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Reliability Analysis of Consecutive-*k*, *r*-Out-Of-*n*: DFM System using GERT

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Abstract—Koutras (1997) analyzed reliability of a consecutive-k, r-out-of-n: DFM system consisting of n components linearly arranged which fails if and only if at least k consecutive components are failed-open or at least r consecutive components are failed-short. In this paper Graphical Evaluation and Review Technique (GERT) has been applied to model and analyze the reliability of the above system. One of the strengths of the GERT network is the graphical representation, which is intuitive and easy to understand. The components are assumed to be i.i.d. Furthermore, numerical computations are conducted using Software Mathematica to determine the actual computation times, which are almost negligible.

Keywords—Consecutive-k, r-out-of-n: DFM system, Reliability, GERT

ACRONYMS

$C_{DFM}(k,r,n)$: Consecutive-k, r-out-of-n: DFM system
GERT: Graphical Evaluation and Review Technique

NOTATIONS

n: number of components
p: survival probability of a component
q_1: probability of a component in failed-open mode
q_2: probability of a component in failed-short mode
$R_{k,r}(n)$: Reliability of the system

Further, the three modes of operation (working, failed-open, failed-short) of a component are supposed to be mutually exclusive and exhaustive, i.e.

$p + q_1 + q_2 = 1$

1. INTRODUCTION

The study of dual failure mode (DFM) or three state devices has received continuing research interest since mid-1950s (Dhillon and Rayapati (1986), Jenney and Sherwin (1986), Malon (1989), Page and Perry (1987, 1988, 1989), and Satoh et al. (1993)). Several redundant structures as well as methods of calculating system reliability have been researched in order to improve their reliability. The major areas of substantial advance are the reliability evaluation and optimal design of various redundant DFM structures. These systems have wide applicability in nuclear industry where the common terminology used is "failure to safety" and "failure to danger"; in fluid flow control networks where a defective valve could be either "stuck open" or "stuck closed"; in electronic/electrical engineering studies, where the modes of failure are usually labeled as "failed-open" and "failed-short" (Koutras (1997)).

Many research results have been reported on reliability evaluation of consecutive-k-out-of-n systems; for example, see Chiang and Niu (1981), Kuo et al. (1994) and Chao et al. (1995). A survey of consecutive-k systems and its various generalizations can be found in Chang et al. (2000), Kuo and Zuo (2003), and Pham (2003). The consecutive-k, r-out-of-n: DFM system is an extension of well known consecutive-k-out-of-n: F system subject to dual failure mode environment. Koutras (1997) studied the reliability of $C_{DFM}(k, r, n)$ in which the system fails if and only if at least k consecutive components are failed-open or at least r consecutive components are failed-short with independent but not necessarily identical components providing recurrence relation. Further, upper and lower bounds are also derived, for a quick assessment of the order of magnitude of the system's reliability.

In this paper, $C_{DFM}(k, r, n)$ (Koutras, 1997) has been analyzed through GERT. The components are assumed to be i.i.d. It can be observed that GERT not only provides the visual picture of the system but also helps to determine the generating function for the reliability of the system in a much easier way. GERT is easier to use than minimal cut set method. In GERT one has to evaluate a W function, the generating function of the waiting time for the occurrence of the system failure, whereas in minimal cut set method one has to enumerate all possible minimal cut sets leading to system failure. Numerical computations to obtain reliability corresponding to different sets of values of n, k, r, p, q_1 and q_2 have been carried out using Software Mathematica. Further, expected time to system failure when probabilities p, q_1 and q_2 are not known for different

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sets of values of the parameters involved are also given. CPU times involved are almost negligible.

2. BRIEF REVIEW OF GERT AND DEFINITIONS

For the sake of completion, a brief description about GERT (Whitehouse (1973), Cheng (1994)) is given below:

GERT is a procedure, which combines the disciplines of flow graph theory, MGF (Moment Generating Function) and PERT (Project Evaluation and Review Technique) to obtain a solution to stochastic networks having logical nodes and directed branches. Each branch has a probability that the activity associated with it will be performed. It, therefore, besides providing visual picture of the system makes it possible to analyze the given system in a less inductive manner. The results can be obtained in a straightforward manner based on MGF using Mason's formula, which takes care of all possible products of transmittances of non-intersecting loops described later.

