	BBB
	(36)	HOCHIKI	
	(37)	JAPAN RADIO	A
	(38)	MATSUSHITA ELECTRIC INDUSTRIAL	AA+
	(39)	SHARP	AA
	(40)	ANRITSU	A–
	(41)	FUJITSU GENERAL	
	(42)	KOKUSAI ELECTRIC	BBB+
	(43)	SONY	AA+
	(44)	TOKIN	BBB
	(45)	AIWA	BBB–
	(46)	TDK	
	(47)	TEIKOKU TSUSHIN KOGYO	
	(48)	SANYO ELECTRIC	A+
	(49)	KENWOOD	
	(50)	MIYAKOSHI	
	(51)	MITSUMI ELECTRIC	
	(52)	TAMURA	BBB
	(53)	ALPS ELECTRIC	A–
	(54)	IKEGAMI TSUSHINKI	
	(55)	PIONEER	A
	(56)	NIHON DEMPA KOGYO	BBB–
	(57)	MATSUSHITA COMMUNICATION IND.	
	(58)	KYUSHU MATSUSHITA ELECTRIC	
	(59)	MATSUSHITA-KOTOBUKI ELECTRONIC	
	(60)	NIPPON COLUMBIA	
	(61)	VICTOR COMPANY OF JAPAN	A–
	(62)	SANSUI ELECTRIC	
	(63)	FOSTER ELECTRIC	
	(64)	CLARION	
	(65)	SMK	
	(66)	TOKO	BBB
	(67)	AKAI ELECTRIC	
	(68)	TEAC	BBB–
	(69)	HOSIDEN	
	(70)	HIROSE ELECTRIC	
	(71)	JAPAN AVIATION ELECTRONICS IND	
	(72)	SHINTOM	
	(73)	HITACHI MAXELL	A+
	(74)	UNIDEN	
	(75)	ALPINE ELECTRONICS	
	(76)	YOKOGAWA ELECTRIC	A
	(77)	SHINDENGEN ELECTRIC MFG.	BBB
	(78)	YAMATAKE	
	(79)	NIHON KOHDEN	
	(80)	CHINO	

Table 1. Continued

	No.	Firms	Rating
Non-default firms for Portfolio & Default Analysis	(81)	OHKURA ELECTRIC	[ ]
	(82)	HORIBA	[A–]
	(83)	ADVANTEST	[A+]
	(84)	ONO SOKKI	[ ]
	(85)	TABAI ESPEC	[BBB–]
	(86)	SAWAFUJI ELECTRIC	[ ]
	(87)	DENSO	[ ]
	(88)	HITACHI MEDICAL	[ ]
	(89)	TOKO ELECTRIC	[ ]
	(90)	STANLEY ELECTRIC	[A–]
	(91)	IWASAKI ELECTRIC	[ ]
	(92)	USHIO	[A+]
	(93)	JAPAN STORAGE BATTERY	[ ]
	(94)	YUASA	[ ]
	(95)	SHIN-KOBE ELECTRIC MACHINERY	[ ]
	(96)	FURUKAWA BATTERY	[ ]
	(97)	ZUKEN	[ ]
	(98)	KINSEKI	[ ]
	(99)	JEOL	[ ]
	(100)	CASIO COMPUTER	[A]
	(101)	FDK	[ ]
	(102)	CMK	[BBB+]
	(103)	ROHM	[AA]
	(104)	MITSUI HIGH-TEC	[ ]
	(105)	SHINKO ELECTRIC INDUSTRIES	[A–]
	(106)	GRAPHTEC	[ ]
	(107)	NIPPON CONLUX	[ ]
	(108)	KYOCERA	[ ]
	(109)	SUMITOMO SPECIAL METALS	[ ]
	(110)	TAIYO YUDEN	[A]
	(111)	MURATA MFG.	[AA]
	(112)	U-SHIN	[ ]
	(113)	NITTO DENKO	[ ]
	(114)	HOKURIKU ELECTRIC INDUSTRY	[ ]
	(115)	MATSUSHITA ELECTRIC WORKS	[AA]
	(116)	TOKAI RIKA	[ ]
	(117)	NICHICON	[ ]
	(118)	NIPPON CHEMI-CON	[A–]
	(119)	KOA	[ ]
	(120)	MITSUBA	[BBB+]
	(121)	DAINIPPON SCREEN MFG.	[BBB–]
	(122)	CANON	[AA+]
	(123)	RICOH	[AA]
	(124)	MUTOH INDUSTRIES	[ ]
Non-default firms for Default Analysis	(1)	NIDEC	[BBB+]
	(2)	ENERGY SUPPORT	[BBB–]
	(3)	TOA	[BBB]
	(4)	SUNX	[BBB–]
	(5)	SYSMEX	[A–]
	(6)	NIPPON CERAMIC	[BBB]
	(7)	HAMAMATSU PHOTONICS	[BBB+]
	(8)	SANKYO SEIKI MFG.	[BBB]
	(9)	ZOJIRUSHI	[BBB–]
	(10)	TOKYO ELECTRON	[A+]
Default firms for Default Analysis	(1)	NIKKO ELECTRIC INDUSTRY	[D]
	(2)	FUJIYA ELECTRIC	[D]
	(3)	TOKYO SANYO ELECTRIC	[D]
	(4)	UNIDEN21	[D]
	(5)	HOKUSHIN ELECTRIC WORKS	[D]
	(6)	SHINKOH COMMUNICATION INDUSTRY	[D]
	(7)	AIDEN	[D]
	(8)	FUJITSU TOWA ELECTRON	[D]
	(9)	SHIN CHUO KOGYO	[D]
	(10)	HITACHI FERRITE	[D]

