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Inventory Models Considering Post-Production Holding Time and Cost

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Abstract—This paper proposes inventory models for an environment where the approval time of the production batches is an important problem variable. The model is motivated by industries, such as the Pharmaceutical, where a batch is produced and then withheld for a certain period pending release and disposition. The paper proposes a series of cost functions that combine the classical EOQ model with a post-production hold time cost component when considering a single-tier, and a dual-tier manufacturing system. Optimal batch sizes are derived for various cases of the post-production hold time and numerical examples are presented. Finally, we present a practical application example where the proposed inventory model is utilized to support business decision-making.

Keywords—Inventory control, Batch size, Post-production hold time

1. INTRODUCTION

The pharmaceutical industry is faced with new challenges as it must now focus on cost reduction and improvements in manufacturing efficiency. In the past, reliance on blockbuster drugs with large profit margins resulted in complacency with relatively inefficient manufacturing operations. Today, pharmaceutical companies are looking into the implementation of lean and six sigma concepts, and the development of models and software tools that will allow higher resource utilization, cost reduction and better customer service. This research is motivated by a production problem in the pharmaceutical industry where the release time (post production hold time) is related to the batch size. While in the traditional economic production quantity (EPQ) model consumption can occur at the same time as production, in a variety of settings such as the pharmaceutical, products undergo quality evaluations that hold the completed batch until released for transportation and customer use, with this post-production hold time possibly being a function of the batch size, thus smaller batches could require just a few hours, while larger batches could require days. This research contributes to the literature in inventory models by addressing the case where the post-production holding time represents a considerable cost and therefore should be taken into consideration in the batch size decision-making process. It is common in the pharmaceutical industry to observe cases where the post-production hold time is longer than the actual manufacturing time.

Our model extends the economic production quantity problem to include post-production hold time. The EPQ problem has been extensively studied in operations research. From the the basic single-stage, single-item lot sizing models a number of variants and improved versions have been developed and published (Beltran and Krass (2002)) and multiple researchers have developed multi-item or multi-stage models to relax the single-item constraint (Kaminsky and Simchi-Levi (2003)). For example, Kreng and Wu (2000a) developed a multi-item model that considers the production rate as an adjustable parameter in the determination of the economic production quantity. Kreng and Wu (2000b) also developed an EPQ model that includes a setup reduction capability in the decision process. Chiu (2003) considered a variant in which a proportion of defective items are produced and are withheld for posterior repair or disposition, with backorders permitted. Hsu (2003) developed an economic lot size model for perishable products with age-dependent inventory and backorder costs. Giri and Chaudhuri (1998) developed EOQ models for perishable products where the demand is a function of the on-hand inventory. Sarker and Parija (1996) and Parija and Sarker (1999) developed models that combines the ordering policy for raw materials with the determination of the batch size for the manufacturing of a product to be delivered on fixed intervals. Hall (1996) integrated the distribution system (i.e., cost of transportation) into the total cost function to examine its effects on EPQ decisions.

Recent models aim at integrating various elements of the supply chain in the inventory optimization problem. Sarker and Khan (2001) worked on a problem that considers the raw material ordering quantities and the finished product production quantities in a single model

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that minimizes the combined system costs. Lee (2005) considers a single product supply chain and considers the batch sizes for delivery to the customer, for production of the finished good, and for the manufacturer of the main raw material component. Thus the model seeks to minimize the ordering/setup costs and the holding costs for the buyer, manufacturer and supplier. This research is in line with the supply chain perspective as it considers two levels of production (supplier and manufacturer). This problem is also relevant in the Pharmaceutical industry as the production of the active ingredient is tightly linked/ coordinated to the production of the finished product (e.g. tablets), and both production stages are often elements of the same corporation.

However, none of the variants found in the literature have considered a production model applicable to industries that have a holding time after completion of a batch, and then ship out the entire batch at once instead of over time. There is a significant cost issue here as small batches with short release times result in high setup costs and small holding costs, with large batches with longer release times result in reduced setup costs but larger holding costs. This paper proposes a model to address this relevant issue. The paper is organized as follows. Section 2 presents the basic problem. Section 3 describes the model when considering a single-tier system. Section 4 provides a numerical example of the single-tier problem. Section 5 describes the two-tier problem. Section 6 discusses two cases of the two-tier problem and presents numerical examples. Finally, Section 7 summarizes the work and presents directions for future work.

2. BASIC PROBLEM DEFINITION

We consider single-tier and two-tier inventory control problems where one finished product is manufactured. The manufacturer produces the product in batches and a batch cannot be released to the customers until a release time is complete. Thus the model assumes no consumption during production; until the complete batch is produced and released. Three costs are considered: the production setup cost, the holding cost during production, and the holding cost during post production waiting disposition and release. Our objective is to develop economic lot size models to minimize the total costs to the system considering various release time functions. The inventory pattern is illustrated in Figure 1.

The following notation is used throughout the remaining of the paper.