A review of the steps employed in applying GERT is as follows:

(1) Convert a qualitative description of a system or problem to a model in stochastic network form.
(2) Collect necessary data to describe the transmittances of the network.
(3) Apply Mason's rule to determine the equivalent function or functions of the network.
(4) Convert the equivalent function into the following two performance measures of the network:
 (i) The probability that a specific node is realized.
 (ii) The moment generating function of the time associated with a node if it is realized.
(5) Make inferences concerning the system under study from the information obtained in (4).

2.1 Definitions

Path: A path is a series of branches, which join two nodes and does not pass through any node more than once. The value of a path is the product of the transmittances along the path.

Loop: A loop is a series of branches, which lead from a node, and eventually returns to that node without passing through any node more than once. The value of a loop is equal to the product of the transmittances around the loop. A **first order loop** can be viewed as a loop having consecutive path of arrows leading from a node and returning to the same node.
A **self-loop** can be viewed as a degenerate first order loop.
Loop of order *n* is represented by a set of *n* disjoint first order loops.

Mason's Rule (Whitehouse (1973)): In an open flow graph, write down the product of transmittances along each path from the independent to the dependent variable. Multiply its transmittance by the sum of the nontouching loops to that path. Sum these modified path transmittances and divide by the sum of all the loops in the open flow graph yielding transmittance T as:

$$T = \frac{\left[\sum(path * \sum nontouching\ \ loops)\right]}{\sum loops} \tag{1}$$

where

$$\begin{aligned}&\sum nontouching\ \ loops\\&=1-(\sum first\ \ order\ \ nontouching\ \ loops)\\&+(\sum second\ \ order\ \ nontouching\ loops)\\&-(\sum third\ \ order\ \ nontouching\ \ loops)+...\end{aligned}$$

$$\begin{aligned}&\sum loops=1-(\sum first\ \ order\ \ loops)\\&+(\sum second\ \ order\ \ loops)-...\end{aligned}$$

For example consider the following open flow graph representation (Fig. 1):

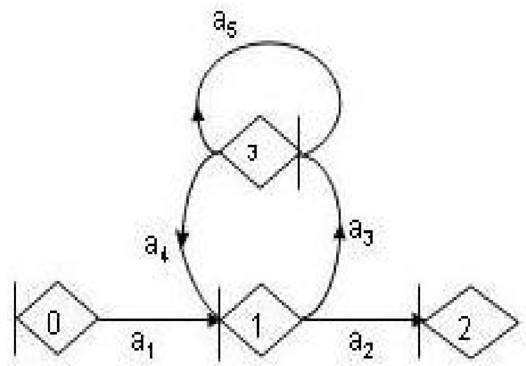

Figure 1. An open flow graph representation.

Then by Mason's rule

$$T=\frac{a_1a_2(1-a_5)}{(1-a_5-a_3a_4)} \tag{2}$$

where

$$\begin{aligned}&\sum nontouching\ \ loops=1-a_5\\&\sum loops=1-(a_5+a_3a_4).\end{aligned}$$

***W* function for GERT (Whitehouse (1973), Cheng (1994)):** In a network G with only GERT nodes, let the random variable Y_{ij} be the duration of activity (i, j) and $f(y_{ij})$ be the conditional probability of the duration y_{ij} of activity (i, j). Then conditional MGF of the random variable Y_{ij} is defined as:

$$M_{ij}(s)=E[e^{sY_{ij}}]=\sum e^{sy_{ij}}f(y_{ij}) \tag{3}$$

If Y_{ij} is constant, say a, with probability 1, then $M_{ij}(s) = e^{sa}$. When $a = 0$, then $M_{ij}(s) = 1$.

