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An Approach for Solving the Multi-product Newsboy Problem

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***Abstract*—**We propose a solution methodology for the multi-product newsboy model with constraints that is based on quadratic programming and a triangular presentation of the area under the cumulative probability distribution function of the demand. The methodology allows easier application of this important inventory control model that could be of particular interest in supply chain management as well as in offering a portable means to pedagogy in this field.

***Keywords*—**Optimization, Quadratic programming, Newsboy

1. INTRODUCTION

The newsboy problem is becoming increasingly relevant in today's global environment. Denardo (2001) states that the classical newsboy model developed earlier by Hadley and Whitin (1963) is considered one of the most important inventory models of the present time. This is because of its numerous applications in different fields ranging from those in fashion industry, airline seats pricing, and drugs' lot-sizing to management of perishable food supplies in super-markets.

Motivated by the interest of the community in Hadley and Whitin's seminal model, researchers have developed this model further covering a wide spectrum of extensions. The interested reader is referred to the reviews by Gallego and Moon (1993), and Khouja (1999). Since the publication of these two reviews, many articles have also appeared addressing variations of the modeling aspects of the earlier problem as well as introducing solution methods to this type of problem. Among these articles that addressed solution methods for the multi-product newsboy are those briefly discussed in the following paragraphs.

Lau and Lau (1995, 1996) have introduced a Lagrangian based numerical method to solve the multi-product multi constraint newsboy problem. To initiate their numerical procedures, the proposed approach requires first obtaining the solution for the unconstrained model. Ben-Daya and Raouf (1993) developed a model for solving the constrained problem when the demand probability density function is uniform. Their approach is also Lagrangian based. Erlebacher (2000) has addressed the model of the capacitated newsboy problem in cases where the cost structure is similar. He developed exact and heuristic solutions depending on the types of the demand probability distribution functions for the considered products. Recently, Mostard et al. (2005) published an article addressing the distribution free newsboy problem for resalable products. Their work is concerned with situations where the probability distribution of the demand is not known and the customers have the option of returning the items.

It should be noted that most of the aforementioned models, except those of Lau and Lau, did not pay enough attention to the lower bounds of the products' demand. And, as Lau and Lau observed, this could lead to negative order quantities particularly when the constraints are tight and the number of product considered is large. Some of the recent works that included the lower bounds of the order quantities are those by Abdel-Malek and Montanari (2005), and Niederhoff (2004). In the first article, Abdel-Malek and Montanari present models based on the dual program of the newsboy for the case with two constraints. In the second one, Niederhoff proposes linear approximation of the model's objective function. Both of the models introduced in these two articles require some effort to apply.

To complement these models, in this paper we develop a model that is easier to implement than those currently published for the multi-product newsboy with constraints while considering order quantities' lower bounds. It is based on triangular presentation of the areas resulting from integrals that are included in the objective function. This facilitates expressing the objective function in quadratic terms. Consequently, one can use familiar linear programming packages to solve the problem.

The rest of the paper is organized as follows. In Section 2 we show the basic newsboy model and the solution methodology development. Section 3 exhibits the application of the developed methodology to different distribution functions. In Section 4, numerical examples are

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shown comparing the developed method to results of some existing models. In Section 5, we present conclusion of the paper.

2. METHODOLOGY DEVELOPMENT

One of the common forms of the multi-product newsvendor model with constraints is

$$Min\ Z = \sum_{\tau=1}^{N}\left[c_\tau x_\tau + h_\tau \int_0^{x_\tau} (x_\tau - D_\tau) f_\tau(D_\tau) dD_\tau + v_\tau \int_{x_\tau}^{\infty} (D_\tau - x_\tau) f_\tau(D_\tau) dD_\tau \right]. \quad (1)$$

Or alternatively,

$$Max\ Z = \sum_{\tau=1}^{N}\left[(v_\tau - c_\tau) x_\tau - (v_\tau + h_\tau) \int_0^{x_\tau} F(D_\tau) dD_\tau - v_\tau E[D_\tau] \right]. \quad (2)$$

Subject to:

$$\sum_{\tau=1}^{N} (c_\tau x_\tau) \le B_g,$$
$$\sum_{i=1}^{M}\sum_{\tau=1}^{N} (\xi_{i,\tau}\ x_\tau) \le R_i,\ i = 1,\ 2,\ \ldots, M \quad (3)$$
$$\overset{N}{\underset{\tau=1}{\forall}} x_\tau \ge L_\tau .$$

where Z is the expected cost to be optimized, N is the number of items, M is the number of resource constraints, τ is the item index, c_τ is the unit cost of item τ, $\xi_{i,\tau}$ is the coefficient of resource i of item τ, L_τ is lower bound of order quantity of item τ, x_τ is the amount to be ordered of item τ, h_τ is the cost incurred per each item leftover at the end of the specified period, D_τ a random variable of item's τ demand, $f_\tau(D_\tau)$ is the probability density function of demand for item τ, $F(D_\tau)$ is the probability cumulative density function (CDF) of demand for item τ, v_τ is the revenue per unit of item τ, μ_τ is the mean of the demand for item τ, B_g is the firm's available budget, and R_i is the amount of available resource i. (See Appendix A for proof of Eq. (2).)