- D Annual demand of the buyer.
- s_j Setup cost per batch for tier j.
- c_j Cost of finished product at tier j.
- Q_j Production batch size at tier j.
- p_j Production time per unit at tier j.
- r_j Release time per unit at tier j.
- tp_j Production time per batch at tier j.
- tr_j Release time per batch, time units at tier j.
- T Total time units per year.
- i Annual capital cost per dollar invested in inventory.
- n Ratio of tier 0 batches to be produced per tier 1 batch.
- $\lceil x \rceil$ Function that results in the nearest integer equal to or exceeding x (e.g. $\lceil 2.2 \rceil = 3$).
- $\lfloor x \rfloor$ Function that results in the nearest integer equal to or not exceeding x (e.g. $\lfloor 2.2 \rfloor = 2$).

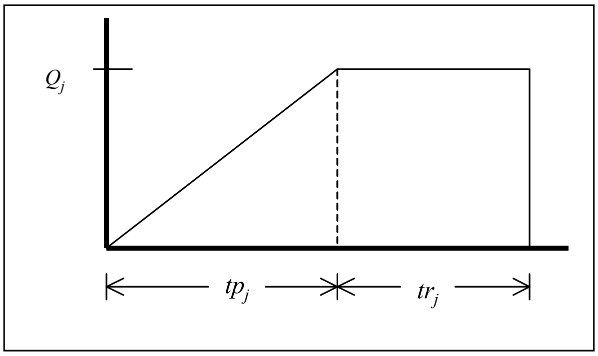

Figure 1. The inventory pattern with post-production hold time.

We assume all costs are independent of order size. The inventory does not change in value with a change in delivery time (it will be accepted by the customers when it is released) and inventory is not perishable (e.g. Pharmaceutical Products have a set life, typically 1-5 years). Production time per unit is assumed to follow a linear relationship ($tp_j = Q_j p_j$). The cost and time required to transport materials between the two-tiers, as well as the setup time are factors not considered in this study.

3. THE SINGLE-TIER PROBLEM COST EQUATIONS

In the case of a single-tier problem, the cost model resembles the traditional EPQ model. We let $j = 0$ represent the single-tier and the costs equations are as follow. Eq. (1) describes the Annual Setup Cost while Eq. (2) describes the Annual Holding Costs. Total costs are the sum of Eqs. (1) and (2).

$$ASC = s_0 D / Q_0 \tag{1}$$

$$AHC = c_0 i D(tp_0 + 2tr_0) / 2T \tag{2}$$

To determine the optimal lot size Q_0^*, we differentiate the total cost equation with respect to Q, and set it equal to zero ($dTC/dQ = 0$). We first assume tr_0 is a constant and not related to the batch size and $tp_0 = Q_0 p_0$. Therefore, for this case, the optimal batch size, Q_0^*, is as in Eq. (3), which becomes the traditional EOQ equation if $T = Dp_0$, implicating constant production (consumption for an EOQ model). Clearly, the time available must be larger that the time used, $T \geq Dp_0$. We now change the assumption regarding the release time tr_0 and model it as $tr_0 = Q_0 r_0$.

In this case, differentiation results in Eq. (4), and Eq. (5) provides Q_0^*, the optimal batch size when $tr_0 = Q_0 r_0$.

$$Q_0^* = (2s_0T/(c_0 ip_0))^{1/2} \quad (3)$$

$$Q_0^{\,2} p_0 + 2Q_0^{\,2} r_0 = 2s_0T/(c_o i) \quad (4)$$

$$Q_0^* = (2s_0T/(c_0 i(2r_0 + p_0)))^{1/2} \quad (5)$$

4. NUMERIC EXAMPLES FOR THE SINGLE-TIER MODEL

We assume D = 2,000, s_0 = \$400, i = 20%, c_0 = \$250, p_0 = 0.225, and T = 500 (some of these values were used by Lee (2005)). The selected experimental values are not intended to represent a particular application or industry, instead to demonstrate the sensitivity of the model to the cost and time parameters. Table 1 presents the results when r_0 is considered at three levels, 0.1, 0.3, and 0.5 time units; s_0 is considered at two levels, \$400 and \$200; and c_0 at two levels, \$250 and \$125. Similar to the traditional EOQ model, a decrease in the setup cost results in a reduction in batch size, while a reduction in per unit cost results in a higher batch size. As the per unit release time increases, the batch size decreases as holding costs increase with an increase in r_0, thus obviously total costs increase as r_0 increases, but in a non-linear fashion.

Table 1. Numerical examples when release time is related to the batch size

s_0	c_0	r_0	Q_0^* PRODUCT	*ASC* \$	*AHC* \$	*TC* \$
400	250	0.1	137.2	5,831	5,831	11,662
		0.3	98.5	8,124	8,124	16,248
		0.5	80.8	9,899	9,899	19,799
	125	0.1	194.0	4,123	4,123	8,246
		0.3	139.3	5,745	5,745	11,489
		0.5	114.3	8,000	6,125	14,125
200	250	0.1	97.0	4,123	4,123	8,246
		0.3	69.6	5,745	5,745	11,489
		0.5	57.1	7,000	7,000	14,000
	125	0.1	137.2	2,915	2,915	5,831
		0.3	98.5	4,062	4,062	8,124
		0.5	80.8	4,950	4,950	9,899