The conditional probability p_{ij} that activity (i, j) will be undertaken given that node i is realized is multiplied by the

MGF to yield a W function such that

$$W_{ij}(s) = p_{ij} M_{ij}(s). \tag{4}$$

The W function is used to obtain the information of a relationship, which exists between the nodes. In the GERT network, the variable z is used to multiply the W function associated with a branch; therefore, the power of z specifies the number of times branches were traversed whose values are multiplied by z. When a branch value is multiplied by z, we say the branch is tagged.

If we define $W(s|r)$, as the conditional W function associated with a network when the branches tagged with a z are taken r times, then the equivalent W generating function can be written as:

$$\begin{aligned} &W(s,z) \\ &= W(s|0) + W(s|1)z + W(s|2)z^2 + \cdots + W(s|r)z^r + \ldots \\ &= \sum_{r=0}^{\infty} W(s|r)z^r \end{aligned} \tag{5}$$

The relationship between the conditional W function and the conditional MGF is:

$$W(s|r) = \xi(r)M(s|r) \tag{6}$$

with

$$W(0|r) = \xi(r) \qquad \text{(since } M(0|r) = 1\text{)},$$

where $\xi(r)$ is the probability that the network is realized when the branches tagged with a z are traversed r times, and $M(s|r)$ is the conditional MGF associated with the network, given that branches tagged with a z are traversed r times. Thus

$$W(s,z) = \sum_{r=0}^{\infty} \xi(r)M(s|r)z^r \tag{7}$$

which implies that

$$W(0,z) = \sum_{r=0}^{\infty} W(0|r)z^r = \sum_{r=0}^{\infty} \xi(r)z^r . \tag{8}$$

The function $W(0,z)$ is the generating function of the waiting time for the network realization.