The objective function given in Eq. (2) can be expressed in the following quadratic form

$$Max\ Z = \sum_{\tau=1}^{N} (A_\tau^{(.)} x_\tau^2 + B_\tau^{(.)} x_\tau + C_\tau^{(.)}). \quad (4)$$

where $A_\tau^{(.)}$, $B_\tau^{(.)}$, and $C_\tau^{(.)}$ are constants to be determined for each product τ according to its demand probability distribution; (.).

In the following, we show in general how to apply what we designate as triangular approach to obtain these constants. We first note that the second term of Eq. (2) includes the integral of the cumulative distribution function. This area can be either expressed or approximated as that of a triangle using the following equation:

$$\int_0^{x_\tau} F(D_\tau) dD_\tau \approx \frac{1}{2}(x_\tau - x_{l,\tau})(\Delta_\tau (x_\tau - x_{l,\tau})) \quad (5)$$

(See Figures 1 and 2)

where, $x_\tau - x_{l,\tau}$ is the length of the triangle base, $F(x_\tau) = \Delta_\tau(x_\tau - x_{l,\tau})$ is the height of the triangle with respect to x_τ, and $\Delta_\tau = [F(x_{u,\tau})/(x_{u,\tau} - x_{l,\tau})]$ represents the slope of triangle. (More details about these parameters and how to obtain them for each probability density function are given in Section 3).

The error of the approximated area can be calculated using the following equation.

$$error = \int_0^{x_\tau} F(D_\tau) dD_\tau - \left[\frac{1}{2}(x_\tau - x_{l,\tau})(\Delta_\tau (x_\tau - x_{l,\tau}))\right] \quad (6)$$

As shown in Eq. (4), substituting Eq. (5) into Eq. (2), one can arrange its terms to obtain a quadratic form. Hence, the values of the coefficients of the objective function can be expressed as follows:

$$A_\tau = -\left(\frac{v_\tau + h_\tau}{2}\right)\Delta_\tau$$
$$B_\tau = -(c_\tau - v_\tau) + (v_\tau + h_\tau)\Delta_\tau\ x_{l,\tau} \quad (7)$$
$$C_\tau = -\left(\frac{v_\tau + h_\tau}{2}\right)\Delta_\tau\ (x_{l,\tau})^2 - v_\tau\ E[D_\tau]$$

It should be noted that the shape of the cumulative distribution function, $F(D_\tau)$, plays a significant role in determining the values of ($x_{l,\tau}$, $x_{u,\tau}$, $F(x_{u,\tau})$). The next section lays out the procedures of obtaining the quadratic form of the objective function for different demand cumulative distribution functions ($F(D_\tau)$).

3. MODELING THE OBJECTIVE FUNCTION FOR DIFFERENT DEMAND DISTRIBUTIONS

As mentioned before, the triangular approach is used for estimating the area under the curve of items' τ demand distribution function. Generally, we can divide these curves' shapes into three major silhouettes; the ramp shapes, the parabola shapes with zero lower bounds, and the S shapes with non zero lower bounds (see Appendix B). For each silhouette, the values of the parameters ($x_{l,\tau}$, $x_{u,\tau}$, $F(x_{u,\tau})$) have to be first appropriately defined. The following sub-sections present the necessary explanations. Also, in Table 1, we summarize the coefficients' formulae ($A_\tau^{(.)}$, $B_\tau^{(.)}$, and $C_\tau^{(.)}$) for three major probability distributions; the uniform, the exponential, and the normal.

In addition, based on these formulae, we show the application of a general distribution case in Section 4.4.

3.1 Silhouette I: The ramp shape distribution function

The first silhouette describes the characteristic of the uniform distribution. Eq. (8) and its pictorial in Figure 1 exhibit the CDF. As can be seen, the area under the curve is a right-angle triangle which yields the exact solutions.

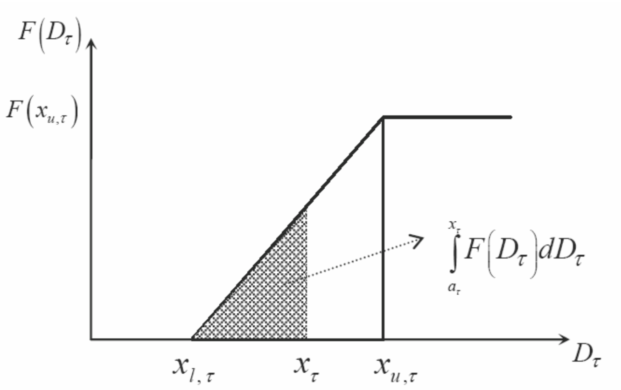

Figure 1. Triangular presentation (shaded area) for uniform distribution.