Consequently, the original eighteen indices are reduced to the three significant indices (Current Ratio, Fixed Assets Turnover and Net Sales) and those three are used for the parameter estimation of the logit model in order to compute non-default probabilities.

Table 3 documents parameter estimates of the logit model. Table 4 summarizes the non-default probability ($\hat{P}_{tj}$) of the 124 firms related to the Japanese electric industry. The probability estimates are all derived from the estimated logit model. The numbers above the probabilities of Table 4 correspond to the firm identification numbers of Table 1.

Finally, it is important to note that the maximum likelihood method is widely used to estimate the parameters of the logit model. See Davidson and MacKinnon (1993). This research has used "Visual Stat" (1996) produced by Design Technologies Inc. as software to obtain both regression estimates and maximum likelihood estimators of the logit model. When we apply it to the current data set, the computation times for the two statistical analyses are less than 1 second, respectively. The proposed approach (16) is solved by "Lindo" (1997) that is produced by Lindo Systems Inc. The computation time is less than 1 second.

Table 2. Regression analysis

Financial Index	Simple Regression Analysis			Multiple Regression Analysis			
	Constant	Parameter			Parameter		Standard
				Constant	−1.314		1.312
Interest Coverage(IC)	0.208	**0.294**	*	IC	0.053		0.771
Interest-bearing Debt Ratio(IDR)	2.682	**0.479**	**	IDR	0.301		0.219
Capital Asset Ratio(CAR)	−1.044	−0.204		CAR			0.832
Ratio of Capital to Fixed Assets(RCFA)	−0.251	−0.156		RCFA			0.786
Fixed Ratio(FR)	−0.174	**−0.297**	*	FR	0.826		2.163
Debt Ratio(DR)	−0.234	**−0.323**	**	DR	−0.767		1.700
Current Ratio(CR)	1.759	**0.349**	**	CR	**0.514**	*	0.323
Quick Ratio(QR)	0.204	**0.240**	*	QR	−0.087		1.384
Operating Incom / Net Sales(OpI/NS)	6.940	**0.315**	**	OpI/NS	0.346		0.050
Ordinary Incom / Net Sales(OdI/NS)	6.556	**0.328**	**	OdI/NS	−0.337		0.060
Return On Equity(ROE)	3.393	0.222		ROE			0.145
Return On Asset(ROA)	8.018	**0.258**	*	ROA	0.061		0.081
Fixed Assets Turnover(FAT)	−2.090	**−0.397**	**	FAT	**−0.565**	**	0.407
Total Capital Turnover(TCT)	−1.237	−0.158		TCT			0.194
Account Receivable Turnover(ART)	0.131	0.176		ART			1.663
Cash Flows(CF)	0.099	0.173		CF			2.484
Net Sales(NS)	1.074	**0.677**	**	NS	**0.949**	**	0.762
Total Asset(TA)	0.667	**0.682**	**	TA	−0.405		1.486

Note: the superscripts ** and * stand for the level of 1% and 5% significance. $R^2 = 0.080$

Table 3. Parameter estimates (logit model)

	Constant	Current Ratio	Fixed Assets Turnover	Net Sales
Estimate	−3.530	7.596	−5.776	3.724
t-score	−0.508	2.670 *	−2.879 *	2.556 *

5.4 Measurement of average rate of return

Table 5 lists the annual average rates of return ($\overline{r}_{i\cdot j}$) of all the firms from 1995 to 1999, where $\overline{r}_{i\cdot j} = \left(\sum_{m=1}^{b} r_{imj}\right)\Big/ b = \left(\sum_{m=1}^{12} r_{imj}\right)\Big/ 12$. These annual average rates of return are used for the first set of equations of (16).