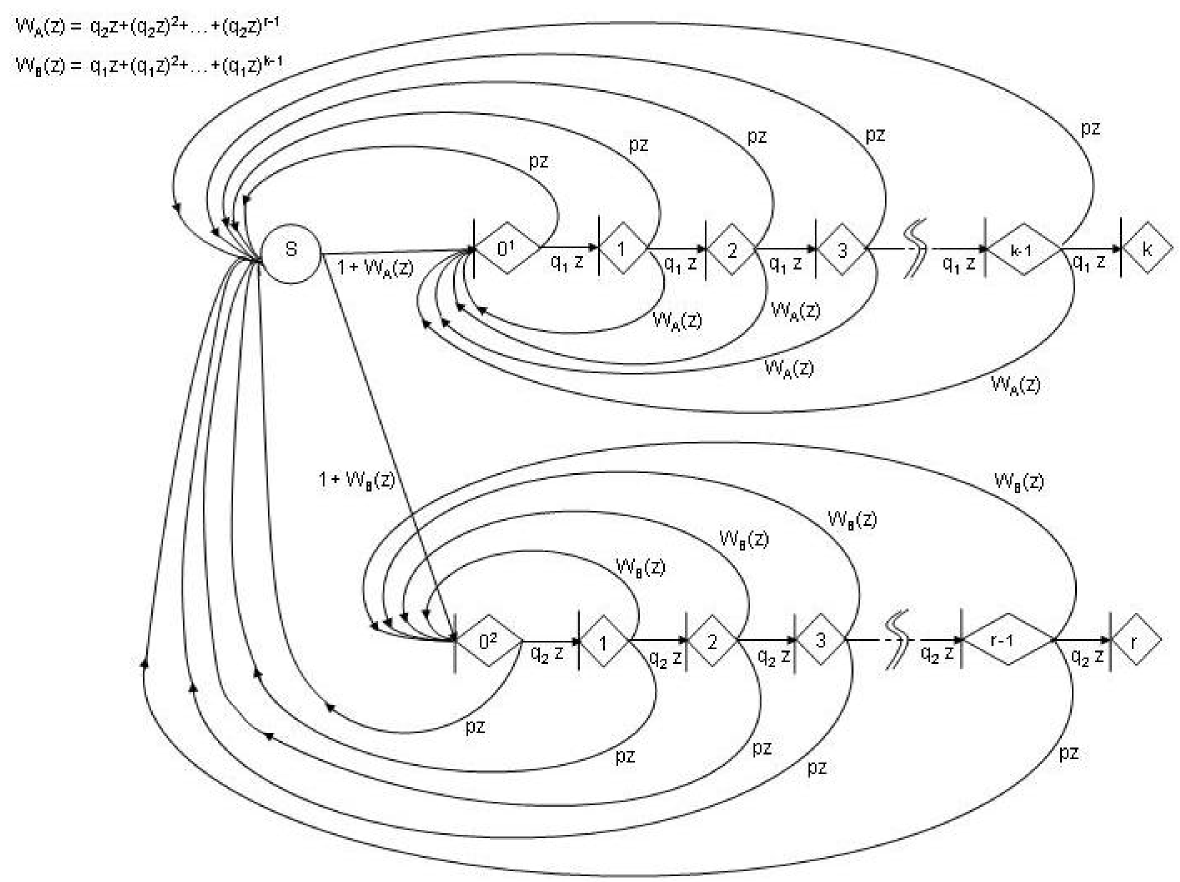

Figure 2. GERT network representing $C_{DFM}(k, r, n)$.

3. MODEL

The GERT network for $C_{DFM}(k, r, n)$ is given in Fig. 2, where each node represents a specific state as described below:

- S: starting node.
- 0^1: no component in failed-open mode.
- 0^2: no component in failed-short mode.
- 1: one component in failed-open or failed-short mode.
- 2: two consecutive components in failed-open or failed-short mode.
- $\vdots$ $\quad\vdots$

$k-1$ (or $(r-1)$): $k-1$ (or $(r-1)$) consecutive components in failed-open mode (or in failed-short mode).
k (or r): k (or r) consecutive components in failed-open mode (or in failed-short mode).

For $C_{DFM}(k, r, n)$, GERT network can be explained as follows: There are two paths from the starting node S one leading to terminal node k in which system fails due to open mode and second leading to terminal node r in which system fails due to short mode. The first path can be summarized as: System on occurrence of first open mode failure (which may be preceded by sub strings consisting of t consecutive short-mode failures with probability q_2^t, $t = 1, 2, \ldots, r-1$ and/or normal component with probability p) moves to state 1 via 0^1 from S. Further, if the next contiguous component also fails due to open mode then system moves to state 2 from state 1 with conditional probability q_1. Otherwise, system either moves to state S with conditional probability p (in case next contiguous component is normal) or to state 0^1 with conditional probability q_2^t, $t = 1, 2, \ldots, r-1$ (in case there occur t contiguous short-mode failures). In this way same procedure is followed until state k is reached, i.e. k contiguous open mode failures occur. Similar procedure is followed for the second path in which system fails due to short mode failure.

Theorem 1. For positive integers k, r, n ($n \geq \max(k, r)$), the $W_{DFM}^{k,\,r}(0,z)$, i.e., generating function of the waiting time for the occurrence of system failure is given by:

$$W_{DFM}^{k,\,r}(0,z) = \frac{(q_1z)^k(1-(q_2z)^r)(1-q_1z)+(q_2z)^r(1-(q_1z)^k)(1-q_2z)}{1-z+(1-q_1)\,z\,(q_1z)^k+(1-q_2)\,z\,(q_2z)^r-(1+pz)(q_1z)^k(q_2z)^r} \tag{9}$$

and the generating function $R_{k,r}(0,z)$ for the reliability $R_{k,r}(n)$ of the system is given by :

$$R_{k,r}(0,z) = \sum_{n=0}^{\infty} R_{k,r}(n)z^n = \frac{(1-(q_1z)^k)(1-(q_2z)^r)}{1-z+(1-q_1)z(q_1z)^k+(1-q_2)z(q_2z)^r-(1+pz)(q_1z)^k(q_2z)^r} \tag{10}$$

It matches with the generating function $G(z)$ of Koutras (1997).

