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The Uncapacitated Facility Location Problem: Some Applications in Scheduling and Routing

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Abstract—The uncapacitated facility location problem (UFLP) represents a particular structure in integer linear program, and has widespread applications in real life. In this paper, the applicability of UFLP-model is explored in problems arising in non-locational context. Three seemingly unrelated problems from the area of scheduling and routing are chosen for the purpose and the reported works in which their relationship with the UFLP has been studied are reviewed. These problems are found to have structures similar to a UFLP, and based on this, computationally competitive solution procedures could be developed for them. The study shows that several important problems, quite diverse in application, share common structures with the UFLP, and identification of this commonality can be beneficial from both modeling as well as algorithmic development points of view.

Keywords—Uncapacitated facility location problem, Dynamic lot-sizing, Job-scheduling, Bus route design problem, Set covering problem

1. INTRODUCTION

Among the myriads of formulations considered in the literature for location problems, the uncapacitated facility location problem (UFLP) seems to have attracted most attention (Krarup and Pruzan (1983)). This may be due to its wide-ranging applicability to real-life decision-making. Apart from decision problems concerning location of plants or facilities (e.g., factories, schools, and warehouses), the underlying structure of UFLP and other related models also captures the characteristics of several non-locational problems which include, among others, ingot size selection (Vasco et al. (1988)), metallurgical grade assignment (Vasco et al. (1989)), archaeological settlement analysis (Bell and Church (1985)), data base management (Pirkul (1986)), production lot sizing (van Oudheusden and Singh (1988)) and vendor selection problem (Current and Weber (1994)). This paper presents a review of reported works in which this model's relationship with important problems in the area of scheduling and routing has been studied. The purpose here is mainly twofold; one, to show that the structure of UFLP applies to many seemingly unrelated decision problems arising in real life, and two, to identify the linkage among these problems which makes it possible to devise similar solution procedures for them. As the area of scheduling and routing is vast, the paper chooses three important problems (viz., dynamic lot-sizing, job-scheduling in a production line, and the bus route design problem) from the area for study, and identifies commonality in their structures with that of the UFLP. The analyses of the above mentioned problems are found to have benefited from UFLP formulation as UFLP-based algorithms provided computationally competitive solution procedures for these problems.

The selection of the above three problems is motivated by many factors. First, each of these has wide applications in real life e.g., in production and inventory planning, scheduling and others, and they are found to arise in several actual settings. Second, many versions of these belong to difficult class of problems and hence offer considerable research challenge, and third, their similarity in structure with the UFLP provides us with modeling alternatives for many practical problems. Also, applicability of the UFLP-model to situations outside the locational context is explicitly explained.

The contribution of this study can be explained as follows. A review of research which studies the UFLP's relationship with several important problems outside the domain of locational decision-making (particularly from scheduling and routing) is presented, and its findings are synthesized to identify the implicit similarity among these apparently unrelated problems. This provided us with conclusions having implications on modeling and algorithmic development of many practical problems. While some of the problems studied could readily get easy exact solution procedures, some other got alternative procedures with better average-case performance.

It is to be noted, however, that this paper unlike many important survey articles on UFLP (e.g., Krarup and Pruzan (1983), Francis et al. (1983), Aikens (1985), Brandeau and Chiu (1989)) does not attempt to present a review of all solution procedures developed to date, nor does it study its special structure in depth. This tries to

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study the commonality in structures of various problems from the area of scheduling and routing, and identifies their linkage with the UFLP so that comparatively easy solution procedures can be developed for them. The remainder of the paper is organized as follows. Section 2 presents the statement and a mixed integer programming formulation to the UFLP. Then each of the above mentioned problems viz., dynamic lot sizing, job-scheduling in a production line, and bus route design problem is studied in relation to UFLP in Sections 3, 4 and 5, respectively. Finally, the paper concludes in Section 6 synthesizing the findings of the above study and giving indication for some future research direction.

2. THE UNCAPACITATED FACILITY LOCATION PROBLEM (UFLP)

The UFLP deals with the supply of a single product from a subset of facilities (i = 1, 2, ..., m) to a set of customers (j = 1, 2, ..., n) with a pre-specified demand, d_j, for that product. The adjectives "uncapacitated" and "simple" subsume identical meaning and are interchangeably used in facility (plant) location literature. Given the cost structure (i.e., fixed cost f_i associated with facility i, and c_{ij}, the variable cost of supplying customer j's demand from facility i), it is sought to identify a minimum cost transportation plan which satisfies each customer's demand. Defining X_{ij} as the fraction of customer j's demand to be supplied from facility i, the UFLP can be represented as:

$$Min\left[\sum_{i=1}^{m} f_i Y_i + \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} X_{ij}\right] \tag{1}$$

$$\text{subject to } \sum_{i=1}^{m} X_{ij} = 1,\ j\ = 1,\ldots,n \tag{2}$$

$$Y_i - X_{ij} \ge 0, i = 1,\ldots,m; j = 1,\ldots,n \tag{3}$$

$$X_{ij} \ge 0, i = 1,\ldots,m; j = 1,\ldots,n \tag{4}$$

$$Y_i \in \{0,1\}, i = 1,\ldots,m \tag{5}$$

The objective function (1) represents the sum of fixed as well as variable costs which is to be minimized. The constraint set (2) specifies that each customer j's demand is met. Constraints (3) ensure that shipment to any customer is possible only from an existing facility. Y_i is a Boolean variable which takes a value 1 if facility i is open (in existence), and zero if it is closed (non-existent).

Several solution procedures, approximate as well as exact (e.g., by Kuehn and Hamburger (1963), Efroymson and Ray (1966), Khumawala (1972), Hansen (1972)), for this problem have been suggested in the literature. Some others are by Cornuejols et al. (1977), Karkazis (1985), Guignard (1988), Galvao (1993), Klose (1998). Some recent algorithms, approximate but very effective for large-size problems, are due to Kratica et al. (2001), Recende and Werneck (2006), Sun (2006). A comparatively recent review of facility location models and their solution procedures has been presented in Klose and Drexl (2005). The dual-based algorithms by Bilde and Krarup (1977) and Erlenkotter (1978) have been found to be very effective. Goldengorin et al. (2003) presented enhanced branch and bound algorithms for the problem. Erlenkotter's computer code, DUALOC, which was later improved significantly in average-case efficiency by Körkel (1989), is considered to be the fastest code described in the literature so far for the exact solution of UFLP. Availability of such effective algorithms has proved to be a major motivating factor for the present study.

3. THE DYNAMIC LOT-SIZING PROBLEM (DLSP)

In a generic DLSP, a facility manufactures a single product to satisfy known integer demands (d_j for each period j) over a pre-specified planning horizon divided into "n" discrete time-periods. Capacity restrictions on production and inventory are ignored. A fixed cost, f_i, is incurred if production is set up in period i. Marginal production cost, c_i (in period i), and inventory holding cost, h_t (for carrying inventory from period t to t + 1), are linear. Unsatisfied demands may (or may not) be backlogged, and shortage cost, if any, is linear. The problem is to determine an overall production schedule in order to satisfy each demand at minimum total cost.

Due to their importance in production planning and inventory control, lot-sizing problems have been widely studied. Starting with Wagner and Whitin (1958) who developed an $O(n^2)$ algorithm based on dynamic programming (DP), several researchers (e.g., Zabel (1964), Eppen et al. (1969), Zangwill (1969), Blackburn and Kunreuther (1974) among others) developed efficient algorithms for different versions of the above problem. Krarup and Bilde (1977), however, presented an integer linear programming formulation which made this problem equivalent to a UFLP. Not only this, they also developed an algorithm, "PLP-B", which runs in $O(mn)$ time. In DLSP → UFLP transformation, each period j of the given planning horizon is considered as a possible site for facility location, and also it represents a customer with demand, d_j. The production setup cost, f_i in period i, is equivalent to the fixed cost of locating facility at that site. Hence, an n-period lot-sizing problem is equivalent to a UFLP of dimension (n × n) with demand of each of the customers equal to the demand in each of the n periods, and fixed costs of locating facilities equal to production setup costs. The variable cost, c_{ij}, defined as the cost of satisfying period j's demand from the production in period i, is calculated as follows:

$$c_{ij} = \begin{cases} d_j\left(c_i + \sum_{t=i}^{j-1} h_t\right) & \text{for } i < j \\ d_j c_i & \text{for } i = j \\ \infty & \text{for } i > j \end{cases} \tag{6}$$

Then the DLSP will be represented by the formulation (1) to (5) where X_{ij} denotes fraction of period *j*'s demand to be supplied from the production in period *i*. The objective function (1) represents the total cost that is to be minimized. Constraints (2) ensure that each period's demand is fully met and the constraint set (3) provides that setup cost is incurred only when there is production at positive level. Y_i takes value 1 if production is set up in period *i*, and 0 otherwise.

The linear programming (LP) relaxation of the above formulation of DLSP is guaranteed to have optimal solution in integers because the coefficient matrix, c_{ij}, has a special structure (totally unimodular). PLP-B, however, does not solve the LP problem directly. It solves the dual of the LP-relaxation of the above problem. The procedure is so simple that comparatively large-size problems are claimed to have been solved by hand. Although, in the worst case "PLP-B" gives rise to the same order of computations [$O(n^2)$] as the DP algorithm does for the DLSP without backlogging, the UFLP formulation is claimed to be better on account of being more user-friendly and simple. Later, Wagelmans et al. (1992) developed an $O(n\log n)$ algorithm based on UFLP formulation for this problem. The same complexity results using different approach were independently obtained by Federgruen and Tzur (1991) and Aggarwal and Park (1993). For a detailed review of various solution procedures for single item lot sizing problems, one may refer to Brahimi et al. (2006).

Also, due to its above relationship with the LP, UFLP formulation of lot-sizing problems has resulted in more efficient sensitivity analysis of such problems (van Hoesel and Wagelmans (1990, 1991)). The LP-formulation with some valid inequalities has the advantage that it provides with a complete linear description of the convex hull of feasible solutions for the DLSP. Barany et al. (1984) and Pochet and Wolsey (1988) used this to reformulate and solve multi-item capacitated lot-sizing problems.

Van Oudheusden and Singh (1988) studied the modeling and computational aspects of DLSP-UFLP relationship. They presented formulations in which the entire area of lot-sizing could be studied as a particular field of facility location. First, they modeled DLSP with backlogging as an instance of UFLP, and performed computational experiments based on alternative implementation of DP and UFLP-based algorithms on various sizes of these problems. In case of backlogging, (6) gets modified to include

$$c_{ij} = d_j\left(c_i + \sum_{t=j}^{i-1} b_t\right) \qquad \text{for } i > j \tag{7}$$

where b_t is the unit shortage cost for period *t*. Computational results presented on the basis of solving a large number of problems of various time periods showed that the UFLP-based algorithms performed much better in case of all the problems solved. It is worth noting here that the complexity of DP algorithm for DLSP with backlogging is $O(n^3)$ whereas UFLP-based algorithm for such problems, in worst case, is not bound to run in polynomial time. However, the latter performed consistently better for all cases of the problems solved.

In a closely related work, Singh and van Oudheusden (1992) considered a case of DLSP with price-sensitive demands in which pricing and production decisions are made simultaneously. They formulated the problem as a general location model that was found equivalent to the classical UFLP. This formulation had the advantage that such problems with backlogging, which to our knowledge have not been earlier considered in the literature, could be easily dealt with through this approach.

Apart from the above wherein average case performance of UFLP-approach is consistently found superior in a wide range of lot-sizing decisions, it is emphasized here that this approach also offers alternative formulations to many practical problems. As c_{ij} does not require any specific structure, several variants of the standard DLSP can be easily dealt with by adjusting the c_{ij} values. We may recall (Bitran and Yanasse (1982)) that if cost coefficients, c_{ij} and f_i, in an UFLP are replaced by $c'_{ij} = kc_{ij} + k'_j$ and $f'_i = kf_i$, respectively, for some $k > 0$ and any k', then the optimal solution remains unchanged. This provides a lot of flexibility under this framework for the determination of lot sizes for perishable items which cannot be stored beyond a certain period, and also some other products essentially requiring some storage time before being fit for use.

One particular situation in which UFLP framework could be of immense use is where various cost estimates are not available. Let us consider a situation in which the production manager can estimate an appropriate number of production setups over a given planning horizon. If it is reasonable to accept that setup, and production and inventory costs do not vary over time, then the problem can be formulated as a *p*-Median problem (*p*MP), a problem closely related to UFLP. In *p*MP, the sum of supply costs incurred is minimized whereas a closely related *p*-Center problem (*p*CP) will minimize only the maximum supply cost. The coefficients c_{ij} in case of *p*MP can be calculated as:

$$c_{ij} = \begin{cases} d_j \sum_{t=i}^{j-1} 1 & \text{for } i < j \\ 0 & \text{for } i = j \\ \infty & \text{for } i > j \end{cases} \tag{8}$$

The *p*MP, as is evident from above, can be solved without having to make even a single cost estimate! Similarly, *p*-Center problem (*p*CP) will minimize the maximum production level in the time period under consideration if (8) is replaced by

$$c_{ij} = \begin{cases} \sum_{t=i}^{j-1} d_t & \text{for } i < j \\ 0 & \text{for } i = j \\ \infty & \text{for } i > j \end{cases} \tag{9}$$

This may be useful in the context of capacitated production planning when, for instance, the overall capacity limitation is rather flexible. This idea can be easily extended to problems with backlogging as well.

It is not difficult to show that the *p*MP and the *p*CP as applied to the above lot-sizing problems are easy to solve. Hence, all *p*-medians or *p*-centers can be calculated in polynomial time. Also, sensitivity analysis on the value of p, the appropriate number of production setups, is possible without much computational effort. This shows that this approach can be seen as a real alternative to the classical modeling of lot-sizing problems under dynamic demand conditions.

To summarize, the dynamic lot-sizing and uncapacitated facility location problems share a common mathematical structure and identification of this commonality could be advantageous from both computational as well as modeling viewpoints. Due to this relationship, alternative solution procedures become readily available for DLSP and several of its variants and extensions (Robinson and Gao (1996), Martel and Gascon (1998)). Not only this, UFLP framework offers modeling alternatives to many real-life lot-sizing problems involving perishable items and cases where cost data are not precisely available. As a result of DLSP–UFLP linkage, the facility location area may also get benefited. The knowledge about the availability of $O(n^4)$ and $O(n^3)$ algorithms (Florian and Klein (1971), Florian et al. (1980), van Hoesel and Wagelmans (1996)) for lot-sizing problems with equal, finite capacity in each time period could be valuable in finding an efficient solution for a capacitated version of the facility location problem. It is noteworthy that the latter has been found equivalent to a matching problem in a graph (Cornuejols et al. (1991)).

4. JOB-SCHEDULING IN A PRODUCTION LINE

In this section, we identify another application of UFLP i.e., in a job-scheduling problem. The problem is stated as follows: A batch of jobs is to be processed on a single facility (i.e., production line or multi-purpose machine). Each job may be processed in one of several "states" of the facility but at varying production costs. One state may be sufficient for processing many jobs. Sequence-dependent changeover costs are incurred when the state of the facility (line) changes. It is required to determine the sequence of states of the facility and allocation of jobs to states that minimize the total cost.

The above is a real-life problem and there are indications in the literature (e.g., see Allahverdi et al. (1999) for a detailed survey of such problems) that such problems do arise in various settings. For example, Burstall (1966) encountered the above problem in a firm which manufactured steel tubes. He developed a heuristic procedure to solve the problem. Buzacott and Dutta (1971) presented a similar problem that generally occurs in a machine shop consisting of multi-purpose machines. Their DP-based algorithm allowed the resolution of problems of up to 12 jobs and any "practical" number of states. For problems of more than 12 jobs they suggested a heuristic using the method of successive approximation.

In the present paper, an implicit enumeration algorithm based on branch and bound technique is proposed. The purpose here is to show how a UFLP-based procedure can be helpful in finding solution to the above problem. The basic idea underlying this algorithm is to dissociate the two aspects of the problem viz., selection of states of the line (facility) to be used, and determination of the sequence in which they occur. For the selection of states, the similarity of the above stated job-scheduling problem with the UFLP is exploited and an algorithm for the latter is used, and for determining the sequence of selected states an algorithm for the traveling salesman problem (TSP; Little et al. (1963)) is used. Both of these are successfully embedded into one under a branch and bound framework. Computational experience with the algorithm on solving fairly reasonable size problems has been quite encouraging (see e.g., Singh (2007)).

The problem may be characterized by the following assumptions. There is a single production line (or only one multi-purpose facility) which has "m" different states. At one time the facility cannot be in more than one state. There are "n" jobs to be processed. For each job there is at least one state of the facility in which it can be processed. In general, a job can be processed when the facility is in one of several states but at different costs. All jobs are of equal importance. None of the states of the line is capable of processing all the jobs at the minimum cost. Two types of costs are considered: c_{ij}, the cost of processing job j when the facility is in state i, and h_{ik}, the changeover cost from state i to state k of the facility. The two special cases can be easily derived. One, if $h_{ik} = C$ (a constant) for all i and k, then the job scheduling problem reduces to a UFLP. On the other hand, if the states of the line can process only one job each then the problem becomes very similar to a TSP.

Introducing Boolean variables, X_{ij}: equal to 1 if job j is processed when the facility is in state i, and 0 otherwise; and Y_{ik}: equal to 1 if the kth state of the facility is used and immediately follows the ith state in the sequence, and 0 otherwise, the job-scheduling problem as stated above can be represented as follows:

$$Min \; \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} X_{ij} + \sum_{i=1}^{m}\sum_{k=1}^{m} h_{ik} Y_{ik} \tag{10}$$

$$\text{subject to } \sum_{i=1}^{m} X_{ij} = 1 \quad j = 1, \ldots, n \tag{11}$$

$$X_{ij} \le \sum_{k=1}^{m} Y_{ik} + \sum_{l=1}^{m} Y_{li} \tag{12}$$

$$X_{ij} \in \{0,1\} \tag{13}$$

$$Y_{ik} \in \{0,1\} \tag{14}$$
$$\{i = 1,2,...,m;\ j = 1,2,...,n; k = 1,2,...,m\}$$

and denoting the set of variables $Y_{ik} \neq 0$ by $Y_{i_p k_p}$ $\{p = 1,\ 2,\ ...,\ m' \leq m-1\}$, there must exist a permutation of the values of the index "p" such that

$$k_p = i_{p+1}\ \{p = 1,2,...,m'-1\} \tag{15}$$

The objective function (10) represents the sum of processing and changeover costs. The constraints (11) require that each job must be processed when the facility is in exactly one state. Constraints (12) ensure that only the states in the optimal sequence need be used. Constraints (13) and (14) express that jobs as well as changeover must be done either completely or not at all, and (15) provides for exactly one ordered sequence of states used for processing jobs.

In a closely related paper by Singh (2007), the formulation, (10) to (15), has been shown to have a structure similar to that of the traveling purchaser problem (TPP). Hence an algorithm to solve TPP (see Singh and van Oudheusden (1997) for details) with some modifications is used to solve the above problem. The main difference between TPP and the above job-scheduling problem is that whereas the former requires a tour to start from and end at the same city, the latter requires only an ordered sequence of the states of the facility finally selected. This could be dealt with by creating a dummy state of the line in which a dummy job (may be n+1th) is processed at cost zero. Also, changeover costs from this dummy state to other states of the line are set equal to zero.

Computational results based on the implementation of the above algorithm (Singh (2007)) on various sizes of randomly generated job-scheduling problems indicate that problems with as many as 100 jobs and up to 10-20 states can be solved in reasonable computational effort. This represents a quantum improvement over the performance of any earlier known algorithm for this problem (e.g., Burstall's heuristic could solve problems of up to 8 states and 19 jobs, and Buzacott and Dutta could solve optimally problems of up to 12 jobs and any 'practical' number of states). Later, Laporte et al. (2003) developed an improved algorithm for the TPP but the algorithm proposed by Singh and van Oudheusden (1997) is sufficient to solve any practical job-scheduling problem of the type stated above. This algorithm can also be used as a heuristic by terminating the computation as and when first feasible solution is found. In many cases it has been found that this solution is reasonably close to an optimal solution and requires very small computation time.

To conclude the section, a useful application of UFLP-model is identified in a job-scheduling problem that is encountered in many real settings. An integer programming formulation for the problem showed that its structure is similar to the TPP. A variant of the TPP algorithm is used to solve the problem and computational experience shows that problems with 100 jobs and up to 20 states can be easily solved. This seems to have considerably improved the results earlier obtained by Burstall, and Buzacott and Dutta.

5. BUS ROUTE DESIGN PROBLEM

In this section we identify an application of the UFLP-model in the area of routing. A bus route design problem (BRDP) is addressed through an interactive use of UFLP. In fact, a standard BRDP as such does not seem to exist. Each BRDP will be characterized by the issues and criteria involved therein. The problem considered here may be stated as follows: Given the road network and demand for travel by bus from one area (point or zone) in a city to another, it is required to determine optimal set of routes for a bus service in the city which maximizes profits to the operator. A case of minimization of cost to the operator with an acceptable level of service to the users was considered by van Oudheusden et al. (1987). They modeled the problem as a set covering problem (SCP), a problem closely related to UFLP.

The solution to the above problem involves two quite independent steps: i) generation of a set of potentially suitable bus routes and ii) selection of optimal routes from the given set. The generation of alternative bus routes may be done by a heuristic procedure in which major traffic generators are linked by shortest routes passing through the points (areas) where demands are higher than the average demand. Areas with less traffic are then connected to the nearest routes. Thus a comparatively large set of suitable routes is identified. In the second step, "optimal" election of routes is made out of routes identified earlier in step (i) through the use of optimization models (UFLP in the present case), and this is the main concern of this section. During the selection process no alteration/modification of initial routes is done. However, it is possible and many a time desirable to alter the initial set of routes according to the new insights gained from the selection phase. The demand is considered to be sensitive to the level of service. The case of comparatively insensitive demand is also relevant to many of the developing countries with few alternative modes of transport, and almost "captive" ridership. But the case considered here is generally appropriate for developed countries where transit demand is sensitive to the level of service provided, and the main issue to be addressed is how to attract 'choice' riders. The level of service may get expressed through the walk distance to a bus route and the frequency of service available on the route. Hence, the operator will have to improve the level of service (e.g., by increasing the frequency of service or providing service on more number of routes etc.) to attract more and more people. This will get reflected on the income accrued to the operator. However, better service would incur more operating cost. If income is considered as a negative cost to the operator then an optimal design would be the one that minimizes the total cost (or maximizes total profits) to the operator. Let us define:

c'_{ij} = total income from users of zone j if they use route i

X_{ij} = $\begin{cases} 1 & \text{if users of zone } j \text{ use route } i \\ 0 & \text{otherwise} \end{cases}$

f_i = cost associated with route i, and this is estimated from the data on capital cost, maintenance cost, running cost and the cost of the crew. A route with different frequency of service will have different costs and may be considered (for analysis purposes) as different routes

m = total number of potential routes indexed by i = 1, 2, ..., m

n = number of traffic areas (demand points) the city is divided in, indexed by j = 1, 2, ..., n

Then the problem may be expressed as:

$$Min \sum_{i=1}^{m} f_i Y_i - \sum_{i=1}^{m}\sum_{j=1}^{n} c'_{ij} X_{ij} \tag{16}$$

subject to $$\sum_{i=1}^{m} X_{ij} = 1 \qquad j = 1, \ldots, n \tag{17}$$

$$Y_i - X_{ij} \geq 0 \qquad i = 1,\ldots,m;\ j = 1,\ldots,n \tag{18}$$

$$X_{ij} \in \{0,1\} \qquad i = 1,\ldots,m;\ j = 1,\ldots,n \tag{19}$$

$$Y_i \in \{0,1\} \qquad i = 1,\ldots,m \tag{20}$$

If we define $c_{ij} = -c'_{ij}$, the above problem becomes a UFLP, and hence can be solved by any of the available approaches. The constraints (17) ensure that each zone is served by some route, whereas constraints (18) ensure that a zone can be assigned to a route only when that route is selected in the optimal set of routes.

