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A New Ant Colony Optimization Approach for the Single Machine Total Weighted Tardiness Scheduling Problem

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Abstract—In this paper the *NP*-hard single machine total weighted tardiness scheduling problem in presence of sequence-dependent setup times is faced with a new Ant Colony Optimization (ACO) approach. The proposed ACO algorithm is based on a new global pheromone update mechanism, which makes the pheromone trails asymptotically range between two bounds arbitrarily fixed and the ACO learning mechanism independent of the values of the objective function of the considered problem. Other features of the algorithm include a diversification mechanism for the solution construction phase based on a local pheromone update rule whose effects are restricted to the single iterations, and a cumulative option for the global pheromone update rule. An experimental campaign, carried out on a benchmark available from the literature, was executed to evaluate the proposed ACO and the effectiveness of its optional features. In particular, the obtained results were compared with the ones of a recently proposed ACO algorithm for the same problem by Liao and Juan (2007). The analysis of the outcomes showed the competitiveness of the new ACO approach, which was able to improve about 72% of the best known results for the benchmark. Finally, a further investigation on a different benchmark set of instances without setup times showed the robustness of the proposed ACO algorithm.

Keywords—Ant colony optimization, Metaheuristics, Scheduling, Total weighted tardiness

1. INTRODUCTION

Ant Colony Optimization (ACO) is a recent metaheuristic approach which aims at exploiting the successful behaviour of real ants in cooperating to find shortest paths to food for solving combinatorial problems (Dorigo and Stützle (2002), Dorigo and Blum (2005)). In most of the real species ants have an effective indirect way to communicate each other which is the most promising trail, and finally the optimal one, towards food: ants produce a natural essence, called pheromone, which they leave on the followed path to food in order to mark it. The pheromone trail evaporates over time and it disappears on the paths abandoned by the ants. On the other hand, the pheromone trail can be reinforced by the passage of further ants: thus, effective (i.e., shortest) paths leading to food are finally characterized by a strong pheromone trail, and they are followed by most of ants. The ACO metaheuristic was first introduced by Dorigo, Maniezzo and Colorni (1991 and 1996) and Dorigo (1992), and since then it has been the subject of both theoretical studies and applications. ACO combines both *Reinforcement Learning* (RL) (Sutton and Barto (1998)) and *Swarm Intelligence* (SI) (Kennedy and Eberhart (2001)) concepts:

- each single agent (an ant) takes decisions and receives a reward from the environment, so that the agent's policy aims at maximizing the cumulative reward received (RL);
- the agents exchange information to share experiences and the performance of the overall system (the ant colony) emerges from the collection of the simple agents' interactions and actions (SI).

ACO metaheuristic has been successfully applied to several combinatorial optimization problems, from the first travelling salesman problem applications (Dorigo et al. (1991 and 1996)), to vehicle routing problems (Bullnheimer et al. (1999), Reinmann et al. (2004)), and to single machine and flow shop scheduling problems (den Besten et al. (2000), Gagné et al. (2002), Ying and Liao (2004)).

This paper proposes a new Ant Colony Optimization (ACO) approach to face one among the most important scheduling problems, i.e., the single machine total weighted tardiness scheduling with sequence-dependent setup times (STWTSDS) problem. The choice of the STWTSDS problem as reference application for the proposed ACO approach is motivated by the relevance of the considered problem for manufacturing industries. The importance of performance criteria involving due dates, such as (weighted) total tardiness or total earliness and tardiness (E-T), as well as the explicit consideration of sequence-dependent setups, has been widely recognized in many real industrial contexts. Both in the survey on US manufacturing practise by Wisner and Siferd (1995) and in the one by Panwalkar et al. (1973) meeting the due dates is identified as the most