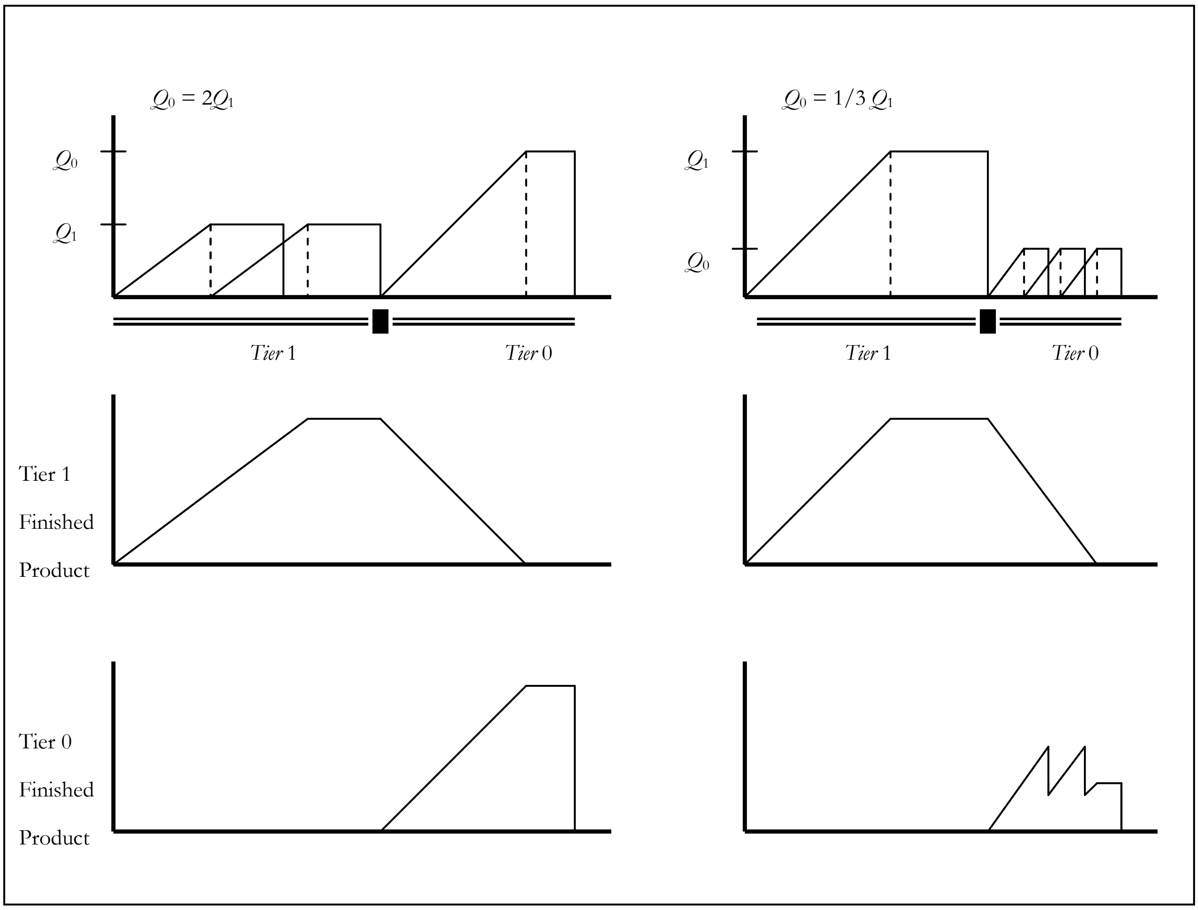

Figure 2. The two-tier problem.

5. THE TWO-TIER PROBLEM COST EQUATIONS

This model assumes a first tier manufacturer (j = 1) feeding the end product manufacturer (j = 0). The model assumes that either the batch size of the tier 1 manufacturer is an integer multiplier of the batch size of the tier 0 manufacturer or vice versa (e.g. $1/n$ is an integer value). An example for both cases is presented in Figure 2. As in the single-tier case, three costs are considered: setup costs, production holding costs, and post-production holding costs. The decision variables are Q_1 and n given $Q_0 = nQ_1$.

The Annual Setup Costs per tier are $s_j D / Q_j$ given there are D/Q cycles. Therefore Eq. (6) includes all the costs with n and Q_1 as the decision variables. When n and Q_1 increase, setup costs decrease.

$$ASC = s_1 D / Q_1 + s_0 D / (nQ_1) = (ns_1 + s_0) D / (nQ_1) \tag{6}$$

The holding costs for the Tier 1 element of the chain are incurred during production, during release time, and finally during consumption by the Tier 0 element. The three areas to be included in the holding cost equation when $n \geq 1$ are presented in Figure 3. Given the maximum inventory amount when $n \geq 1$ is Q_0, the three areas are defined as: $a_1 = Q_0 n t p_1 / (2T)$, $a_2 = Q_0 t r_1 / T$, and $a_3 = Q_0 t p_0 / (2T)$. There are D/Q_0 cycles per year and thus the Annual Holding Cost for the Tier 1 manufacturing process if $n \geq 1$ is given by Eqs. (7) (substituting for $tp_1 = Q_1 p_1$ and $tp_0 = nQ_1 p_0$).