$$F(D_\tau) = \begin{Bmatrix} 0 & D_\tau < a_\tau \\ \dfrac{(D_\tau - a_\tau)}{(b_\tau - a_\tau)} & a_\tau \le D_\tau \le b_\tau \\ 1 & D_\tau \ge b_\tau \end{Bmatrix} \qquad (8)$$

Since the values of both a_τ and b_τ are known (i.e. $x_{l,\tau} = a_\tau$, $x_{u,\tau} = b_\tau$, thus, $F(x_{u,\tau}) = 1$), we can determine in a straightforward manner the parameters of the triangle (the value of the integral) as follows:

$$\begin{aligned} &base = x_\tau - a_\tau , \\ &height(F(x_\tau)) = \Delta_\tau (x_\tau - a_\tau), \\ &slope(\Delta_\tau) = \left[\frac{1}{b_\tau - a_\tau} \right]. \end{aligned} \qquad (9)$$

Then, we obtain the area of triangle for this case as follows:

$$\int_0^{x_\tau} F(D_\tau)\, dD_\tau \approx \frac{(x_\tau - a_\tau)^2}{2(b_\tau - a_\tau)}. \qquad (10)$$

Consequently, substituting Eq. (10) into Eq. (2), we obtain formulae for the coefficients of the objective function.

$$\begin{aligned} A_\tau &= -\left(\frac{v_\tau + h_\tau}{2(b_\tau - a_\tau)} \right), \\ B_\tau &= -c_\tau + \frac{(h_\tau a_\tau + v_\tau b_\tau)}{(b_\tau - a_\tau)}, \\ C_\tau &= -\frac{(h_\tau a_\tau{}^2 + v_\tau b_\tau{}^2)}{2(b_\tau - a_\tau)} \end{aligned} \qquad (11)$$

Table 1. Summary of the coefficients of the objective function for common probability distributions

Distribution Function	$A_\tau^{(\cdot)}$	$B_\tau^{(\cdot)}$	$C_\tau^{(\cdot)}$
Uniform Dist. (exact)	$-\dfrac{(h_\tau + v_\tau)}{2(b_\tau - a_\tau)}$	$-c_\tau + \dfrac{(h_\tau a_\tau + v_\tau b_\tau)}{(b_\tau - a_\tau)}$	$-\dfrac{(h_\tau a_\tau{}^2 + v_\tau b_\tau{}^2)}{2(b_\tau - a_\tau)}$
Exponential Dist. (approximate)	$-\dfrac{(v_\tau + h_\tau)}{2x^*_\tau}\left(1 - e^{\frac{-x^*_\tau}{\mu_\tau}}\right)$	$-(c_\tau - v_\tau)$	$-v_\tau \mu_\tau$
Normal Dist. (approximate)	$-\left(\dfrac{v_\tau + h_\tau}{2}\right)\Delta_\tau$	$-(c_\tau - v_\tau) + (v_\tau + h_\tau)[\Delta_\tau]x_{l,\tau}$	$-\left(\dfrac{v_\tau + h_\tau}{2}\right)(\Delta_\tau)(x_{l,\tau})^2 - v_\tau(\mu_\tau)$

3.2 Silhouette II: The parabola shape distribution function with zero lower bound

Among the members of probability distribution functions that belong to this family of shapes are the Exponential, the Weibull and the Lognormal distributions. The steps to determine the coefficients of the objective function for these types are as follows:

1) Calculate $F(x^*_\tau) = \theta_\tau = \dfrac{(v_\tau - c_\tau)}{(v_\tau + h_\tau)}$
2) Calculate $x^*_\tau = F^{-1}(\theta_\tau)$, where x^*_τ denotes the unconstrained optimal solution (If constraints are redundant, unbinding, the approach will give the solutions equal to that obtained in *step* (2).)
3) Set the values of the parameters for the triangle area as follows:

$x_{l,\tau} = 0$, $x_{u,\tau} = x^*_\tau$, $F(x_{u,\tau}) = F(x^*_\tau)$.
(see Figure 2)

Then, one can proceed in the similar fashion as mentioned in the previous sub-section to obtain the triangle's parameters. The coefficients of the objective function are shown in Eq. (12) and (13) respectively.

$$\begin{aligned} &base = x_\tau , \\ &height(F(x_\tau)) = \Delta_\tau x_\tau , \\ &slope(\Delta_\tau) = \left[\frac{F(x^*_\tau)}{x^*_\tau} \right], \\ &area\left(\int_0^{x_\tau} F(D_\tau)\, dD_\tau \right) = \frac{x^2_\tau (F(x^*_\tau))}{2(x^*_\tau)}. \end{aligned} \qquad (12)$$

$$A_\tau = -\left(\frac{v_\tau + h_\tau}{2(x_\tau^*)}\right)(F(x_\tau^*)),$$
$$B_\tau = (v_\tau - c_\tau),$$
$$C_\tau = -v_\tau(\mu_\tau). \tag{13}$$

To illustrate further the application for this type of silhouette, let us consider the case of an exponentially distributed demand function. Its distribution function is given by Eq. (14).