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important scheduling objective. Criteria weighting both the early and the tardy completion of jobs with respect to their due dates, the so-called *non-regular* objectives (Baker and Scudder (1990)), have been considered to encompass the just-in-time (JIT) philosophy aiming at reducing the level of inventories. Among the regular objectives based on due dates, the minimization of the total weighted tardiness is the subject of a very large amount of literature on scheduling; however, the coverage of the problem including an explicit consideration of sequence-dependent setups is not so extended. Setup operations are necessary to prepare production resources (e.g., machines) for the job to be executed next, and whenever they depend on the (type of) preceding job just completed they are called sequence-dependent setups. In the survey by Panwalkar et al. (1973) the presence of sequence-dependent setups has been observed in a relevant number of industrial scheduling contexts. Nevertheless, it is often assumed the setup times independent of the sequence of jobs on the machine, including them into processing times; alternatively, setup times are simply disregarded and eventually inserted in the so found solutions. In a review on machine scheduling problems involving setups, Allahverdi et al. (1999) provided a number of industrial examples including sequence-dependent setups and they indicated the importance of taking into account setup times separately from job processing times. In addition, it was underlined in Rubin and Ragatz (1995) how the difficulty of total tardiness scheduling on a single machine is increased by the presence of sequence-dependent setups, since dominance conditions used for simple tardiness problems do not hold true. In addition, although in general weighted tardiness problems with sequence-dependent setups may originate from single or multi-machine contexts, it was observed that the solution of single machine problems is often required even in more complex environments (Pinedo (1995)).

The work presented in this paper aims at analysing the behaviour of an ACO approach which mainly differs from previous ones in the literature for a new pheromone trail model based on an original global pheromone update (GPU) rule. In particular, (a) pheromone values are independent of the problem cost (or quality) function and they are bounded within an arbitrarily chosen and fixed interval; (b) the new GPU rule implements the ant colony learning system by exploiting the solution quality as a sort of signal driving the reward mechanism, and updating the pheromone values accordingly; in addition, this GPU rule makes the pheromone values asymptotically increase (decrease) towards the upper (lower) bound without requiring any explicit cut-off, differently from the *Max-Min* Ant System (MMAS) (Stützle and Hoos (2000)), where upper and lower bounds for pheromone values are used as well; finally, (c) a diversification strategy is adopted which is based on a temporary perturbation of the pheromone values performed by a local pheromone update (LPU) rule within any single iteration.

The rest of the paper is organized as follows: Section 2 introduces a formal definition of the STWTSDS problem and reviews the relevant literature for this problem and for previous related ACO approaches. Section 3 illustrates the basic aspects of the ACO approach, discussing how it can be applied to the STWTSDS problem and highlighting the new features introduced. Section 4 shows the extended experimental campaign performed on the benchmark set generated by Cicirello (2003), whose best known results have been very recently updated both in Cicirello (2006) and in Lin and Ying (2006), and it compares the proposed ACO with the one presented in Liao and Juan (2007); in addition, the results obtained to test the robustness of the proposed ACO on a single machine total weighted tardiness scheduling problem benchmark available from ORLIB are presented. Finally, Section 5 draws some conclusions.

2. PROBLEM DEFINITION AND LITERATURE REVIEW

Formally, the STWTSDS problem requires the scheduling of a set of n independent jobs, which are all ready for processing at time zero, on a single machine which is continuously available and can process only one job at a time. For each job $j = 1, ..., n$, the following quantities are given: a processing time p_j, a due date d_j and a weight w_j. A sequence-dependent setup time s_{ij} should be waited before starting the processing of job j immediately sequenced after job i. The job tardiness is defined as $T_j = \max(0, C_j - d_j)$, being C_j the completion time of job j, and the job is said *tardy* if $T_j > 0$. A schedule corresponds to a feasible sequencing of the jobs on the machine: due to the *regularity* of the problem objective (Baker and Scudder (1990)), having fixed a feasible sequencing, each job must complete at its earliest completion time. The scheduling objective is the minimization of the total weighted tardiness, i.e.,