$$AHC_{1(n\geq 1)} = Dc_1 i(nQ_1 p_1 + 2tr_1 + nQ_1 p_0)/(2T) \tag{7}$$

When $n \leq 1$ (and $1/n$ an integer) the areas to be considered are presented in Figure 4. The maximum inventory amount is Q_1 and the three areas are defined as follows: $a_1 = Q_1 t p_1 / (2T)$, $a_2 = Q_1 t r_1 / T$, and $a_3 = Q_1 t p_0 / (2Tn)$. Given there are D/Q_1 cycles per year, the Annual Holding Cost for the Tier 1 manufacturing process if $n \leq 1$ is given by Eqs. (8). When $n = 1$, $AHC_{1(n\geq 1)} = AHC_{1(n\leq 1)}$.

$$AHC_{1(n\leq 1)} = Dc_1 i(Q_1 p_1 + 2tr_1 + Q_1 p_0)/(2T) \tag{8}$$

We assume once a batch completes its post-production hold, it is automatically shipped to the customers and therefore eliminated from the inventory. The Annual Holding Cost of the Tier 0 manufacturer is as in Eq. (9). The total costs equations for the two conditions are presented in Eqs. (10) and (11).

$$AHC_0 = Dc_0 i(nQ_1 p_0 + 2tr_0)/(2T) \tag{9}$$

$$TC_{(n\geq 1)} = ASC + AHC_{1(n\geq 1)} + AHC_0 \tag{10}$$

$$TC_{(n\leq 1)} = ASC + AHC_{1(n\leq 1)} + AHC_0 \tag{11}$$

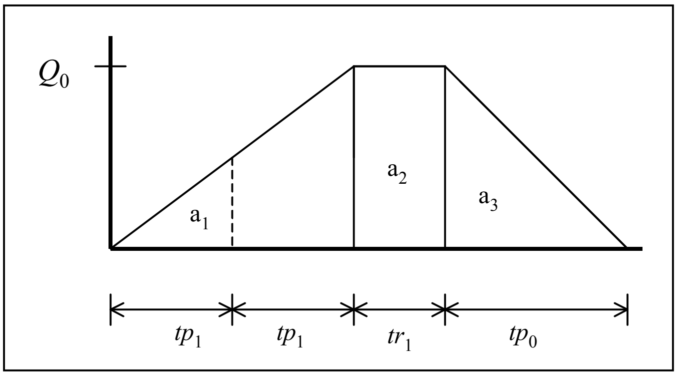

Figure 3. Holding cost areas for tier 1 manufacturer when $n \geq 1$.

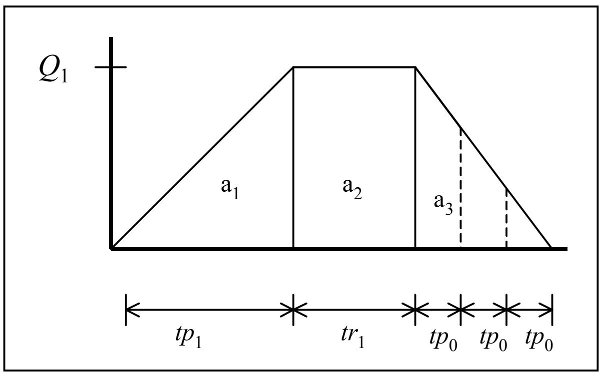

Figure 4. Holding cost areas for tier 1 manufacturer when $n \leq 1$.

6. TWO-TIER PROBLEM CASES AND THE OPTIMAL SOLUTION

As in the case of the single-tier system, we consider two cases of the release time in the determination of an optimal solution. In case 1 the release time is fixed and not related to the batch size, in case 2 the release time is linearly related to the batch size (as production time).

6.1 Case 1: Fixed release time

This case assumes the release times are not related to the batch size, and therefore the annual holding costs associated with release times are a fixed value. The problem in this case is then the optimization of total costs during the production times, constrained by the post-production release times. To determine the optimal (Q_1, n) the traditional approach is to differentiate the total cost equation with respect to Q, and set it to zero ($dTC/dQ = 0$). However, as in Sarker and Khan (2001), given n is an unknown integer variable, no differentiation is possible. However, assuming a fixed (an optimal value) for n, Eq. (12) shows the result of the differentiation and Eq. (13) the optimal batch size (with no time constraints) when $n \geq 1$.

$$(ns_1 + s_0)/(nQ_1) = c_1 i/(2T)(nQ_1 p_1 + nQ_1 p_0) + c_0 i/(2T)(nQ_1 p_0) \tag{12}$$

$$Q'_{1(n\geq 1)} = (2T(ns_1 + s_0)/(in^2(c_1 p_1 + c_1 p_0 + c_0 p_0)))^{1/2} \tag{13}$$

Eq. (13) reduces to the EOQ equation if $T = Dp_1$ (i.e. equivalent to continuous production during the time T), $n = 1$, and all Tier 0 variables are set to 0. Eq. (13) also reduces to the traditional EOQ formula if we set $T = Dp_0$, $n = 1$ and all Tier 1 variables are set to 0. Eq. (14) provides the optimal solution (assuming a known optimal value of n) in the case $n \leq 1$. Eq. (14) also reduces to the EOQ equation if $T = Dp_1$, $n = 1$ and all Tier 0 variables are set to 0 or if $T = Dp_0$, $n = 1$ and all Tier 1 variables are set to 0.