$$F(D_\tau) = 1 - e^{\frac{-D_\tau}{\mu_\tau}} \qquad 0 < D_\tau < \infty \tag{14}$$

Using Eq. (13), we obtain coefficients of the objective function as follows:

$$\Delta_\tau = \left[\frac{1 - e^{\frac{-x_\tau^*}{\mu_\tau}}}{x_\tau^*}\right],$$

$$A_\tau = -\left(\frac{v_\tau + h_\tau}{2(x_\tau^*)}\right)\left(1 - e^{\frac{-x_\tau^*}{\mu_\tau}}\right),$$
$$B_\tau = -(c_\tau - v_\tau),$$
$$C_\tau = -v_\tau(\mu_\tau). \tag{15}$$

The error of the approximated area in this case is

$$error = x_\tau - \mu_\tau\left(1 - e^{\frac{-x_\tau}{\mu_\tau}}\right) - \frac{1}{2}\frac{x_\tau^2}{x_\tau^*}\left(1 - e^{\frac{-x_\tau^*}{\mu_\tau}}\right) \tag{16}$$

where, $0 \le x_\tau \le x_\tau^*$. Thus, the lower bound of the error of the approximated area is given by taking $\lim_{x_\tau \to 0}$ and the upper bound of the error is given by taking $\lim_{x_\tau \to x_\tau^*}$. The result shows in the following Eq. (17).

$$0 \le error \le \frac{1}{2}x_\tau^*\left(1 + e^{\left(\frac{-x_\tau^*}{\mu_\tau}\right)}\right) - \mu_\tau\left(1 - e^{\left(\frac{-x_\tau^*}{\mu_\tau}\right)}\right) \tag{17}$$

3.3 Silhouette III: The S shape distribution function with non zero lower bound

Among the CDFs that belong in this category are the Normal, the Student (*t*), and the Beta distributions. Their silhouettes look similar to that which is shown in Figure 3. From that figure, one can see that setting the value of $x_{l,\tau} = 0$ is not suitable. Therefore, we have to find the appropriate value for $x_{l,\tau}$. It should be noted that by allowing $x_{l,\tau} > 0$, we are truncating the tail of the distribution function. Hence, the range of possible optimal solutions of item τ will be within $x_{l,\tau} \le x_\tau \le x_\tau^*$.

Because of the different nature of this type of distribution function, we propose two approximation procedures. One can apply both and then compare which of them yields less cost The first approach is based on Taylor expansion of the demand distribution function, while the second is based on the calculation of the triangular area for the following specific values of $F(x)$; (0.001 and 0.9). Note that the triangular area which is calculated for that range of $F(x)$ in essence covers the area under the CDF.

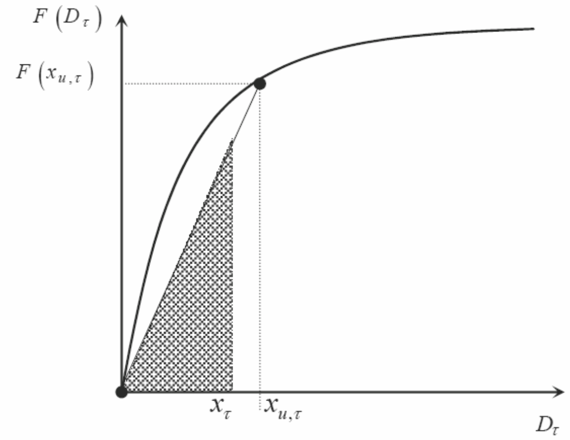

Figure 2. Triangular approximation (shaded area) for exponential distribution.

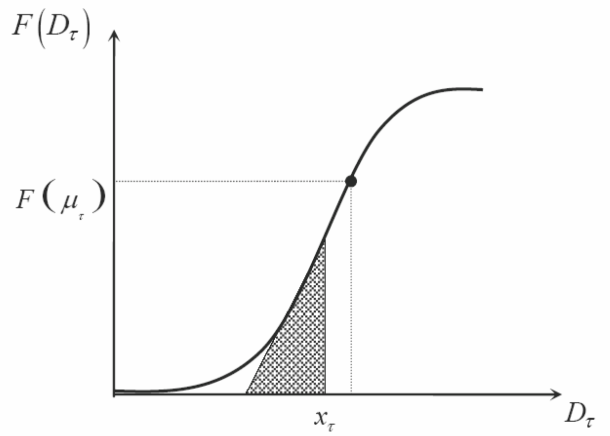

Figure 3. Triangular approximation (shaded area) obtained for Taylor's expansion.