$$Z = \min \sum_{j=1}^{n} w_j T_j \tag{1}$$

This problem, which is denoted as $1/s_{ij}/\Sigma w_j T_j$, is strongly *NP*-hard since it is a special case of the $1//\Sigma w_j T_j$ that has been proved to be strongly *NP*-hard by Lawler (1997); the complexity of the considered problem, confirmed by the fact that the special case without setups and with unitary weights is still *NP*-hard (Du and Leung (1990)), justifies the research of heuristic approaches for its solution in practical cases. Nevertheless, exact algorithms based on branch and bound (B&B) or dynamic programming approaches have been proposed, but they are able to tackle instances of reduced dimensions. For example, an early B&B algorithm for the STWTSDS problem was proposed in Rinnooy Kan et al. (1975); Abdul-Razaq et al. (1990) faced the single machine total tardiness scheduling with sequence-dependent setups (STTSDS) problem, whereas the algorithm in Potts and van Wassenhove (1985) was able to solve up to 40-job instances for the sequence-independent problem; Luo and Chu (2006) recently devised a B&B algorithm for the

STTSDS solving up to 30-job instances with reduced computational times. Most of the recent research on total tardiness scheduling with sequence-dependent setups is focused on the development of heuristics: in particular, *constructive* heuristics, generally corresponding to dispatching rules, *improvement* heuristics and *metaheuristics*. The well-known apparent tardiness cost with setups (ATCS) heuristic, proposed by Lee et al. (1997), appears to be the best constructive approach for the STWTSDS problem; such heuristic extends to the case of sequence-dependent setups the time-dependent apparent tardiness cost (ATC) rule defined a decade before by Vepsalainen and Morton (1987). Constructive heuristics usually require a small computational effort (for this reason they may be preferred in industrial applications), but they are outperformed by improvement approaches, as well as metaheuristics, which, in turn, are usually much more computational time demanding. Improvement approaches consist of local search algorithms that, starting from an initial solution produced by a simple constructive rule, explore a succession of neighbouring solutions until no further improvement is possible. As noted in the paper by Cicirello and Smith (2005), which includes a survey of heuristic approaches for the STWTSDS problem, also Lee et al. (1997) proposed a local search procedure based on a reduced set of swap and insert moves to improve the solution generated by the ATCS rule. The dominance of improvement approaches over constructive ones is witnessed, for example, in Potts and van Wassenhove (1991), where the effectiveness of simple pair-wise interchange methods against dispatching rules for the single machine total weighted tardiness problem was shown; more recently, constructive heuristics were compared to a memetic algorithm in França et al. (2001), or also in Anghinolfi and Paolucci (2006), where a hybrid metaheuristic was proposed for a similar parallel machine case. Cicirello and Smith (2005) analysed the behaviour of several stochastic search procedures for the STWTSDS, showing the effectiveness of introducing randomization. In particular, the authors developed several algorithms, a value-biased stochastic sampling (VBSS), a VBSS with hill-climbing (VBSS-HC) and a simulated annealing (SA), that were compared to limited discrepancy search (LDS) and heuristic-biased stochastic sampling (HBSS) for a 120 benchmark problem instances defined by Cicirello (2003) and available on the web. Several metaheuristic approaches have been proposed for the STTSDS problem: genetic algorithms (GA) in Rubin and Ragatz (1995) and in Armentano and Mazzini (2000); a memetic algorithm combining GA with local search in França et al. (2001); a SA approach (Tan and Narasimhan (1997)); a greedy randomized adaptive search procedure (GRASP) in Feo et al. (1996). Tan et al. (2000) compared four implementations of B&B, GA, random-start pair-wise interchange (RSPWI) and SA proposed for the STTSDS in previous works by the same authors, concluding that SA and RSPWI are suitable approaches to face larger instances, whereas the GA shows the worst performance. In recent times, the Cicirello's best known results were independently improved in Lin and Ying (2006) and in Cicirello (2006). Lin and Ying (2006) developed three approaches for the STWTSDS, i.e., a SA, a GA and a tabu search (TS), whose best results over 10 runs were compared against the Cicirello and Smith (2005) best known ones; the results reported by the authors show that all the three algorithms were able to improve the previous best known results for more than 71% of the instances with an average computation time for each single run of 27s. Cicirello (2006) presented a GA approach for the STWTSDS problem based on a new non-wrapping order crossover (NWOX) operator, derived from the well-known order crossover (OX) operator, whose purpose is to propagate to the offspring not only the jobs' order but also their absolute positions in the sequences; this new NWOX operator appeared well-suited for the STWTSDS problem, and the GA presented in Cicirello (2006) was able to improve 49 of the 120 best known results of the Cicirello's (2003) benchmark.