$$Q'_{1(n\geq 1)} = (2T(ns_1 + s_0)/(in(c_1 p_1 + c_1 p_0 + c_0 p_0)))^{1/2} \tag{14}$$

In the fixed release case, the optimal solution is found with $n \leq 1$ as described next. By plugging the $Q'_{1(n\geq 1)}$ equation into the ASC Eq. (6) with $n = 1$ and comparing it with $n > 1$, it can be shown that as n increases from 1, ASC increases and therefore total costs increase (given the total costs are based on the point where $AHC = ASC$). This simple result demonstrates that in the fixed release case, the optimal solution will not have $n > 1$. On the other hand, when comparing ASC ($n = 1$) with ASC ($n < 1$), ASC may increase or decrease (and therefore total costs). Therefore the optimal TC will be found when $n \leq 1$. Using an approach similar to Sarker and Khan (2001) we substitute $Q'_{1(n\geq 1)}$ in $TC_{(n\leq 1)}$ and obtain Eq. (15). When $TC^2_{z(n\leq 1)}$ is minimized, $TC_{(n\leq 1)}$ is minimized. Assuming n to be a continuous variable and $m = 1/n$ we differentiate $TC^2_{z(n\leq 1)}$ with respect to n and equate to zero, obtaining Eq. (16).

$$X = (ns_1 + s_0), Y = c_1 p_1 + c_1 p_0 + c_0 p_0$$

$$TC^2_z = X^2 D^2 / n^2 Q_1^2 + D^2 i^2 Q_1^2 Y^2 / (4T^2) + D^2 iXY/(2Tn) \tag{15}$$

$$m^* = (c_0 p_0 s_1 / (c_1 p_1 s_0 + c_1 p_0 s_0))^{1/2} \tag{16}$$

To obtain the optimal n, we need to determine the two integer values of the inverse, thus let $m_a = \lfloor m^* \rfloor$ and $m_b = \lceil m^* \rceil$. Next, we find the $Q'_{1(n\leq 1)}$ for $n = 1/m_a$ (if $m_a > 0$) and for $n = 1/m_b$, and then the corresponding total costs. The optimal solution is one of these two solution points.

To illustrate the behavior of the model we present some examples employing a variable set similar to that used in Section 4. We assume the following values: $D = 2{,}000$, $i = 20\%$, $s_0 = \$400$, $c_0 = \$250$, $c_1 = \$125$, $p_0 = p_1 = 0.0625$, $tr_0 = 4$, $tr_1 = 8$, and $T = 500$. Table 2 presents the optimal value of m from Eq. (16), the m_a and m_b values, the corresponding $Q'_{1(n\leq 1)}$, and the corresponding total costs for s_1 values of 200, 800, 4,000, and 8,000. Given m is an integer, there are two instances where the total costs are minimized by both adjacent solution points. Figures 5 to 8 illustrate the Total Cost and Q'_1 values as n changes from 1 to 1/7 respectively. Intuitively, as s_1 increases the optimal solution would include fewer setups in that tier, which is equivalent to a reduction in n.

6.2 Case 2: Release time a linear function of Q

As in Section 4, this case assumes tr_j is related to the batch size by the function $r_j Q_j$. Similar to Section 6.1, we fix the value of n and differentiate for the appropriate total cost equation (Eqs. (10) and (11)). The two equations needed to obtain the optimal batch sizes are presented next.

Table 2. Example data for the relationship between n, Q_1, and total cost

s_1	m^*	m_a	m_b	Q'_1 (m_a)	Q'_1 (m_b)	$TC(m_a)$	$TC(m_b)$
200	0.71	0	1	--	309.8	--	**9346.0**
800	1.41	1	2	438.2	584.2	**12554.5**	**12554.5**
4,000	3.16	3	4	1117.1	1197.3	**20219.0**	20308.3
8,000	4.47	4	5	1567.7	1633.0	**26094.9**	**26094.9**

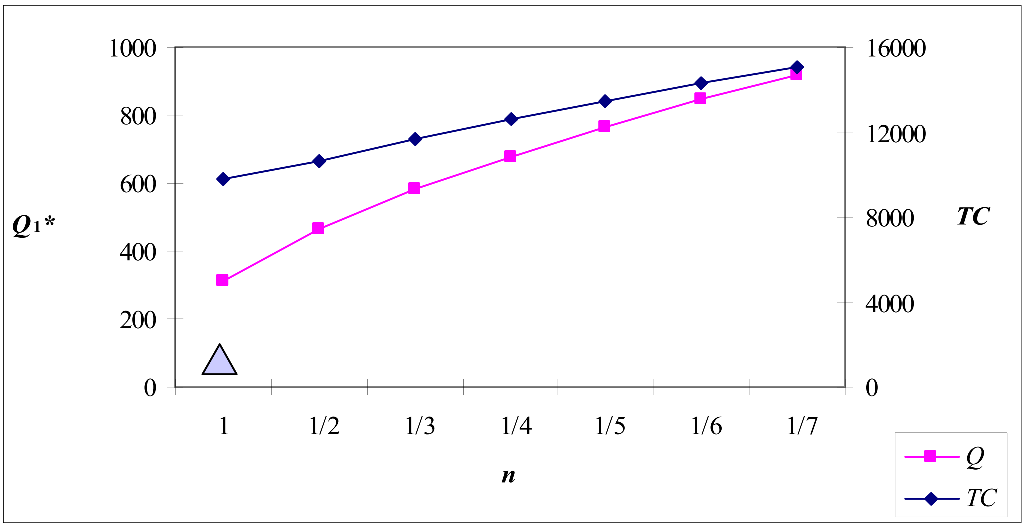

Figure 5. Example relation between n, Q_1^*, and total cost with s_1 = 200.