3.3.1 The first approach (Taylor series expansion)

We expand the CDF of the demand using Taylor series around the expected value for item τ.

$$F(x_\tau) = F(\mu_\tau) + f(\mu_\tau)(x_\tau - \mu_\tau) \tag{18}$$

where $F(x_\tau)$ represents the CDF, $F(\mu_\tau)$ represents the value of the CDF at μ_τ, $f(\mu_\tau)$ represents the value of density function at μ_τ. Letting $F(x_\tau) = 0$ in Eq. (18), we obtain the formulae for $x_{l,\tau}$ and Δ_τ as shown

$$x_{l,\tau} = \mu_\tau - \frac{F(\mu_\tau)}{f(\mu_\tau)}, \quad slope(\Delta_\tau) = f(\mu_\tau) \tag{19}$$

Hence, the parameters and the area of the triangle can be approximated as follows:

$$base = x_\tau - \mu_\tau + \frac{F(\mu_\tau)}{f(\mu_\tau)},$$

$$height\big(F(x_\tau)\big) = f(\mu_\tau)\left(x_\tau - \mu_\tau + \frac{F(\mu_\tau)}{f(\mu_\tau)}\right), \text{ and} \tag{20}$$

$$area\left(\int_0^{x_\tau} F(D_\tau)\,dD_\tau\right) = \frac{f(\mu_\tau)}{2}\left(x_\tau - \mu_\tau + \frac{F(\mu_\tau)}{f(\mu_\tau)}\right)^2.$$

Substituting Eq. (20) into Eq. (7), we obtain the objective function coefficients as shown in Eq. (21-23).

$$\begin{aligned} A_\tau &= -\left(\frac{v_\tau + h_\tau}{2}\right) f(\mu_\tau), \\ B_\tau &= (v_\tau - c_\tau) + (v_\tau + h_\tau)K_{1,\tau}, \\ C_\tau &= (v_\tau + h_\tau)K_{2,\tau} - v_\tau(\mu_\tau) \end{aligned} \tag{21}$$

$$K_{1,\tau} = \big(f(\mu_\tau)\,\mu_\tau - F(\mu_\tau)\big) \tag{22}$$

$$K_{2,\tau} = \frac{-\left[\big(f(\mu_\tau)\mu_\tau - F(\mu_\tau)\big)^2\right]}{2f(\mu_\tau)} \tag{23}$$

3.3.2 The second approach (Covering range approach)

For this approach, Δ_τ and $x_{l,\tau}$ are calculated by using the following equations:

$$\Delta_\tau = \frac{(0.9 - 0.001)}{\left(F^{-1}(0.9) - F^{-1}(0.001)\right)} \quad \text{and} \quad x_{l,\tau} = F^{-1}(0.9) - \frac{0.9}{\Delta_\tau}$$

In a similar fashion, we can obtain the coefficients of the objective function. Figure 4 and the numerical examples show the details of these approaches.

4. NUMERICAL EXAMPLES

This section illustrates numerically the applications of the proposed model and compares them, when applicable, to existing ones. These examples are extracted from Lau and Lau (1996) and Abdel-Malek et al. (2004). (In these papers, however, the only constraint considered was that of the budget and the lower bounds, L_τ's of the demands are set to be zero. Therefore, $\xi_{i,\tau}$ which are the coefficients of the additional constraints and L_τ's are set to zero in our analysis for comparison purposes). The first example demonstrates the application to silhouette I, the second to silhouette II, the third to silhouette III, and the last example considers a mix of all silhouettes. These examples bring to focus the generality of the proposed models as compared to the existing ones and its structured way of application as well as the quality of the solutions rendered.

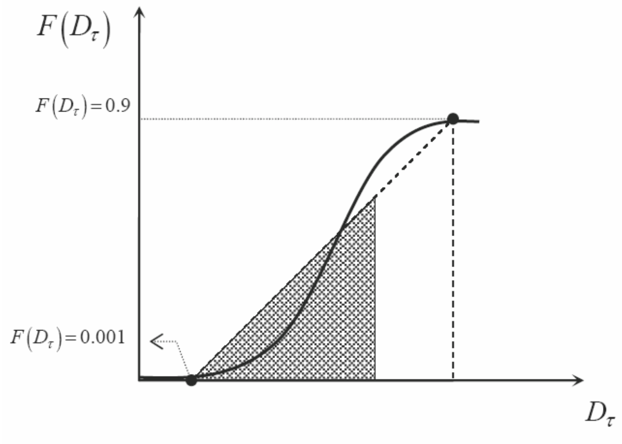

Figure 4. Triangular approximation (shaded area) for second approach.

4.1 Uniform distribution

Lau and Lau (1996) consider a three-product case. Their pertinent data are shown in Table 2. Applying the steps introduced in Section 3.1, the values of the coefficients of the objective function, Eq. (11), are calculated and exhibited in Table 2.

Table 2. Example's numerical data and calculated coefficients of the objective function

Item	v_τ	h_τ	c_τ	a_τ	b_τ	A_τ	B_τ	C_τ
1	4	1	1	5	195	−0.0132	4.1316	−400.3289
2	3	2	1	15	585	−0.0044	3.1316	−900.9868
3	2	6	2	10	190	−0.0222	6.4444	−602.2222

It should be noted that, in their paper, Lau and Lau denote c_τ as the unit usage capacity instead of unit cost and also they vary the level of available capacity between 50 and 804 units. The comparison of the results between Lau and Lau (1996) and the triangular approach is shown in Table 3.