Proof. For $C_{DFM}(k, r, n)$ there exist two paths leading to system failure. The first path corresponds to the case in which system fails due to open mode and second path corresponds to system failure due to short mode and are given as:

No.	Paths	Value
1	S to 0^1 to 1 to 2 to $\cdots$ to $(k-1)$ to k	$(1+W_A(z))(q_1z)^k$
2	S to 0^2 to 1 to 2 to $\cdots$ to $(r-1)$ to r	$(1+W_B(z))(q_2z)^r$

where

$$W_A(z) = q_2z+(q_2z)^2+\cdots+(q_2z)^{r-1} \quad \text{and}$$
$$W_B(z) = q_1z+(q_1z)^2+\cdots+(q_1z)^{k-1}$$

Each of the paths consists of only first order loops, as second and higher orders do not exist. For Path No. 1:

$$\sum \textit{first order loops} = pz\,(1+\sum_{i=1}^{k-1}(q_1z)^i)(1+\sum_{j=1}^{r-1}(q_2z)^j)+(q_1z)(q_2z)\,(\sum_{i=0}^{k-2}(q_1z)^i)(\sum_{j=0}^{r-2}(q_2z)^j)$$

$$= \frac{pz(1-(q_1z)^k)(1-(q_2z)^r) + (q_1z)(q_2z)(1-(q_1z)^{k-1})(1-(q_2z)^{r-1})}{(1-q_1z)(1-q_2z)} \tag{11}$$

Similarly, we obtain **Σfirst order loops** for Path No. 2, which is same as (11). Since **Σnontouching loops** do not exist for both the paths, therefore, by the use of Mason's rule, $W_{DFM}^{k,\ r}(0,z)$ function is obtained as:

$$W_{DFM}^{k,\ r}(0,z) = \frac{(q_1z)^k(1-(q_2z)^r)(1-q_1z) + (q_2z)^r(1-(q_1z)^k)(1-q_2z)}{(1-q_1z)(1-q_2z) - pz(1-(q_1z)^k)(1-(q_2z)^r) - q_1q_2z^2(1-(q_1z)^{k-1})(1-(q_2z)^{r-1})}$$
$$= \frac{(q_1z)^k(1-(q_2z)^r)(1-q_1z) + (q_2z)^r(1-(q_1z)^k)(1-q_2z)}{1-z+(1-q_1)\,z\,(q_1z)^k + (1-q_2)\,z\,(q_2z)^r - (1+pz)(q_1z)^k(q_2z)^r}.$$

Hence (9) is proved.

Thus, the generating function $R_{k,r}(0,z)$ for the reliability $R_{k,r}(n)$ of the system (Feller, 1985) is given by:

$$R_{k,\ r}(0,z) = \frac{1-W_{DFM}^{k,\ r}(0,z)}{1-z}, \tag{12}$$

which on solving yields (10). This completes the proof.

Further, for $q_2 = 0$, the generating function $R_{k,r}(0,z)$ reduces to reliability generating function of ordinary consecutive-*k*-out-of-*n*: *F* system. It can be observed that using GERT, the generating function for the reliability of the $C_{DFM}(k, r, n)$ can be obtained in a much easier way than Koutras (1997).

Example 1. Consider $C_{DFM}(3, 3, n)$ consisting of *n* components linearly ordered which fails whenever at least 3 consecutive components are failed open or at least 3 consecutive components are failed short. The GERT network is shown in Fig. 3 and explained below:

There are two paths from the starting node: first in which system fails due to open mode and second in which due to short mode. The first path in which system fails due to open mode can be summarized as follows: System on occurrence of first open mode failure, which may be preceded by normal component having conditional probability *p* and/or by substrings consisting of *t* contiguous short mode failures having conditional probability q_2^t, where $t = 1, 2$; moves to state 1 via 0^1 from *S*. Further, if the next contiguous component fails due to open mode then it moves to state 2 from 1 with conditional probability q_1. Otherwise it moves back to state *S* with conditional probability *p* in case of normal component or to state 0^1 in case of occurrence of *t* contiguous short mode failures, $t = 1, 2$. Again if the third contiguous component is in open mode failure then system moves to state 3 from state 2 otherwise similar procedure is repeated as above. System fails due to open mode as soon as three contiguous open mode failures occur. Similar procedure follows for the system to fail corresponding to short mode failures.