5.5 Portfolio allocation under market risk and credit risk

Table 6 summarizes optimal portfolio allocations (in a percentage expression) of the proposed VaR model (16) under eleven different weight combinations on (w_1, w_2). The numbers listed within parentheses indicate a group of firms to which we allocate our fund. For instance, when $w_1 = 0.5$ and $w_2 = 0.5$, we need to allocate 38.5%, 36.5% and 25.0% of our total fund to Firm (13: Toshiba Tec), Firm (74: Uniden) and Firm (106: Graphtec), respectively. Such investment produces 1.5% portfolio return (in 2000), as listed in the last row of Table 6. The computation of each portfolio allocation is run on Lindo produced by Scientific Press. The computational time is less than 1 second.

In reviewing Table 6, we need to add the following two comments: First, if a market risk is measured by a variance of return, as formulated in the mean-variance analysis; the market risk may be reduced by allocating a fund to many different assets. However, the proposed approach does not look for such a fund allocation, rather looking for the optimal allocation that maximizes both the lower bound of an expected portfolio return and that of an expected portfolio allocation weighted by non-default probabilities. Consequently, the number of fund allocations may be limited in Table 6. Second, it is also true that if the proposed Model (16) incorporates a lower bound for each portfolio allocation, then more portfolio allocations can be found in Table 6. The incorporation of such a lower bound on fund allocation needs practical consideration or prior information.

Finding 1: The best combination, in Table 6, can be found in $w_1 = 0.1$ and $w_2 = 0.9$, implying that the credit risk is nine times as important as the market risk in this case. The investment produces 14.7% yield in the total portfolio return. It is important to note that both $w_1 = 1$ (in the second column) and $w_2 = 1$ (in the last column) produce negative results (−20.5% and −8%). This result indicates the importance of a simultaneous integration of both market and credit risks in portfolio allocation.

The optimal portfolio allocations are obtained by applying (16) to the training samples (1995-1999). The fund allocations can be extended to a validation sample (2000) in order to examine the performance of the proposed approach. Such a comparative analysis is documented in Figure 3. The figure visually describes a time trend of total portfolio returns under five different weight combinations on (w_1, w_2). The time trend is weekly measured in 2000. As depicted in Figure 3, the portfolio return measured under $w_1 = 0.1$ and $w_2 = 0.9$ (more weight on credit risk) exhibits the highest performance among the five combinations, while the combination between $w_1 = 1$ and $w_2 = 0$ (100% weight on market risk) is the worst

Table 4. Non-default probabilities

No. / year	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
1999	0.9949	0.9989	0.9998	0.9996	0.9995	0.9898	0.9214	0.6416	0.8008	0.6961	0.9986	0.9967	0.9766	0.8544	1.0000
1998	0.9882	0.9967	0.9998	0.9996	0.9989	0.9917	0.8344	0.4828	0.8260	0.6622	0.9986	0.9945	0.9736	0.6919	0.9998
1997	0.9679	0.9962	0.9998	0.9993	0.9989	0.9823	0.9179	0.5158	0.8478	0.6968	0.9961	0.9906	0.9658	0.6935	1.0000
1996	0.9922	0.9953	0.9998	0.9993	0.9985	0.9767	0.9321	0.5110	0.8612	0.8533	0.9968	0.9780	0.9607	0.7049	0.9999
1995	0.9954	0.9944	0.9997	0.9995	0.9988	0.9780	0.9486	0.6040	0.8764	0.8608	0.9973	0.9961	0.9736	0.6858	0.9999

No. / year	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)
1999	0.6785	0.9408	0.7934	0.9647	0.9990	0.9677	0.9673	0.9998	0.9998	0.9982	0.7264	0.8445	0.7669	0.9623	0.9422
1998	0.6349	0.9041	0.7591	0.9434	0.9984	0.9589	0.9657	0.9996	0.9998	0.9981	0.7896	0.8512	0.8017	0.9605	0.9683
1997	0.5891	0.9351	0.7569	0.8247	0.9978	0.9440	0.9564	0.9996	0.9997	0.9972	0.8741	0.9103	0.7137	0.9483	0.9464
1996	0.5664	0.8729	0.7718	0.9858	0.9983	0.9488	0.9542	0.9997	0.9997	0.9973	0.8851	0.9151	0.8019	0.9453	0.8506
1995	0.6284	0.9335	0.7923	0.9782	0.9991	0.9329	0.9536	0.9997	0.9998	0.9977	0.8139	0.9176	0.9410	0.8275	0.9298