In recent years, several ACO approaches have been proposed to face total tardiness scheduling problems which may include or not sequence-dependent setups. A first implementation was studied in Bauer et al. (1999), where the authors adapted the Ant Colony System (ACS) (Dorigo and Gambardella (1997)) to the single machine total tardiness problem, showing that their algorithm outperforms a set of leading heuristics for this problem. An analysis of the combination of different local search strategies with an ACS algorithm for the total weighted tardiness problem was proposed by den Besten et al. (2000), who highlighted as dominant strategy the use of solution neighbourhoods based on the concatenation of simple moves; the ACO algorithm in den Besten et al. (2000) tested over the ORLIB benchmark (www.ms.ic.ac.uk/info.html) found always the best known solution even for the largest instances (100 jobs) with 6.99s as average CPU time. Merkle and Middendorf (2000 and 2003) defined a new approach of evaluating pheromone values, called *Pheromone Summation* (PS) rule, in an ACO algorithm that extends to total weighted tardiness scheduling problems, the ACS proposed for traveling salesman problem (TSP) in Dorigo and Gambardella (1997). Since for standard ACO approaches the probability $p(h, j)$ of a job j of being scheduled in a sequence place h depends on single pheromone value associated with the pair (j, h), the aim of the PS rule is to avoid a too much delayed scheduling of jobs which fail to be sequenced in their most favourite place. Similarly Merkle and Middendorf (2001) pointed out that for permutation scheduling problems the sequential solution construction procedure usually adopted in ACO algorithms can be based on a probability $p(h, j)$ that should take into account the previous decisions for the places preceding h; hence the authors devised an ACO approach which alternates iterations where "random" ants consider the sequence places in random order to iterations where "sequential" ants assign the jobs in the sequence order but including also a suitable heuristic information in the selection probability. In Merkle and Middendorf (2002) it was

remarked the need of methods like the PS rule for permutation problems where good solutions show a so-called "similarity property", i.e., they usually differ for a small number of places; in alternative to the PS rule the authors defined a new *Relative Pheromone Evaluation* (RPE) method, based on a normalization of pheromone values, that for a single machine total earliness with multiple due dates scheduling problem outperformed both standard and PS rule pheromone evaluation approaches. Gagné et al. (2002) showed the effectiveness of an ACO algorithm for the STTSDS problem which includes a lookahead information, obtained from a lower bound described in Tan et al. (2000), in the heuristic component of the transition rule used to select the next job to be included in a partial schedule. Quite recently, the ACO algorithm by Gagné et al. (2002), together with B&B and other metaheuristic approaches, has been outperformed by different variants of GRASP for the STTSDS proposed by Armentano and Bassi de Araujo (2006) and by Gupta and Smith (2006). An ACO algorithm for the STWTSDS has been proposed in Liao and Juan (2007), where the authors showed the appropriateness of their approach by improving about 86% of the best results obtained with the set of improvement heuristics in Cicirello (2003) for the 120 benchmark problem instances; also this algorithm is based on ACS, but it imposes a minimum pheromone value similarly to the MMAS (Stützle and Hoos (2000)), and adopts a new parameter for the pheromone initialization and a different timing for local search execution. Two final remarks may emerge from the literature review of ACO approaches to scheduling. Firstly it can be observed that the presence of sequence-dependent setups has been mainly considered into the heuristic information exploited by the ACO algorithms: for example, in Gajpal et al. (2006) sequence-dependent setups influence the heuristic used to generate a starting solution that, after a local search enhancement, is used to initialize the pheromone trails. Secondly, the role of the local search appears basic to improve the behaviour of ACO algorithms.