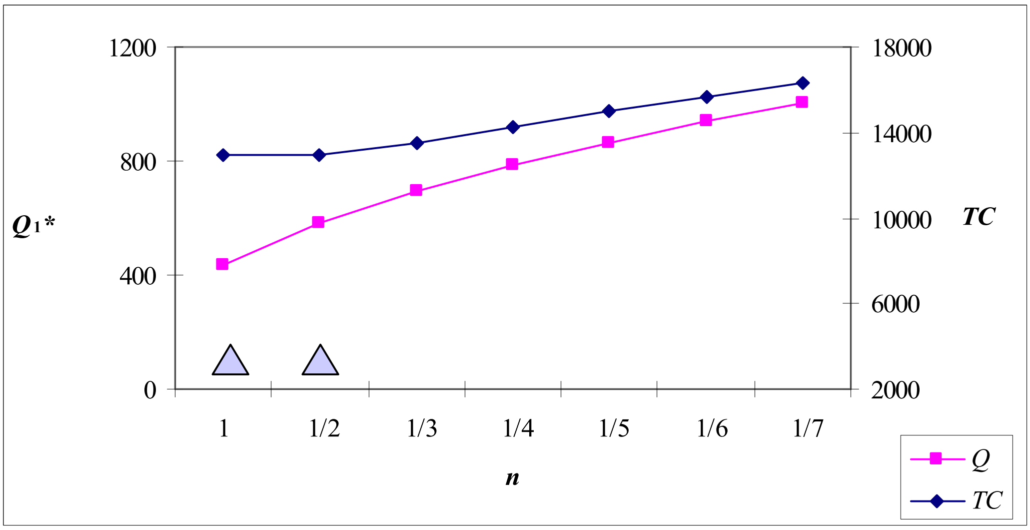

Figure 6. Example relation between n, Q_1^*, and total cost with s_1 = 800.

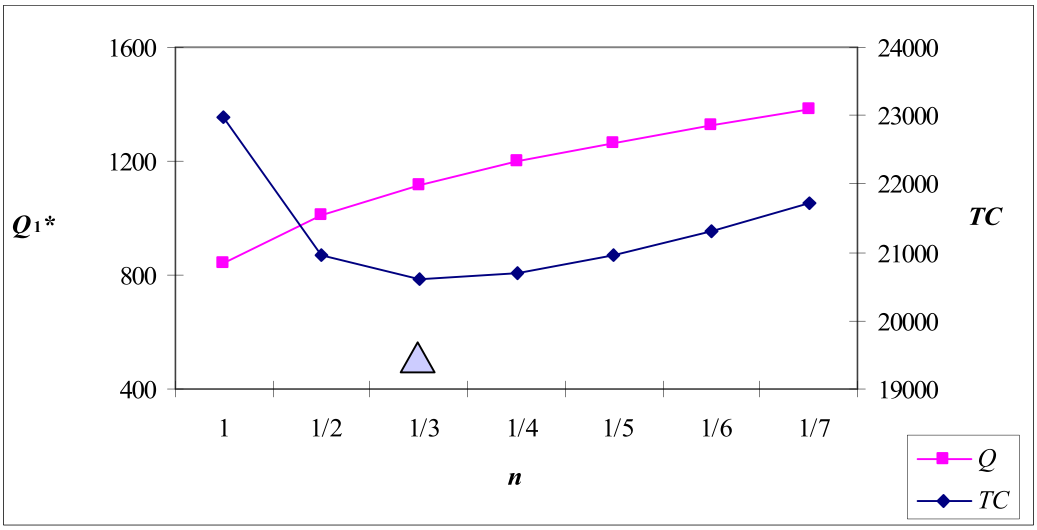

Figure 7. Example relation between n, Q_1^*, and total cost with s_1 = 4,000.