No. / year	(31)	(32)	(33)	(34)	(35)	(36)	(37)	(38)	(39)	(40)	(41)	(42)	(43)	(44)	(45)
1999	0.7227	0.8351	0.7774	0.6462	0.8602	0.3950	0.9254	0.9999	0.9997	0.9990	0.4169	0.9921	0.9999	0.8869	0.9723
1998	0.4452	0.8282	0.7068	0.6571	0.8009	0.4819	0.9451	0.9999	0.9994	0.9978	0.3321	0.9898	0.9999	0.8743	0.8882
1997	0.5162	0.7559	0.5500	0.5891	0.7631	0.4862	0.8552	0.9999	0.9993	0.9949	0.3441	0.9604	0.9999	0.8756	0.8533
1996	0.4586	0.6977	0.7180	0.5426	0.8475	0.4601	0.9599	0.9999	0.9994	0.9868	0.3336	0.9021	1.0000	0.8733	0.7912
1995	0.5946	0.6999	0.5715	0.5404	0.8008	0.5775	0.9598	0.9999	0.9996	0.9922	0.4245	0.9457	0.9998	0.8174	0.6219

No. / year	(46)	(47)	(48)	(49)	(50)	(51)	(52)	(53)	(54)	(55)	(56)	(57)	(58)	(59)	(60)
1999	0.9998	0.9518	0.9997	0.8851	0.5209	0.8925	0.9951	0.9951	0.9322	0.9998	0.9507	0.9884	0.9965	0.9815	0.6231
1998	0.9997	0.9153	0.9997	0.7792	0.4486	0.8480	0.9916	0.9928	0.9701	0.9991	0.9916	0.9884	0.9945	0.9838	0.7955
1997	0.9996	0.9572	0.9997	0.8492	0.5263	0.8996	0.9884	0.9943	0.9681	0.9996	0.9896	0.9799	0.9980	0.9530	0.9067
1996	0.9996	0.9482	0.9999	0.8724	0.4566	0.8686	0.9952	0.9967	0.9731	0.9996	0.9885	0.9861	0.9989	0.9751	0.8937
1995	0.9997	0.9641	0.9997	0.8938	0.3064	0.8167	0.9579	0.9965	0.9838	0.9994	0.9737	0.9891	0.9991	0.9735	0.8841

No. / year	(61)	(62)	(63)	(64)	(65)	(66)	(67)	(68)	(69)	(70)	(71)	(72)	(73)	(74)	(75)
1999	0.9974	0.2430	0.3931	0.9027	0.8627	0.9399	0.9836	0.3049	0.9420	0.9955	0.8118	0.0132	0.9998	0.9921	0.9119
1998	0.9942	0.2724	0.3896	0.8756	0.8581	0.9781	0.9408	0.3757	0.8190	0.9681	0.8522	0.0154	0.9997	0.9880	0.9093
1997	0.9974	0.3404	0.3665	0.8757	0.8343	0.9807	0.9590	0.4024	0.8881	0.9557	0.8249	0.0248	0.9999	0.9976	0.8780
1996	0.9942	0.4803	0.3312	0.8983	0.8751	0.9784	0.7276	0.3773	0.9440	0.9690	0.7916	0.0452	0.9999	0.9997	0.8856
1995	0.9949	0.8637	0.3030	0.9037	0.8904	0.9815	0.8464	0.4476	0.8997	0.9759	0.8144	0.1814	0.9999	0.9999	0.8621

No. / year	(76)	(77)	(78)	(79)	(80)	(81)	(82)	(83)	(84)	(85)	(86)	(87)	(88)	(89)	(90)
1999	0.9998	0.9748	0.9428	0.9056	0.7621	0.5570	0.9991	0.9998	0.9276	0.9748	0.0578	0.9999	0.9770	0.4295	0.9819
1998	0.9986	0.9637	0.9607	0.9485	0.6703	0.6994	0.9858	0.9930	0.8824	0.9561	0.0512	0.9998	0.9748	0.4278	0.9906
1997	0.9997	0.9453	0.9456	0.9653	0.6199	0.6000	0.9984	0.9954	0.9251	0.9534	0.0481	0.9997	0.9700	0.3920	0.9865
1996	0.9996	0.9285	0.9428	0.8542	0.6075	0.6969	0.9963	0.9917	0.7368	0.9182	0.0584	0.9998	0.9802	0.4005	0.9969
1995	0.9989	0.8832	0.9880	0.9354	0.6111	0.8271	0.9970	0.9961	0.9772	0.9762	0.0637	0.9999	0.9727	0.4050	0.9946