3. THE PROPOSED ACO APPROACH

This section presents the characteristics of the new ACO approach proposed for the STWTSDS problem. For this purpose, some notation must be introduced. In general a solution x of a single machine scheduling problem of a set of n independent jobs is represented by a sequence $\sigma(x) = (x[1], ..., x[n])$, where $\sigma(x[h])$ or simply $[h]$, $h = 1, ..., n$, denotes the index of the job that in solution x is sequenced in the h-th position on the machine, e.g., $j = \sigma(x[h]) = [h]$, with $j = 1, ..., n$. In addition, the position-job pairs (h, j), $h, j = 1, ..., n$, determined by a sequence $\sigma(x)$ are denoted as *solution components* of x.

The core of the approach proposed in this paper for ACO (denoted in the following with ACO_{AP}) is mainly based on the Ant Colony System (ACS) (Dorigo and Gambardella (1997)), and it includes concepts inspired to the MMAS (Stützle and Hoos (2000)) and to the approaches in Merkle and Middendorf (2000 and 2003); however, how it will be detailed in the following, in the ACO_{AP} algorithm such concepts are encapsulated in a new pheromone model and exploited in a real different manner. In addition, the developed algorithm may be also compared to the one in Liao and Juan (2007) (denoted hereinafter as ACO_{LJ}), whose results have been taken as main reference to evaluate the ACO_{AP} effectiveness.

3.1 The overall ACO_{AP} algorithm description

Figure 1 reports the very high level structure of the ACO_{AP} algorithm.

```
Initialization;
k = 1;
While <termination condition not met>
{
    For each ant a ∈ A
     {
        Construction of solution x_a^k;
        Local pheromone update;
     }
     Local Search phase;
     Global pheromone update;
     k = k + 1;
}
```

Figure 1. The overall ACO_{AP} algorithm.

A set A of m artificial ants is considered. At each iteration k, every ant a identifies a solution x_a^k building a sequence $\sigma(x_a^k)$ of the n jobs, whose objective value $Z(x_a^k)$ is then simply computed by assigning to each job its feasible (i.e., taking into account both processing times and setups) earliest start time for that sequence. Every ant a builds the sequence $\sigma(x_a^k)$ by iterating n selection stages: first, the set of not sequenced jobs for ant a, U_a^0, is initialized as $U_a^0 = \{1, ..., n\}$; then, at stage $h = 1, ..., n$, the ant a selects one job j from the set U_a^{h-1} to be inserted in the position h of the partial sequence, and updates $U_a^h = U_a^{h-1} \setminus \{j\}$; at stage $h = n$ all the jobs are sequenced and $U_a^n = \varnothing$. The job selection at each stage h of the construction procedure at iteration k is based on a rule that is influenced by the pheromone trail $\tau_k(h, j)$ associated with the possible solution components (h, j), where $j \in U_a^{h-1}$.

A characteristic that distinguishes the proposed algorithm from all the previous approaches is that the pheromone values assigned to $\tau_k(h, j)$ are independent of the objective or quality function values associated with previously explored solutions including the component (h, j). Pheromone trails here represent a sort of measure of the utility of including a component during the construction of "good" solutions that is progressively

learned from the solution space exploration. In addition, an arbitrary range $[\tau_{Min}, \tau_{Max}]$ is adopted for the pheromone values, which is independent of the specific problem or instance considered. Also in MMAS lower and upper bounds are imposed for $\tau_k(h, j)$, but they must be appropriately selected and dynamically updated each time a new best solution is found, taking into account the objective function values. Differently, in the ACO_{AP} algorithm such bounds are independent of the objective function and arbitrarily selected, since any pair of values, such that $\tau_{Min} < \tau_{Max}$, can be chosen. Note that in this way τ_{Max} and τ_{Min} are removed from the set of parameters needed by the algorithm. In addition, the variation of $\tau_k(h, j) \in [\tau_{Min}, \tau_{Max}]$ during the exploration process, i.e., the ant colony learning mechanism, is controlled by a new GPU rule (described in the following) that imposes a smooth variation of $\tau_k(h, j)$ within these bounds such that both extremes are asymptotically reached. Note that such a characteristic is different from MMAS, where the lower and upper bounds are used as cut-off thresholds. The new pheromone-based learning mechanism of the ACO_{AP} algorithm relies on three features: (a) the new kind of *asymptotic pheromone trails* previously described; (b) a local pheromone update (LPU) rule that induces a pheromone perturbation to favour a stronger intra-iteration diversification mechanism than in standard ACS, but it keeps the scope of such perturbation restricted to each single iteration; (c) a new *unbiased global pheromone update* rule. The features (b) and (c) will be detailed in the following where, in order to make simpler and more readable the expressions, a relative pheromone value $\tau'_k(h, j) = \tau_k(h, j) - \tau_{Min}$, such that $\tau'_k(h, j) \in [0, \tau'_{Max}]$, where $\tau'_{Max} = \tau_{Max} - \tau_{Min}$, is used. The ACO_{AP} algorithm in Figure 1 can now be detailed.