For estimating c'_{ij}, transit demand is estimated first. As explained earlier, it is assumed to be a function of two attributes – service frequency and walk distance. The variation in transit usage is taken to be same as given in Kocur and Hendrickson (1982). Now transit demands being available and assuming a uniform fare for all trips, c'_{ij} can be easily evaluated.

The above modeling approach was used to determine an optimal route network in a city of 3 million people in India (refer to van Oudheusden et al. (1987) for details). The city was divided into 60 zones, and travel demand between various origin-destination (OD) pairs was collected through mass transit survey. At the first instance, some 31 routes were generated out of which 13 were selected in the optimal solution. After optimization the solution was checked to ascertain whether the assumed frequency of service on routes was sufficient to serve the demand. This way vehicle size restriction could be taken in to consideration. In case of need, frequency was accordingly adjusted. The same route with different frequencies of service over time, as it normally happens during morning and evening peak and noon-time lull periods, is treated as different routes in the analysis.

Unlike SCP where a given zone j is covered by a route i if the route provides a minimum acceptable level of service, there are no zones (areas) which are required to be 'covered' in UFLP-modeling. A single route optimal solution is theoretically possible in this case. However, if it is required that a given minimum number of commuters of a zone j must be served, this can be ensured by manipulating corresponding c_{ij} values. By manipulating the c_{ij} values corresponding to various zones simultaneously, pre-specified levels of demand coverage can be ensured.

The use of SCP and UFLP for a bus route design problem is really innovative. They provide simple and flexible modeling approach to urban transportation planning. Although, they consider single objective and are mainly used for "many to one" travel pattern but it is possible to adapt them to multi-objective case and "many to many" travel pattern. In short, an interactive use of these models can support a decision process in which several criteria and issues that cannot be modeled explicitly may be considered.

6. CONCLUSION

Application of the uncapacitated facility location model to some important problems in the area of scheduling and routing is studied. Several important conclusions can be drawn based on this study. First, it is evident that the uncapacitated facility location problem is not merely a location model rather it represents a mathematical structure that also applies quite well to many important problems outside the domain of locational decision-making. In each of the three problems studied we could identify two aspects: one, an ordered discrete principal aspect (e.g., periods to set up production in, states of production line to be in the optimal solution, and the routes to be finally chosen); and the other, a non-ordered subservient aspect that depends on the first (e.g., demands to be supplied from a particular period's production, jobs to be processed when the line is in a particular state, and zones to be served by each selected route). It is this commonality in characteristic that provides the three seemingly unrelated problems with a structure similar to a UFLP. In fact, it is our conjecture that UFLP can provide modeling alternatives to many problems having the above characteristics. Second, it may also be concluded from the above study that for structured problems, the mixed integer programming formulations can be more flexible and tree search algorithms having the same or even more forbidding worst-case behavior can outperform other (e.g., dynamic programming based) algorithms. This may seem to be in conflict with our intuitive belief, but we can clearly see this coming true in the last two sections.

At a more specific level, it can be said that alternative solution approaches become readily available as a result of UFLP formulation of the lot-sizing problems. From the study made, it is quite evident that these solution approaches are computationally more competitive than traditional DP-based approaches. Not only this, many of the practical problems in the area of lot sizing where cost estimates are not available could be modeled as *p*MP, and solved by a polynomial time algorithm. Also, the job-scheduling problem encountered by Burstall in a real setting could be easily solved by a branch and bound algorithm that solves a related UFLP for determining bounds. The methodology proposed for the bus route design through the interactive use of optimization models viz., set covering and uncapacitated facility location problems is quite innovative.

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