From Table 3, one can see that the triangular approach gives a lower total cost than that of Lau and Lau. It should be noted that despite the approximate and simple nature of our approach, as opposed to the numerical iterative approach of Lau and Lau, our results are similar or slightly better.

4.2 The exponential distribution

Let us consider again the example of Lau and Lau (1996) for the case of exponentially distributed demand function. The pertinent data are shown in Table 4. Implementing the steps introduced in Section 3.2, the values of coefficients of the objective function are calculated and also shown in Table 4.

Table 3. Comparison between that of Lau & Lau and triangular approach

Capacity	Item 1 (x_1^{**})		Item 2 (x_2^{**})		Item 3 (x_3^{**})		Total Cost ($)		% Diff
	Lau & Lau	Triangular Approach	Lau & Lau	Triangular Approach	Lau & Lau	Triangular Approach	Lau & Lau	Triangular Approach	
>804	157	157	357	357	145	145	553	553	0.000
80	43.31	43.31	15.94	15.94	10.37	10.37	1636	1636	0.000
70	43	27.14286	7	0	10	42.85714	1666	1565	6.062
50	43	21.42857	0	0	3.5	28.57143	1726	1654	4.172

Note: x_1^{**} is the constrained optimal solution.

Table 4. Example's numerical data and calculated coefficients of the objective function

Item	v_τ	h_τ	c_τ	μ_τ	A_τ	B_τ	C_τ
1	4	1	1	100	−0.0124	4	−400
2	1	1	1	500	−0.0014	1	−500
3	2	2	1	300	−0.0048	2	−600

The comparison of the results between those of Lau and Lau and the triangular approach is shown in Table 5. As can be seen, the results obtained from the two models are almost identical. However, the proposed approach is more systematically structured.

4.3 The Beta distribution

To illustrate the triangular approach for a Beta distribution function, we consider the example given in Abdel-Malek et al. (2004) where an iterative method called GIM is used. In their example, six products are examined with a budget constraint of $ 6,500.00. We implement the steps of Section 3.3. Table 6 shows the pertinent data and the obtained optimum results. From the table, one can see that the solution obtained by the triangular approach is better and requires less computational effort.

4.4 General distributions

This numerical example is also extracted from Lau and Lau (1996). It considers a newsvendor problem with seven products and five constraints. The products parameters are given in Table 7, and those of resources and their usage for each product are shown in Table 8.

In this example we can first group items into two sets based on their distribution function shapes. The first set of items {1, 3, 5, 6, 7} has the parabola distribution shape, silhouette II given in section 3.2, and the second set of items {2, 4} has the S shape distribution, silhouette III which is given in section 3.3. After grouping the set of items, we can implement the steps described for each type of silhouette. And noting that there are two possible approaches to silhouette III set of products, we apply both of them and present the results of both in Tables (9 and 10). The obtained coefficients of the objective function ($A_\tau^{(\cdot)}, B_\tau^{(\cdot)}$, and $C_\tau^{(\cdot)}$) are shown in Table 9 and the optimal results as well as the comparison to those of Lau and Lau are exhibited in Table 10.

Table 5. Comparison between that of Lau & Lau and triangular approach

Capacity	Item 1 (x_1^{**})		Item 2 (x_2^{**})		Item 3 (x_3^{**})		Total Cost ($)		% Diff
	Lau & Lau	Triangular Approach	Lau & Lau	Triangular Approach	Lau & Lau	Triangular Approach	Lau & Lau	Triangular Approach	
> 1755.2	161.9	160.94	346.6	346.57	207.9	207.94	923.41	923.40	0.00%
1300	155	157.72	236.5	235.66	198.9	199.63	936.92	936.95	0.00%
1000	150.5	155.6029	164.4	162.56	191.9	194.14	962.46	962.62	0.02%
500	142.1	152.07	44.9	40.73	178.4	185.01	1041	1042	0.10%
25	25	25	0	0	0	0	1414	1414	0.00%

Table 6. Comparison between the iterative GIM and the triangular approach

item	v_τ	h_τ	c_τ	$x_{\tau\, min}$	$x_{\tau\, max}$	α_τ	β_τ	A_τ	B_τ	C_τ	GIM (x_τ^{**})	Triangular Approach (x_τ^{**})	Total Cost ($) GIM	Total Cost ($) Triangular Approach	% Diff
1	7	1	4	100	300	2	1	-0.03	11.74	-2357.32	206.83	207.83	1094.12	1093.43	
2	12	2	7	50	250	1	1.2	-0.04	8.54	-1772.63	95.69	95.53	1298.74	1298.94	
3	30	4	15	75	150	1	2	-0.33	61.94	-4670.91	90.10	89.96	1617.57	1617.06	
4	17	3	10	50	200	2	2	-0.10	21.66	-2669.56	100.12	100.45	1517.38	1516.79	
5	27	5	15	50	200	2	3	-0.19	36.72	-3779.24	90.072	90.15	2027.19	2026.98	
6	10	2	6	73	275	0.8	0.2	-0.04	17.56	-3596.91	209.35	215.65	1699.30	1693.12	
Summation of total cost ($)													9254.29	9246.31	0.09%

Table 10 reveals that the quality of the optimum solutions yielded by the developed methods. In addition, it should be that among the differences between the methodology proposed here and that of Lau and Lau that the latter seems to consider only products when their purchase costs are equal as opposed to the former approach where there is the flexibility of accommodating products with varying purchase costs; and that Lau and Lau method is based on numerical iterative models developed for this particular purpose in contrast to the method here that could utilize familiar linear programming software.