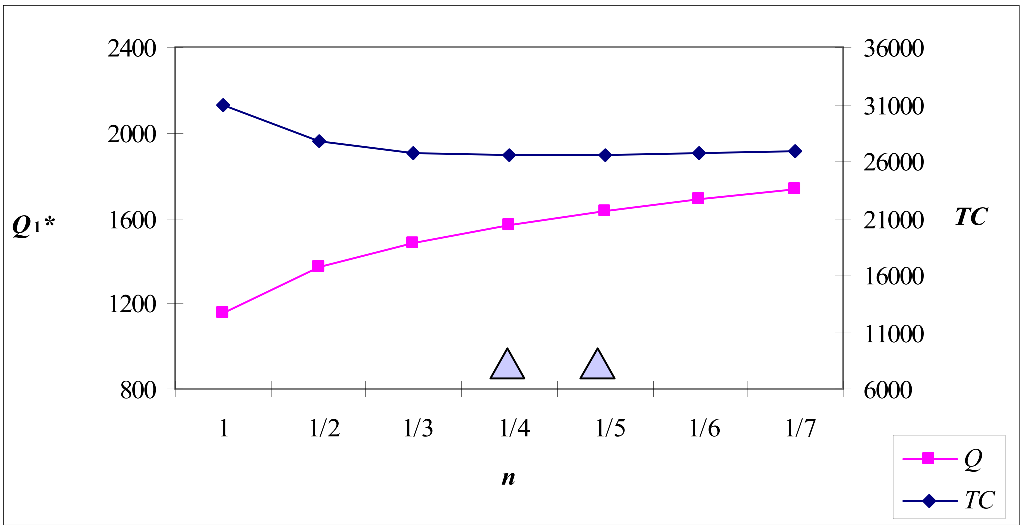

Figure 8. Example relation between n, Q_1^*, and total cost with s_1 = 8,000.

$$Q'_{1(n\geq 1)} = (2T(ns_1 + s_0) \\ /(in(nc_1p_1 + nc_1p_0 + 2c_1r_1 + nc_0p_0 + 2c_0r_0)))^{1/2} \tag{17}$$

$$Q'_{1(n\leq 1)} = (2T(ns_1 + s_0) \\ /(in(c_1p_1 + c_1p_0 + 2c_1r_1 + nc_0p_0 + 2c_0r_0)))^{1/2} \tag{18}$$

In the case where the release time is a linear function of the batch size, all integer values of n and m ($m = 1/n$) are possible optimal solutions. Following the processes performed in Section 6.1, we solve for n and m by plugging each Q'_1 equation into the corresponding TC equation. The optimal n and m values are given in Eqs. (19) and (20).

$$n^* = (2s_0c_1p_1/(s_1c_1p_1 + s_1c_1p_0 + s_1c_0p_0 + 2s_1c_1r_0))^{1/2} \tag{19}$$

$$m^* = ((s_1c_0p_0 + s_1c_0r_0)/(s_0c_1p_1 + s_0c_1p_0 + 2s_0c_1r_1))^{1/2} \tag{20}$$

Given the integer nature of n and m, we evaluate both n_a and n_b for n^* and both m_a and m_b for m^* by $n_a = \lfloor n^* \rfloor$, $n_b = \lceil n^* \rceil$, $m_a = \lfloor m^* \rfloor$ and $m_b = \lceil m^* \rceil$. One of these four points (with the related batch size) provides the optimal solution.

Table 3 presents three example instances for the linear case with D = 2,000, i = 20%, s_0 = 400, s_1 = \$200, c_0 = \$250, c_1 = \$125, $p_0 = p_1$ = 0.2, and T = 500. In the first instance, $r_0 = r_1 = p_0 = p_1$ = 0.2, and for this instance both m^* and n^* are less than 1, and the solution with $n = m = 1$ (Q = 110), thus clearly we only need to consider the solution with n = 1. In the second instance the value of r_0 is increased to 2, which results in m^* = 1.66 and n^* = 0.41, thus the optimal will be when n = 1 or 1/2, with solution (67, 1/2) being the optimal. The third instance presented in Table 3 has r_0 = 0.2 and r_1 = 2, resulting in m^* =0.30 and n^* = 2.58, thus clearly n = 1, 2, or 3 will result in the optimal solution, with solution (47, 2) being the optimal. Clearly, as demonstrated by these experiments, the optimal solution is sensitive to the release time variable.

Table 4 presents two additional instances where we set $r_0 = r_1$ = 0.2 and vary the values of the setup cost and item cost with the objective of demonstrating that cost parameters will have an effect on the optimal solution. In the first instance presented in Table 4 with s_1 = 2,000 and s_0 = 400, the value of n^* = 0.26 and m^* = 2.24, thus the optimal solution will have one of the following values for n: 1, 1/2, or 1/3. The optimal solution is (272, 1/3), which is intuitive given as s_1 increases it is preferable to have fewer setups in the tier 1 operation (the baseline value with s_1 = 200 had n = 1, Q'_1 = 110). In the second instance with c_0 = 2,500, the value of n^* = 0.41 and m^* = 2.24, thus as in the previous instance, the optimal solution will have one of the following values for n: 1, 1/2, or 1/3. The optimal is (61, 1/2), noting how the size of the batch sizes decreased (when compared to the baseline of Q'_1 = 110) with the increase in the item cost of the tier 0 operation, an intuitive result due to the increase in holding costs.

7. APPLICATION OF THE MODEL

In many industrial environments, including the Pharmaceutical manufacturing sector, equipment capabilities and production recipes determine the production batch sizes options. Further, production times are not a linear function of the batch size and the post-production time is based on historical performance of the analyzed product (or similar products when the information is not available). However, the proposed cost models can be used to analyze the possible batch combinations. Table 5 presents a sample data case where all batch sizes are integer multiples of each other while Table 6 presents the costs results associated with each batch combination assuming c_1 = 1,000, c_0 = 2,500, $s_1 = s_0$ = 1,000, D = 30,000, i = 20%, and T = 2,400 hours. Note that all batch combinations presented in Table 6 require less time than the 2,400 hours available. The optimal batch sizes are Q_1 = 1,000 and Q_0 = 500 with a total cost of \$268,750. This simple result illustrates that this model can be quite useful in determining the optimal batch combination when investment options are being analyzed, for example reduction in setup costs, production time, or release time. Furthermore, the proposed cost models could be combined with mathematical programming to find the optimal solution when full enumeration is a cumbersome task.