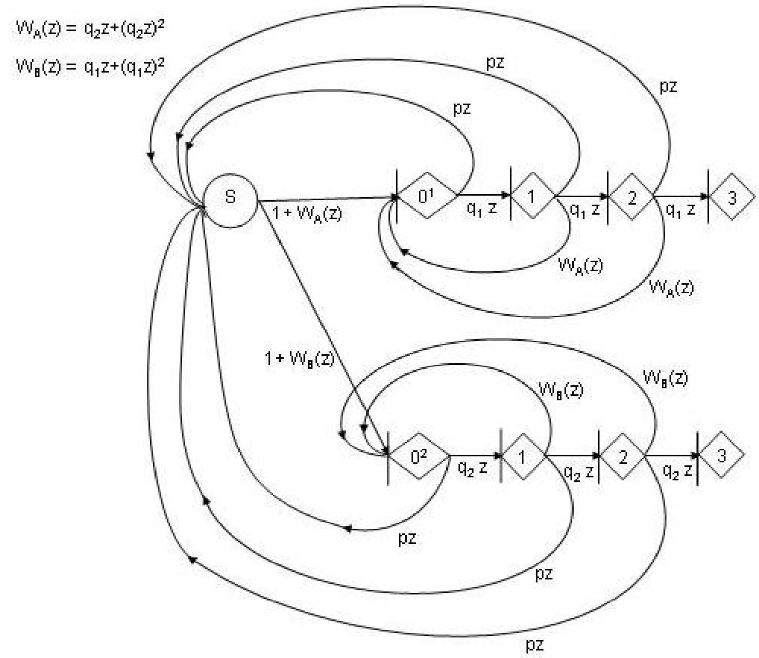

Figure 3. GERT network representing $C_{DFM}(3, 3, n)$.

The two paths leading to system failure state are as detailed below:

No.	Paths	Value
1	S to 0^1 to 1 to 2 to 3	$(q_1z)^3\ (1+q_2z+(q_2z)^2)$
2	S to 0^2 to 1 to 2 to 3	$(q_2z)^3\ (1+q_1z+(q_1z)^2)$

To apply Mason's Rule, we locate all the loops corresponding to each of the paths. However, only first order loops exist. The loops corresponding to Path No. 1 are as given below:

No.	Paths	Value
1	S to 0^1 to S	$pz(1+q_2z+(q_2z)^2)$
2	S to 0^1 to 1 to S	$pz(1+q_2z+(q_2z)^2)q_1z$
3	S to 0^1 to 1 to 2 to S	$pz(1+q_2z+(q_2z)^2)\ (q_1z)^2$
4	0^1 to 1 to 0^1	$q_1z\ (q_2z+(q_2z)^2)$
5	0^1 to 1 to 2 to 0^1	$(q_1z)^2\ (q_2z+(q_2z)^2)$

Thus, **Σfirst order loops** corresponding to open mode failure path is:

$$\begin{aligned}&\sum \textit{first order loops}\\ &= pz(1+q_1z+(q_1z)^2)(1+q_2z+(q_2z)^2)\\ &+(q_1z)(q_2z)(1+q_1z)(1+q_2z).\end{aligned} \tag{13}$$

Similarly, we can obtain first order loops corresponding to the Path No. 2 in which system fails due to short mode failure. The **Σfirst order loops** obtained by second path is same as (13).

Now, applying Mason's Rule (Whitehouse (1973), pp-168, pp-257) we obtain the following generating function $W_{DFM}^{3,\,3}(0,z)$ of the waiting time for the occurrence of system failure:

$$\begin{aligned}W_{DFM}^{3,3}(0,z) &= \frac{(q_1z)^3(1+q_2z+(q_2z)^2)+(q_2z)^3(1+q_1z+(q_1z)^2)}{1-pz(1+q_1z+(q_1z)^2)(1+q_2z+(q_2z)^2)-(q_1z)(q_2z)(1+q_1z)(1+q_2z)}\\ &= \frac{(q_1z)^3(1+q_2z+(q_2z)^2)+(q_2z)^3(1+q_1z+(q_1z)^2)}{(1-q_1z)(1-q_2z)-pz(1-(q_1z)^3)(1-(q_2z)^3)-(q_1z)(q_2z)(1-(q_1z)^2)(1-(q_2z)^2)}\\ &= \frac{(q_1z)^3(1-(q_2z)^3)(1-q_1z)+(q_2z)^3(1-(q_2z)^3)(1-q_2z)}{1-z+(1-q_1)z(q_1z)^3+(1-q_2)z(q_2z)^3-(1+pz)(q_1z)^3(q_2z)^3}\end{aligned} \tag{14}$$

Hence,

$$R_{3,3}(0,z)=\frac{(1-(q_1z)^3)(1-(q_2z)^3)}{1-z+(1-q_1)z(q_1z)^3+(1-q_2)z(q_2z)^3-(1+pz)(q_1z)^3(q_2z)^3} \tag{15}$$

Taking n = 30, p = 0.80, q_1 = 0.10 and q_2 = 0.10 and using Software Mathematica we obtain from (15):

$R_{3,3}(n)$ = 0.950415.

However, in most of the practical applications, probabilities of three modes of operation (working, failed-open, failed-short) are not known and the components follow a certain distribution, say, exponential distribution such that

age specific failure rate of a component is constant, say λ,
failed-open mode rate of a component is constant, say λ_1,
failed-short mode rate of a component is constant, say λ_2.

Then,

$$p=\frac{\lambda}{\lambda+\lambda_1+\lambda_2}$$

$$q_1=\frac{\lambda_1}{\lambda+\lambda_1+\lambda_2}$$

$$q_2=\frac{\lambda_2}{\lambda+\lambda_1+\lambda_2}.$$

Thus the corresponding GERT network is given as Fig. 4 where using (4), we have

$$W_1(s) = \frac{\lambda_1}{\lambda+\lambda_1+\lambda_2}\left(1-\frac{s}{\lambda+\lambda_1+\lambda_2}\right)^{-1}$$

$$W_2(s) = \frac{\lambda_2}{\lambda+\lambda_1+\lambda_2}\left(1-\frac{s}{\lambda+\lambda_1+\lambda_2}\right)^{-1}$$

$$W_3(s) = \frac{\lambda}{\lambda+\lambda_1+\lambda_2}\left(1-\frac{s}{\lambda+\lambda_1+\lambda_2}\right)^{-1}$$

Now, the following generating function $W_{DFM}^{k,r}(s)$ of the waiting time for the occurrence of system failure is obtained:

$$W_{DFM}^{k,r}(s) = \frac{(W_1(s))^k\,(1-W_1(s))\,(1-(W_2(s))^r) + (W_2(s))^r\,(1-W_2(s))\,(1-(W_1(s))^k)}{(1-W_1(s))\,(1-W_2(s)) - W_3(s)(1-(W_2(s))^r)(1-(W_1(s))^k) - W_1(s)W_2(s)(1-(W_1(s))^{k-1})(1-(W_2(s))^{r-1})} \tag{16}$$

such that $W_{DFM}^{k,r}(0) = 1$.

Thus,

$$E(t) = E(\text{time to system failure}) = \left.\frac{dW_{DFM}^{k,r}(s)}{ds}\right|_{s=0} \tag{17}$$

4. COMPUTATIONAL EXPERIMENTS

To study computational efficiency, reliability values of $C_{DFM}(k, r, n)$ for several sets of values of n, k, r, p, q_1 and q_2 are computed on Pentium 4 with a 2.93 GHZ CPU and 248 MB of RAM under Windows XP operating system using Software Mathematica using (10) and are given in Table 1. Further, expected time to system failure for several sets of values of λ, λ_1, λ_2, k and r, computed using (17) are given in Table 2. It may be observed that CPU time involved is almost negligible.