Table 7. Products parameters and costs

Product #	Distribution	Parameters	v_τ [$/item]	h_τ [$/item]
1	Exponential	(335)	3	2
2	Normal	(150, 45)	2.5	1.5
3	Weibull	(1.8, 100)	3	2.5
4	Beta	(50, 850, 3, 4)	4	3
5	Weibull	(2, 60)	5	5
6	Lognormal	(5.19, 0.47)	6	7
7	Exponential	(600)	7	2

Table 8. Numerical data for operating constraints

Constraint #	Resources availability	Resources Usage for Product # 1	2	3	4	5	6	7
1	2800	1	1	4	4	1	2.5	0.5
2	1900	2	2	3	1	1	0.5	0.7
3	2000	3	1	1	1	4	1.5	2.5
4	5800	4	1	2	1.5	5.5	6	4.2
5	2400	1	3	1	2	3	0.5	4

5. CONCLUSION

The multi-product newsboy with constraints has been found suitable for modeling many situations particularly in today's supply chain environment. Motivated by the interest of the community in this problem, in this paper we complement the existing solution methodologies by developing an approach that utilizes the familiar linear programming software as opposed to the existing numerical methods that are mainly Lagrangian based. The developed approach converts the objective newsboy cost function to a quadratic form. The value of each integral of the cost function is estimated by a triangular area which yields exact solutions in case of uniform distribution of the demand probability function, or approximate otherwise.

In addition to the systematic steps of the developed models and the high quality of the solutions rendered, they present the decision maker with the ability to conduct post optimality analysis for the solutions obtained. Moreover, the methods here include the lower bounds of the order quantity which many of the existing methods ignore. This protects against obtaining erroneous solutions particularly when the resources are tight and the number of products considered is large.

Finally, the models here are portable to the class room environment given exposure to students to these important models of the newsboy problem. Also, the methods here have the potential of being extended to cover other cost structures of the original models which is our intent for future development.

Table 9. Coefficients of the objective function

Product #	1st approach $A_\tau^{(\cdot)}$	1st approach $B_\tau^{(\cdot)}$	1st approach $C_\tau^{(\cdot)}$	2nd approach $A_\tau^{(\cdot)}$	2nd approach $B_\tau^{(\cdot)}$	2nd approach $C_\tau^{(\cdot)}$
1	0.00	3.00	-1005.00	-0.0049	3	-1005
2	-0.02	5.82	-530.34	-0.0174	5.7306	-524.6479
3	-0.02	3.00	-300.00	-0.0207	3.6193	-10.0431
4	-0.01	8.96	-2345.63	-0.0101	8.7819	-355.7650
5	-0.04	4.00	-240.00	-0.0632	5.8185	-21.0723
6	-0.01	2.00	-0.94	-0.0231	5.7805	-165.2614
7	0.00	5.00	-3000.00	-0.0033	5	-3000

Table 10. Cost comparison between triangular approach and Lau & Lau

Product #	x_τ^{**} (items) 1st approach	2nd approach	Lau & Lau	Total cost ($) 1st approach	2nd approach	Lau & Lau	% Diff 1st approach	2nd approach
1	210.73	189.64	188.7	644.40	660.25	661.039		
2	95.77	37.85	105.9	145.50	280.71	125.751		
3	63.08	64.73	71.7	125.14	123.44	117.667		
4	356.38	331.90	324.6	506.34	552.567	568.714		
5	16.45	16.59	29.2	150.522	150.05	115.247		
6	95.84	89.40	115.1	223.51	231.77	207.507		
7	257.22	318.45	256.9	2050.00	1907.00	2051		
Summation of total cost ($)				3845.412	3905.787	3846.925	0.04%	1.54%

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APPENDIX A

This Appendix shows how to derive the objective function shown in Eq. (2) from this given in Eq. (1).