No. / year	(91)	(92)	(93)	(94)	(95)	(96)	(97)	(98)	(99)	(100)	(101)	(102)	(103)	(104)	(105)
1999	0.8909	0.9991	0.9046	0.8318	0.6711	0.2661	0.9983	0.9980	0.5810	0.9987	0.4790	0.9956	0.9996	0.9430	0.9926
1998	0.7432	0.9973	0.9250	0.7326	0.6829	0.3565	0.9984	0.9958	0.5540	0.9963	0.4294	0.9627	0.9991	0.9979	0.9659
1997	0.9090	0.9954	0.9139	0.7955	0.6727	0.4110	0.9996	0.9947	0.4506	0.9867	0.4393	0.9945	0.9987	0.9980	0.9455
1996	0.8206	0.9967	0.9259	0.8322	0.6928	0.4462	0.8576	0.9879	0.4238	0.9930	0.5798	0.9716	0.9988	0.9971	0.9725
1995	0.8677	0.9988	0.8995	0.8465	0.7374	0.4251	0.9996	0.9943	0.4075	0.9981	0.5355	0.9959	0.9987	0.9979	0.9351

No. / year	(106)	(107)	(108)	(109)	(110)	(111)	(112)	(113)	(114)	(115)	(116)	(117)	(118)	(119)	(120)
1999	0.9827	0.9926	1.0000	0.9820	0.9971	0.9999	0.9581	0.9979	0.7486	0.9998	0.9680	0.9916	0.9965	0.9169	0.9388
1998	0.9975	0.9862	0.9999	0.9560	0.9970	0.9998	0.9368	0.9964	0.9033	0.9997	0.9665	0.9880	0.9942	0.8208	0.9189
1997	0.9959	0.9876	0.9998	0.9323	0.9979	0.9999	0.9042	0.9978	0.9170	0.9999	0.9506	0.9900	0.9948	0.7989	0.9091
1996	0.9961	0.9950	0.9999	0.8600	0.9959	0.9998	0.9206	0.9981	0.9033	0.9999	0.9455	0.9822	0.9858	0.7146	0.9204
1995	0.9967	0.9932	0.9999	0.8425	0.9956	0.9997	0.9333	0.9979	0.9140	0.9999	0.9292	0.9815	0.9983	0.6871	0.9091

No. / year	(121)	(122)	(123)	(124)
1999	0.9929	0.9996	0.9991	0.9032
1998	0.9866	0.9995	0.9989	0.9344
1997	0.9752	0.9994	0.9984	0.8632
1996	0.9799	0.9995	0.9985	0.9551
1995	0.9833	0.9995	0.9991	0.9894

Table 5. Average rate of return

No. / year	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
1999	-0.0096	0.0110	0.0308	0.0053	0.0224	0.0280	0.0459	-0.0033	-0.0190	0.0323	-0.0099	0.0008	-0.0024	0.0385	0.0262
1998	0.0047	-0.0028	-0.0103	0.0078	0.0022	-0.0114	-0.0169	-0.0050	-0.0195	-0.0109	-0.0028	-0.0083	0.0027	0.0188	0.0096
1997	0.0125	0.0134	-0.0054	-0.0106	-0.0263	-0.0129	-0.0077	-0.0261	-0.0231	-0.0156	-0.0259	-0.0146	-0.0138	-0.0278	0.0047
1996	0.0107	0.0040	0.0014	-0.0038	-0.0027	-0.0028	-0.0069	0.0011	0.0040	0.0024	-0.0044	-0.0019	-0.0053	-0.0189	-0.0035
1995	0.0029	0.0011	0.0018	0.0041	0.0018	-0.0015	-0.0039	-0.0011	-0.0066	0.0138	-0.0022	0.0005	-0.0034	0.0070	-0.0056

No. / year	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)
1999	-0.0041	-0.0010	-0.0064	0.0085	0.0166	-0.0180	0.0237	0.0308	0.0409	0.0214	0.0221	0.0063	0.0048	0.0050	0.0551
1998	0.0010	-0.0031	-0.0006	0.0083	-0.0107	-0.0091	0.0023	-0.0105	0.0026	0.0117	0.0024	0.0026	0.0181	-0.0013	-0.0006
1997	-0.0293	-0.0254	-0.0326	-0.0283	-0.0026	-0.0181	-0.0183	-0.0003	0.0094	-0.0352	-0.0438	-0.0316	-0.0259	-0.0147	-0.0511
1996	-0.0128	-0.0079	-0.0116	-0.0033	-0.0047	0.0168	-0.0004	0.0038	-0.0023	-0.0136	-0.0141	-0.0112	-0.0004	-0.0037	-0.0060
1995	-0.0003	-0.0001	0.0012	-0.0088	0.0106	0.0023	0.0014	0.0036	0.0047	0.0100	0.0046	-0.0071	-0.0077	-0.0027	-0.0103

No. / year	(31)	(32)	(33)	(34)	(35)	(36)	(37)	(38)	(39)	(40)	(41)	(42)	(43)	(44)	(45)
1999	0.0013	0.0455	-0.0037	-0.0019	-0.0067	-0.0088	0.0260	0.0126	0.0341	-0.0087	-0.0035	0.0267	0.0472	0.0001	-0.0123
1998	-0.0023	-0.0097	0.0104	-0.0097	-0.0135	0.0019	-0.0108	0.0016	0.0046	-0.0032	0.0247	-0.0195	-0.0124	-0.0097	-0.0036
1997	-0.0337	-0.0378	-0.0254	-0.0220	-0.0112	-0.0179	-0.0234	0.0004	-0.0220	-0.0081	-0.0289	-0.0130	0.0154	-0.0142	0.0182
1996	-0.0215	0.0073	0.0026	-0.0093	0.0031	-0.0065	0.0043	0.0043	0.0000	0.0040	-0.0065	-0.0089	0.0074	-0.0170	-0.0071
1995	0.0155	0.0204	-0.0068	-0.0015	-0.0129	-0.0129	-0.0107	0.0009	-0.0031	-0.0081	0.0047	0.0043	0.0033	0.0063	-0.0005

No. / year	(46)	(47)	(48)	(49)	(50)	(51)	(52)	(53)	(54)	(55)	(56)	(57)	(58)	(59)	(60)
1999	0.0113	0.0041	0.0062	-0.0072	-0.0085	0.0106	-0.0102	-0.0103	0.0040	0.0128	0.0390	0.0587	0.0078	-0.0062	-0.0090
1998	0.0018	-0.0013	0.0010	0.0165	0.0066	0.0091	0.0058	0.0189	-0.0037	-0.0021	0.0028	0.0154	-0.0019	-0.0107	0.0047
1997	0.0096	-0.0283	-0.0125	-0.0244	-0.0819	-0.0057	-0.0133	-0.0009	-0.0364	-0.0034	-0.0249	0.0054	-0.0084	0.0030	-0.0387
1996	0.0130	-0.0099	-0.0078	-0.0041	0.0378	-0.0048	-0.0058	0.0021	0.0019	0.0057	-0.0075	0.0081	-0.0062	0.0051	-0.0110
1995	0.0032	0.0019	0.0014	-0.0138	-0.0036	0.0208	-0.0046	-0.0035	-0.0144	-0.0086	-0.0143	-0.0033	-0.0114	0.0003	-0.0017

No. / year	(61)	(62)	(63)	(64)	(65)	(66)	(67)	(68)	(69)	(70)	(71)	(72)	(73)	(74)	(75)
1999	-0.0027	-0.0033	0.0135	0.0299	0.0223	0.0078	-0.0211	-0.0141	0.0435	0.0385	0.0073	-0.0045	0.0186	-0.0029	0.0115
1998	-0.0143	-0.0010	0.0130	-0.0014	0.0109	-0.0260	-0.0069	0.0172	0.0260	0.0062	-0.0169	-0.0251	-0.0089	-0.0037	-0.0018
1997	-0.0016	-0.0664	-0.0052	-0.0222	-0.0150	0.0142	-0.0408	-0.0199	0.0068	-0.0002	-0.0107	-0.0453	-0.0039	-0.0136	-0.0148
1996	-0.0047	-0.0033	-0.0112	0.0095	-0.0159	-0.0125	-0.0005	-0.0157	-0.0032	0.0044	-0.0027	-0.0053	0.0129	-0.0044	0.0022
1995	-0.0037	-0.0056	-0.0061	-0.0029	-0.0007	-0.0005	-0.0095	0.0105	-0.0300	0.0007	-0.0005	-0.0163	-0.0016	-0.0135	-0.0036

No. / year	(76)	(77)	(78)	(79)	(80)	(81)	(82)	(83)	(84)	(85)	(86)	(87)	(88)	(89)	(90)
1999	0.0091	-0.0032	-0.0126	-0.0046	-0.0049	0.0063	-0.0070	0.0480	-0.0172	0.0082	0.0206	0.0056	-0.0034	0.0012	0.0100
1998	-0.0132	-0.0101	-0.0095	-0.0064	0.0005	-0.0102	-0.0065	0.0023	-0.0086	-0.0123	-0.0059	-0.0042	-0.0030	-0.0050	0.0016
1997	-0.0078	-0.0104	-0.0107	-0.0262	-0.0278	-0.0369	0.0043	0.0147	-0.0081	-0.0177	-0.0313	-0.0062	-0.0057	-0.0195	-0.0230
1996	0.0009	0.0016	0.0056	-0.0088	-0.0108	-0.0134	-0.0043	0.0043	-0.0076	-0.0084	-0.0053	0.0133	0.0005	-0.0112	0.0033
1995	-0.0023	-0.0044	0.0002	-0.0017	-0.0031	-0.0037	-0.0073	0.0164	-0.0028	-0.0017	-0.0031	-0.0031	-0.0071	0.0037	-0.0070

No. / year	(91)	(92)	(93)	(94)	(95)	(96)	(97)	(98)	(99)	(100)	(101)	(102)	(103)	(104)	(105)
1999	-0.0021	0.0249	0.0183	-0.0145	0.0092	0.0020	0.0536	0.0301	0.0019	0.0007	0.0211	0.0025	0.0514	0.0050	0.0049
1998	0.0059	0.0048	-0.0115	0.0299	0.0081	-0.0069	0.0142	-0.0080	0.0099	-0.0042	0.0111	-0.0092	-0.0086	-0.0029	0.0011
1997	-0.0301	-0.0134	-0.0299	-0.0532	-0.0341	-0.0324	-0.0355	-0.0284	-0.0177	0.0016	-0.0342	-0.0010	0.0197	-0.0025	0.0053
1996	-0.0053	0.0009	-0.0009	-0.0062	-0.0068	-0.0077	-0.0079	-0.0057	-0.0095	-0.0043	-0.0151	0.0007	0.0101	-0.0028	-0.0036
1995	-0.0058	0.0024	-0.0009	-0.0039	-0.0037	0.0034	-0.0142	-0.0070	0.0057	-0.0080	0.0013	-0.0087	0.0112	0.0010	0.0205

No. / year	(106)	(107)	(108)	(109)	(110)	(111)	(112)	(113)	(114)	(115)	(116)	(117)	(118)	(119)	(120)
1999	0.0060	0.0014	0.0539	-0.0180	0.0547	0.0612	0.0062	0.0362	0.0086	-0.0049	0.0233	0.0292	0.0044	0.0367	0.0069
1998	-0.0173	0.0091	0.0003	-0.0124	0.0142	0.0105	-0.0176	-0.0065	-0.0086	0.0008	0.0129	0.0055	0.0070	0.0088	-0.0172
1997	-0.0350	-0.0330	-0.0072	0.0241	-0.0211	-0.0050	-0.0075	0.0101	-0.0388	0.0045	-0.0352	-0.0037	-0.0200	-0.0203	-0.0180
1996	0.0022	-0.0029	-0.0022	-0.0081	0.0137	0.0005	-0.0014	0.0022	-0.0089	-0.0032	-0.0011	-0.0046	-0.0097	-0.0057	0.0029
1995	-0.0123	0.0089	0.0013	-0.0096	-0.0003	-0.0008	-0.0088	0.0005	0.0053	0.0024	-0.0064	0.0015	0.0029	0.0038	-0.0035

No. / year	(121)	(122)	(123)	(124)
1999	0.0279	0.0188	0.0222	0.0579
1998	-0.0273	-0.0083	-0.0160	-0.0038
1997	-0.0128	0.0062	0.0071	-0.0512
1996	-0.0021	0.0114	0.0059	-0.0039
1995	0.0059	0.0037	0.0048	-0.0162

Table 6. Optimal portfolio allocations (%) under different weights

w_1 w_2	1.0 0.0	0.9 0.1	0.8 0.2	0.7 0.3	0.6 0.4	0.5 0.5	0.4 0.6	0.3 0.7	0.2 0.8	0.1 0.9	0.0 1.0
(13)		0.406	0.406	0.406	0.385	0.385	0.371				
(25)								0.598	0.651	0.273	
(38)											0.500
(43)											0.333
(48)									0.260	0.305	
(50)	0.972										
(74)		0.360	0.360	0.360	0.365	0.365	0.436	0.298	0.067		
(76)										0.386	
(89)	0.028										
(97)								0.020	0.002	0.001	
(106)		0.233	0.233	0.233	0.250	0.250	0.193	0.084	0.021	0.034	
(108)											0.166
k_1	0.125	0.096	0.096	0.096	0.096	0.096	0.095	0.077	0.069	0.064	0.000
k_2	0.000	0.983	0.983	0.983	0.984	0.984	0.985	0.995	0.998	0.999	1.000
Portfolio Return	-0.205	0.019	0.019	0.019	0.015	0.015	0.008	0.005	0.105	0.147	-0.080

Note: (13): Toshiba Tec, (25): Oki Electric, (38): Matsushita Electric, (43): Sony, (48): Sanyo, (50): Miyakoshi, (74): Uniden, (76): Yokogawa Electric, (89): Toko, (97): Zuken, (106): Graphtec and (108): Kyocera.

performer among them. This result also confirms the importance of incorporating credit risk into portfolio analysis, as mentioned in Finding 1.

In examining Figure 3, we need to note that Japanese economy (2000) was in a deep recession and hence, the above result discussed in Finding 1 could be anticipated in this study. It might be found that if the economy were in a healthy condition, an opposite result (e.g., the combination between $w_1 = 1$ and $w_2 = 0$ produces the best portfolio return) might be found in this study.

5.6 Performance comparison between two portfolio models

Using the validation sample (2000), Figure 3 visually compares the total portfolio return (%) of our model (16) with that of the L_1 risk model (18) proposed by Konno and Yamazaki (K-Y, 1991).

Finding 2: In the figure, the proposed model outperforms the L_1 risk model. This result indicates that the approach proposed in this study may be an alternative to the conventional L_1 risk model.

It is important to note that the non-default probabilities are incorporated into the computational process of the proposed approach. The incorporation is not found in the L_1 risk model (K-Y, 1991). Such a difference produces a result summarized in Finding 3.

Finding 3: Comparing Figure 3 with Figure 4, we find that if the weight combination between $w_1 = 1$ and $w_2 = 0$ is selected for the proposed model; then it cannot outperform the L_1 market risk model. The weight combination indicates that only the market risk is considered in the proposed model. A use of VaR for a market risk measure may not always provide reliable information for portfolio allocation. Hence, as discussed in this study, the VaR-based fund allocation needs to be used with other information such as the credit risk. In other works, the use of the market risk needs to be combined with the credit risk. Such a combined use can enhance the reliability of portfolio allocation. (See, for example, several works on VaR (e.g., Alexander and Baptisata, 2004).)

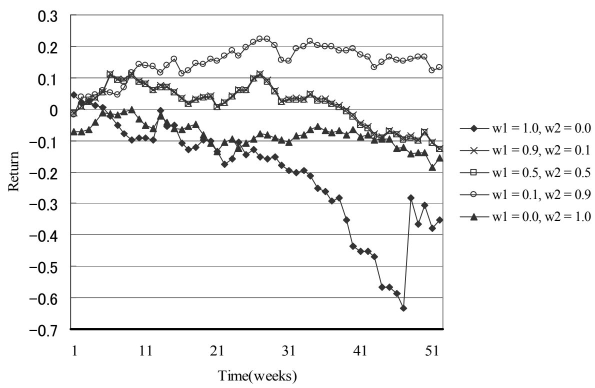

Figure 3. Portfolio return on investment (2000).

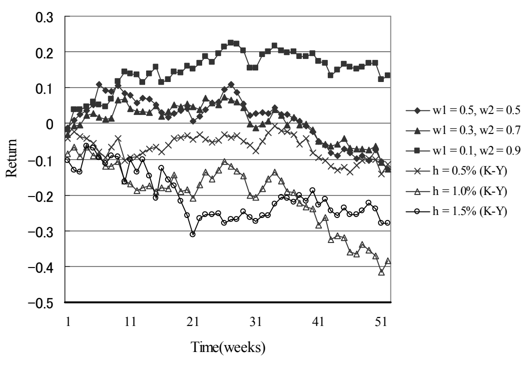

Figure 4. Comparison between two portfolio models (2000).

Finally, a limitation on these empirical findings exists, of course, based on the data set analyzed, namely the Japanese electric industry from 1995 to 2000. An implication derived from the data set is very limited as scientific evidence. However, it should be reconfirmed that the Japanese economy was under a deep recession in the observed time period and therefore, the incorporation of credit risk into portfolio allocation was important in such sluggish economy.

6. CONCLUSION AND FUTURE EXTENSIONS

A VaR-based model is newly proposed for portfolio allocation. Both credit and market risks are simultaneously incorporated into the newly proposed asset allocation model. Using the chance constraint programming technique, the proposed model is reformulated into a linear programming equivalent model. Since linear programming software can solve the formulation (16), investors can apply the proposed model to large scale portfolio optimization problems.

As an illustrative case study, the proposed approach is applied for analyzing how to allocate a total fund to stocks related to the Japanese electric industry. The performance of the proposed approach is compared with that of the conventional L_1 risk model (K-Y, 1991). The methodological comparison implies that the VaR model can be used as an alternative to the L_1 risk model, in particularly for portfolio allocation in sluggish economy (like the current Japanese economy).

As a future research extension, this study needs to pay attention to the following research tasks: First, we need to investigate how to select the best weight combination (w_1, w_2) based upon modern decision theory. Second, a normal distribution is used to transform from a probability model of VaR to its deterministic equivalence. The assumption is used for mathematical convenience. In near future, we need to drop the assumption in our reformulation process. Third, the performance of the proposed approach is compared with only the K-Y model. It is not our intension to describe that the proposed approach is the best one. Rather, this study is an initial step to develop a new approach for VaR and credit risk. Hence, it is expected that we need to methodologically compare the proposed approach with other new VaR approaches, all of which deal with portfolio allocation under the credit risk (e.g., Rockafeller and Uryasev, 2000; Zagst, 2002; Zagst et al., 2003; Alexander and Baptista, 2004). Such a methodological comparison is an important future research task.

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