Table 3. Example instances in a two-tier system and release time linearly related to the batch size

Modified Parameters		n	Q_1'	TC
$r_0 = 0.2$	$n^* = 0.82$	3	51	26,331
$r_1 = 0.2$	$n_a = 0$, $n_b = 1$	2	67	24,000
	$m^* = 0.71$	**1**	**110**	**21,909**
	$m_a = 0$, $m_b = 1$	1/2	149	23,851
		1/3	180	26,362
$r_0 = 2$	$n^* = 0.41$	3	22	59,777
$r_1 = 0.2$	$n_a = 0$, $n_b = 1$	2	30	53,666
	$m^* = 1.66$	1	51	46,989
	$m_a = 1$, $m_b = 2$	**1/2**	**67**	**46,667**
		1/3	79	49,616
$r_0 = 0.2$	$n^* = 2.58$	3	39	34,254
$r_1 = 2$	$n_a = 2$, $n_b = 3$	**2**	**47**	**33,941**
	$m^* = 0.30$	1	65	36,661
	$m_a = 0$, $m_b = 1$	1/2	86	44,754
		1/3	102	51,918

Table 4. Second example instances in a two-tier system and release time linearly related to the batch size

Modified Parameters		n	Q_1'	TC
	$n^* = 0.26$	4	111	75,578
$s_1 = 2,000$	$n_a = 0$, $n_b = 1$	3	128	66,613
	$m^* = 2.24$	2	156	56,285
	$m_a = 2$, $m_b = 3$	1	219	43,818
		1/2	249	39,911
		1/3	272	39,856
		1/4	291	40,746
	$n^* = 0.41$	4	15	77,460
$c_0 = 2,500$	$n_a = 0$, $n_b = 1$	3	19	70,805
	$m^* = 2.24$	2	25	63,498
	$m_a = 2$, $m_b = 3$	1	43	55,426
		1/2	61	53,555
		1/3	74	55,507
		1/4	86	58,286

Table 5. Data for the application case

Code	Batch Size	tp_1	tr_1	tp_0	tr_0
A	500	11h	24h	6h	10h
B	1,000	14h	26h	6h	25h
C	2,000	16h	36h	8h	46h
D	4,000	19h	48h	11h	90h

Table 6. Results for all batch combinations

Comb.	Q_1 UNITS	Q_0 UNITS	n	ASC \$	AHC_1 \$	AHC_0 \$	TC \$
A-A	500	500	1	120,000	81,250	81,250	282,500
A-B	500	1,000	2	90,000	95,000	175,000	360,000
A-C	500	2,000	4	75,000	125,000	312,500	512,500
A-D	500	4,000	8	67,500	183,750	596,875	848,125
B-A	**1,000**	**500**	**1/2**	**90,000**	**97,500**	**81,250**	**268,750**
B-B	1,000	1,000	1	60,000	90,000	175,000	325,000
B-C	1,000	2,000	2	45,000	110,000	312,500	467,500
B-D	1,000	4,000	4	37,500	148,750	596,875	783,125
C-A	2,000	500	1/4	75,000	140,000	81,250	296,250
C-B	2,000	1,000	1/2	45,000	125,000	175,000	345,000
C-C	2,000	2,000	1	30,000	120,000	312,500	462,500
C-D	2,000	4,000	2	22,500	143,750	596,875	763,125
D-A	4,000	500	1/8	67,500	203,750	81,250	352,500
D-B	4,000	1,000	1/4	37,500	173,750	175,000	386,250
D-C	4,000	2,000	1/2	22,500	163,750	312,500	498,750
D-D	4,000	4,000	1	15,000	157,500	596,875	769,375

8. CONCLUSIONS

This paper presents inventory models of direct application to the pharmaceutical industry that consider post-production hold time and cost. The paper proposed optimal models and procedures for single-tier and two-tier manufacturing systems that considers setup and holding costs. The two-tier model considers two levels of a supply chain and assumes these batch sizes can be related by an integer value. The analysis demonstrated that in the case of two-tiers and fixed release times, we only need to consider cases where the batch size of the first tier is larger or equal to the batch size of the second tier. However, when considering the post-production hold time as a function of the batch size, any relationship between the batches is possible. Numerical examples demonstrated that the models are sensitive to the cost and time parameters.

This paper also presents a practical application example where the proposed inventory model is used to support business decision-making. The example assumes actual production and post-production hold time data is available for a variety of batch sizes. By calculating the inventory costs associated with all the batch combinations the optimal batch size combination can be determined. An implementation such as this could be used to perform 'what if analysis', e.g. change in the production time, release time, or in the setup cost.

Future research related to this problem can extend into several directions. For example, the consideration of setup times whose duration is a function of the batch size. Furthermore, in the two-tier case, transportation costs and time could be an important variable that should be included. Finally, the modeling of time parameters as stochastic, for example the post-production hold times, could provide a better representation of some industrial applications.

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