Table 1. Reliability values of i.i.d consecutive-*k*, *r*-out-of-*n*: DFM system

q_1	q_2	k	r	n	Reliability	CPU Time (Sec)
0.10	0.15	3	3	15	0.951117	≈ 0
0.10	0.15	3	3	25	0.915530	≈ 0
0.20	0.30	4	5	30	0.922363	≈ 0
0.20	0.30	5	4	30	0.846773	≈ 0
0.20	0.30	5	5	50	0.912378	≈ 0
0.05	0.15	5	7	75	0.999878	≈ 0
0.05	0.15	5	7	100	0.999835	≈ 0
0.05	0.15	7	7	150	0.999791	≈ 0
0.05	0.15	7	7	200	0.999718	0.005

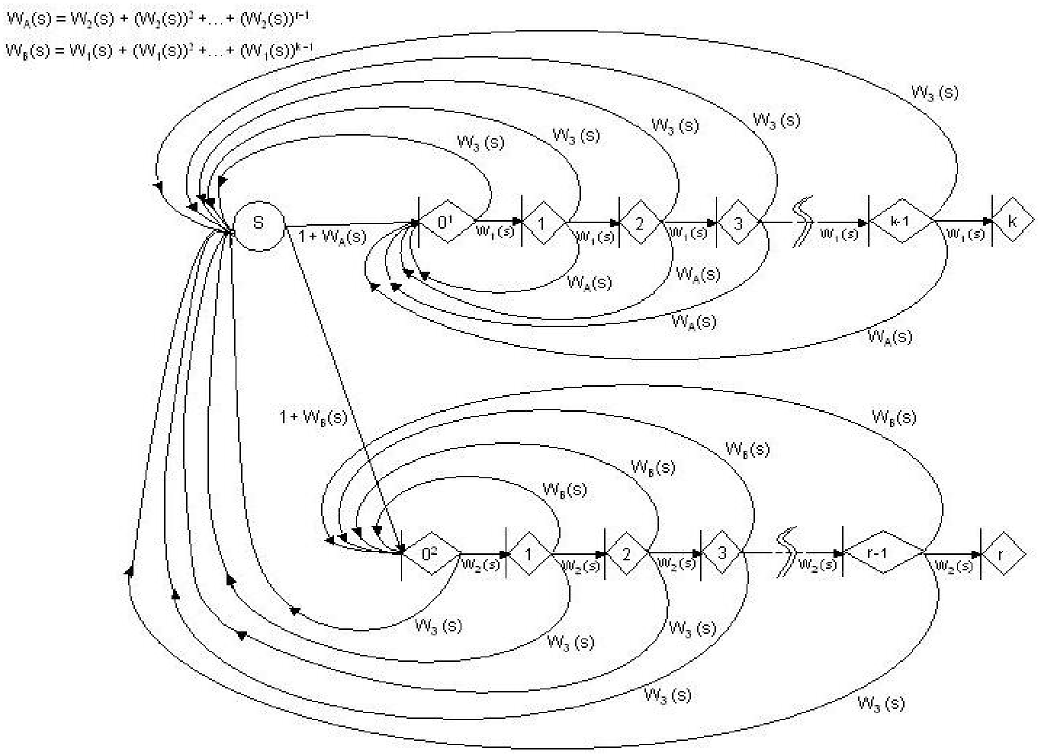

Fig 4. GERT network representing $C_{DFM}(k,r,n)$ when probabilities of three modes of operation are not known.

Table 2. Expected time to consecutive-k, r-out-of-n: DFM system failure

λ	λ_1	λ_2	k	r	$E(t)$	CPU Time (Sec)
0.60	0.15	0.25	3	3	≈ 68	≈ 0
0.75	0.10	0.15	3	3	≈265	≈ 0
0.80	0.10	0.10	4	5	≈ 10100	≈ 0
0.70	0.10	0.20	5	7	≈ 51975	≈ 0.003
0.80	0.10	0.10	5	5	55555	≈ 0
0.70	0.20	0.10	7	7	≈ 96804	≈ 0.006
0.80	0.10	0.10	6	5	101009	≈ 0.003

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