Initialization. For each solution component (h, j), $h, j = 1, ..., n$, an initial value of the pheromone trail is assigned by fixing $\tau_0(h, j) = (\tau_{Max} + \tau_{Min})/2$; in addition, the best current solution x^* is initialized as an empty solution and the associated objective value $Z(x^*)$ is fixed to infinity.

Job selection rule. At a selection stage h of iteration k, an ant a determines which job $j \in U_a^{h-1}$ is inserted in the h-th position of the sequence as follows. First, similarly to the ACS, it is chosen which job selection rule must be used between *exploitation* and *exploration*: a random number q is extracted from the uniform distribution $U[0, 1]$ and if $q \leq q_0$ the exploitation rule is used, otherwise the exploration one. The parameter q_0 (fixed such that $0 \leq q_0 \leq 1$) directs the ants' behaviour towards either the exploration of new paths or the exploitation of the best paths previously emerged. The *exploitation* rule selects the job j in a deterministic way as

$$j = \arg \max_{u \in U_a^{h-1}} \{\tau'_k(h, u) \cdot [\eta(h, u)]^{\beta_k}\} \tag{2}$$

whereas the *exploration* rule according to a *selection probability* $p(h, j)$ computed as

$$p(h, j) = \frac{\tau'_k(h, j) \cdot [\eta(h, j)]^{\beta_k}}{\sum_{u \in U_a^{h-1}} \tau'_k(h, u) \cdot [\eta(h, u)]^{\beta_k}} \tag{3}$$

The quantity $\eta(h, j)$, associated with the solution component (h, j), is an heuristic value which is computed, as done in Liao and Juan (2007), equal to the priority $I_t(h, j)$ of assigning job j in position h at time t according to the ATCS rule (Lee et al. (1997))

$$\eta(h, j) = I_t(h, j) = \frac{w_j}{p_j} \exp\left[\frac{\max(d_j - p_j - t, 0)}{k_1 \bar{p}}\right] \exp\left[-\frac{s_{[h-1]j}}{k_2 \bar{s}}\right] \tag{4}$$

where

$$t = \sum_{i=1}^{h-1} (s_{[i-1][i]} + p_{[i]}) + s_{[h-1]j} \tag{5}$$

$\bar{p}$ and $\bar{s}$ are respectively the average processing time and the average setup time, and k_1 and k_2 are the lookahead parameters fixed as originally suggested in Lee et al. (1997). Therefore, similarly to the ACO approaches previously reviewed, even in the ACO_{AP} algorithm the influence of the sequence-dependent setups is encapsulated in the heuristic values used in the job selection rule. The parameter β_k in (2) and (3) is the relative importance of the heuristic value with respect to the pheromone trail one at iteration k; the initial value β_0 of such parameter is updated at each iteration with the following exponential rule

$$\beta_{k+1} = \varphi \cdot \beta_k \tag{6}$$

where φ is a factor fixed in [0, 1]. The progressive reduction of parameter β_k was not included in previous approaches to the STWTSDS problem, but, to the best authors' knowledge, it has been first introduced in the ACO algorithm proposed by Merkle et al. (2002) for resource constrained scheduling problems; in this way, the influence of the heuristic values $\eta(h, j)$ on the ants' decisions diminishes iteration after iteration, leaving to the pheromone trails the leading role of driving the solution construction process. Finally, note that for this reason the choice of the heuristic to compute the (h, j) values could appear less critical, but still necessary in the initial iterations when the pheromone is equally distributed over all the possible solution components.