$$Z = \sum_{\tau=1}^{N}\left[c_\tau x_\tau + h_\tau \int_0^{x_\tau} (x_\tau - D_\tau) f_\tau (D_\tau) dD_\tau + v_\tau \int_{x_\tau}^{\infty} (D_\tau - x_\tau) f_\tau (D_\tau) dD_\tau \right]$$

$$Z = \sum_{\tau=1}^{N}\left[c_\tau\, x_\tau + v_\tau \int_{x_\tau}^{\infty} D_\tau f(D_\tau) dD_\tau - v_\tau\, x_\tau \int_{x_\tau}^{\infty} f(D_\tau) dD_\tau + h_\tau\, x_\tau \int_0^{x_\tau} f(D_\tau) dD_\tau - h_\tau \int_0^{x_\tau} D_\tau\ f(D_\tau) dD_\tau \right] \tag{A-1}$$

$$Z = \sum_{\tau=1}^{N}\left[c_\tau\, x_\tau + v_\tau \left(E(D_\tau) - \int_0^{x_\tau} D_\tau\ f(D_\tau) dD_\tau \right) - v_\tau x_\tau \left[1 - F(x_\tau)\right] + h_\tau x_\tau F(x_\tau) - h_\tau \int_0^{x_\tau} D_\tau f(D_\tau) dD_\tau \right]$$

$$Z = \sum_{\tau=1}^{N}\left[c_\tau\, x_\tau + v_\tau E(D_\tau) - (v_\tau + h_\tau) \int_0^{x_\tau} D_\tau f(D_\tau) dD_\tau - v_\tau x_\tau \left[1 - F(x_\tau)\right] + h_\tau x_\tau F(x_\tau) \right]$$

Integrating by parts the function, $\int_0^{x_\tau} D_\tau f(D_\tau) dD_\tau$, we obtain

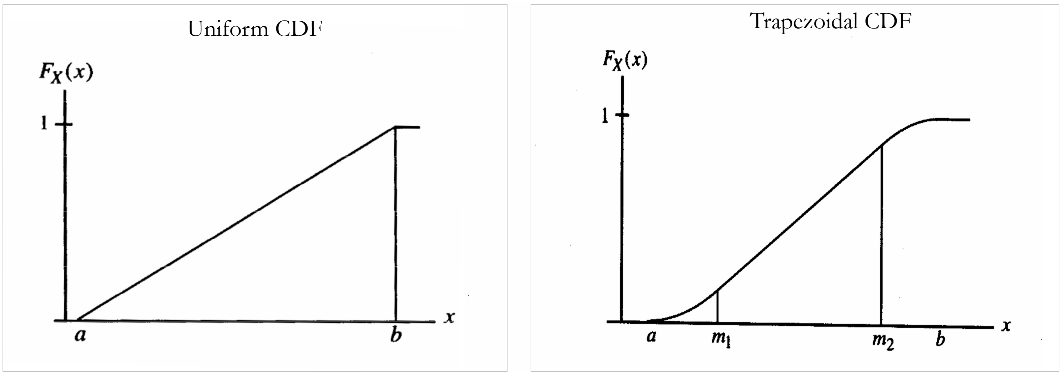
$$\int_0^{x_\tau} D_\tau f(D_\tau) dD_\tau = x_\tau F(x_\tau) - \int_0^{x_\tau} F(D_\tau) dD_\tau \tag{A-2}$$

And Substituting Eq. (A-1) into (A-2) and converting the problem into maximization, we obtain Eq. (2).

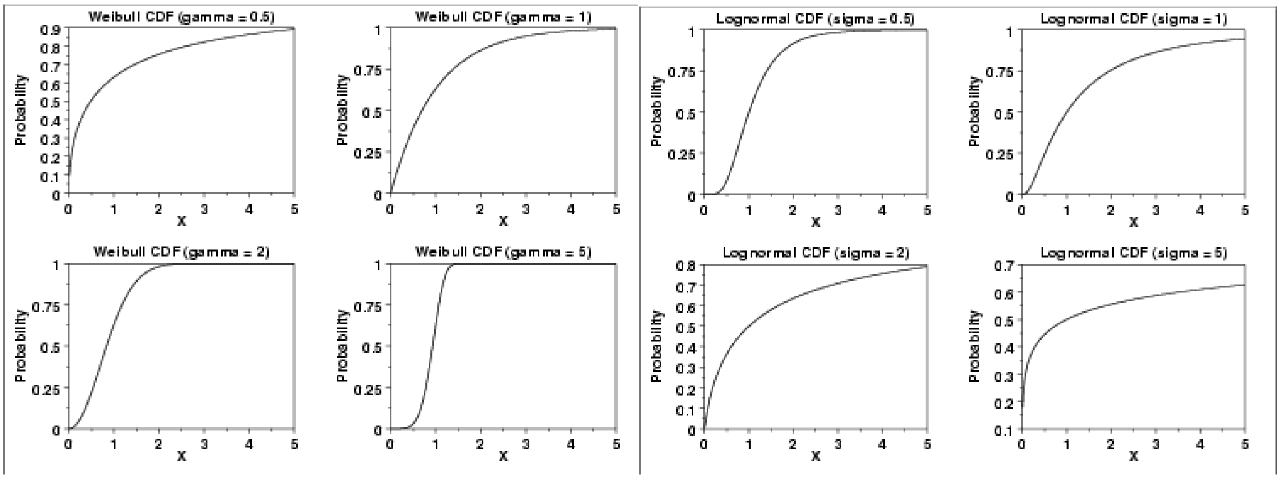
APPENDIX B

This following figures show the silhouettes of the functions for the considered probability distributions.

Silhouette I:



Silhouette II:



Silhouette III: