

# Upper Bounds for Large Scale Integer Quadratic Multidimensional Knapsack Problems

Dominique Quadri<sup>1</sup>, Eric Soutif<sup>2,\*</sup>, and Pierre Tolla<sup>1</sup>

<sup>1</sup>LAMSADE, Université Paris-IX, Place du Maréchal de Lattre de Tassigny, 75775 Paris Cedex 16, France

<sup>2</sup>CEDRIC, Conservatoire National des Arts et Métiers, 292 rue Saint-Martin, 75003 Paris, France

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**Abstract**—We consider the separable quadratic multi-knapsack problem (*QMKP*) which consists in maximizing a concave separable quadratic integer function subject to  $m$  linear capacity constraints. The aim of this paper is to develop an effective method to compute an upper bound for (*QMKP*) from a surrogate relaxation originally proposed in Djerdjour et al. (1988). The quality of three other upper bounds for (*QMKP*) is evaluated and they are compared theoretically and experimentally with the bound we suggest. An effective heuristic method is presented to obtain a good feasible solution for (*QMKP*). Finally, computational experiments are reported. They assess the efficiency of our upper bound for instances up to 2000 variables and constraints.

**Keywords**—Integer programming, Separable quadratic programming, Multidimensional Knapsack Problem, Surrogate relaxation

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\* Corresponding author's email: Eric.Soutif@cnam.fr