Local pheromone update. (*intra-iteration diversification*). As often done in previous ACO approaches to avoid premature convergence of the algorithm, a LPU is

performed after any single ant a completed the construction of a solution x_a in order to make more unlike the selection of the same sequence by the following ants. In the ACO_{AP} the local pheromone update rule adopted is

$$\tau'_k(h,j) = (1-\rho)\cdot\tau'_k(h,j) \quad \forall h = 1, \ldots, n; j = \sigma(x_a[h]) \qquad (7)$$

where ρ is a parameter fixed in [0, 1]. A new characteristic introduced for the ACO_{AP} is to consider such kind of update strictly *local*, i.e., to use it to favour the diversification of the sequences produced by the ants within the same iteration. Note that rule (7) imposes a perturbation on the (relative) pheromone values which is stronger than the one in the standard ACS approach, since it drives pheromone values towards τ_{Min} instead of τ_0. Therefore, here rule (7) is used to temporarily modify the pheromone values only in the single iteration scope, since such changes are deleted before executing the *global pheromone update phase* and starting the next iteration. This feature, said *reset of the local pheromone update* (RLPU), appears consistent with the interpretation of the pheromone values as learned utility, as it assigns only to the GPU the crucial task of modifying the pheromone trails according to the colony exploration experience.

Local search phase. After all the ants have completed the solution construction procedure at an iteration, an *intensification* phase may be performed, which consists of one or more local search (LS) explorations starting from a subset X_{LS} of the solutions found in the iteration. In particular, two rules (said LS *timing rules*) can be used to determine the set X_{LS} and, depending on its cardinality, how many LS explorations must be executed:

- *Best in Iteration* (BI) rule: X_{LS} always includes a single starting solution corresponding to the best solution found in the current iteration, i.e., $x_b^k = \arg\min_{a=1,\ldots,m} Z(x_a^k)$. Then, according to this rule a single LS is always executed in each iteration.
- *Improved Solution Without LS* (ISWLS): let x^*_{WLS} be the best solution found by any ant in the previous iterations without using the LS; then, X_{LS} may include one or more solutions found in the current iteration k improving x^*_{WLS}, ie $x = \{x_a^k : Z(x_a^k) < Z(x^*_{WLS}), a = 1, \ldots, m\}$. With the ISWLS the number of LSs executed in one iteration can vary from zero to m, even if this latter appears a very unlikely case.

In general, any LS algorithm can be used for the intensification phase in ACO_{AP}. In particular, an LS algorithm similar to one in Tasgetiren et al. (2004), which in turn is based on a variant of the variable neighbourhood search (Mladenovic and Hansen (1997)), has been adopted. The LS algorithm, summarized in Figure 2, performs a random neighbourhood exploration allowing both an alternation of random insert and swap moves; in addition the algorithm executes a limited number of random restarts as in the iterated local search. Note that a similar neighbourhood structure is used in Liao and Juan (2007). Random moves consist of picking at random two sequence positions in the current solution and inserting (swapping) the job in the first position after (with) the job in the second one. The algorithm executes an exploration sequence first made of a succession of random insert moves until no improvement is found, and then made of a succession of swap moves: whenever a swap move is not able to find an improved solution, then a new sequence of random insert moves is started and the exploration counter is incremented. After $n\cdot(n-1)$ explorations have been completed, the algorithm executes a random restart from the current best solution. The maximum number of allowed random restarts is bounded by $n/5$, thus the overall complexity of the LS algorithm is $O(n^3)$. As a result of the LS phase, the best current solution x^* is possibly updated.

```
x = x0;
restart_counter = 0;
repeat
{
   x1 = random_insert_move (x);
   exploration_counter = 0;
   repeat
   {
      neighbourhood_counter = 1;
      while neighbourhood_counter ≤ 2
      {
         if neighbourhood_counter = 1
            x2 = random_insert_move (x1);
         else
            x2 = random_swap_move (x1);
          if Z(x2) < Z(x1)
            x1 = x2;
         else
            neighbourhood_counter++;
         }
        exploration_counter++;
      } until (exploration_counter < n*(n − 1))
   restart_counter++;
} until (restart_counter < n/5)
```

Figure 2. The LS algorithm.

Global pheromone update. Two main peculiarities of the ACO_{AP} algorithm, which differentiate it from the previous approaches, are in the meaning given to the pheromone trail values and consequently in the way such values are updated after the completion of an iteration. It has been already pointed out that the (relative) pheromone values $\tau'_k(h,j)$ adopted in the ACO_{AP} range in $[0, \tau'_{Max}]$; such values are changed in the GPU phase with a rule, called *Unbiased Pheromone Update* (UPU) since it neither uses any cost nor quality function, but it performs a smooth update of the pheromone trails associated with a

set of quality solution components. Let Ω_k^* be the set of the best solution components (said, t*he best component set*) determined after the completion of iteration k; then, the UPU rule consists of the three following steps:

1. pheromone evaporation for the solution components not included in Ω_k^*

$$\tau'_{k+1}(h,j) = (1-\alpha)\cdot\tau'_k(h,j) \quad \forall (h,j) \notin \Omega_k^* \tag{8}$$

where $0 \leq \alpha \leq 1$ is a parameter establishing the evaporation rate;

2. computation of the maximum pheromone reinforcement $\Delta\tau'_k(h,j)$ for the solution components in Ω_k^*

$$\Delta\tau'_k(h,j) = \tau'_{Max} - \tau'_k(h,j) \quad \forall (h,j) \in \Omega_k^* \tag{9}$$

3. update of the pheromone trails to be used in the next iteration for the solution components in Ω_k^*

$$\tau'_{k+1}(h,j) = \tau'_k(h,j) + \alpha\cdot\Delta\tau'_k(h,j) \quad \forall (h,j) \in \Omega_k^* \tag{10}$$

The UPU rule guarantees that $\tau'_k(h,j) \in [0, \tau'_{Max}]$ and $\tau'_k(h,j)$ converges towards the bounds asymptotically ($\Delta\tau'_k(h,j)$ is progressively reduced as much as $\tau'_k(h,j)$ approaches to τ'_{Max}, as well as the decrease of $\tau'_k(h,j)$ towards 0 in (8)) with a law similar to the most frequently used cooling schedule for the SA metaheuristic (Kirkpatrick et al. (1983)). An example of the trend due to the UPU rule is depicted in Figure 3. This figure shows, for a fixed solution component $(h,j) = (H,J)$, the effect of the evaporation step (8), with $\alpha = 0.1$, on the associated pheromone $\tau_k(H,J)$ from its initial value τ_0 at iteration $k = 0$, to a value close to τ_{Min} at iteration $k = 40$; then, assuming that the component (H, J) is included in Ω_k^* for $k = 41, ..., 100$, the figure shows the consequent asymptotical increase of $\tau_k(H,J)$ towards τ_{Max} due to steps (9) and (10); finally, $\tau_k(H,J)$ is again subject to the evaporation (8), having assumed $(H,J) \notin \Omega_k^*$ for $k >$ 100.

Two possibilities are available for defining the best component set Ω_k^*:

- *Best-so-far* (BS) solution component set: Ω_k^* includes only the solution components associated with x^*, i.e.,

$$\Omega_k^* = \left\{ (h,j) : h = 1,...,n;\ j = \sigma(x^*[h]) \right\} \tag{11}$$

- *Cumulative BS* (CBS) solution component set: if a component (h,j) is present in x^*, that is, h appeared a "good" sequence position for job j, hence, taking into account that a tardiness cost must be minimized, it should be sensible to consider j even more urgent for successive sequence positions if job j misses to be sequenced as h-th; according to this rationale, the set Ω_k^* is defined as

$$\Omega_k^* = \left\{ (l,j) : l = h,...,n; h = 1,...,n;\ j = \sigma(x^*[h]) \right\} \tag{12}$$

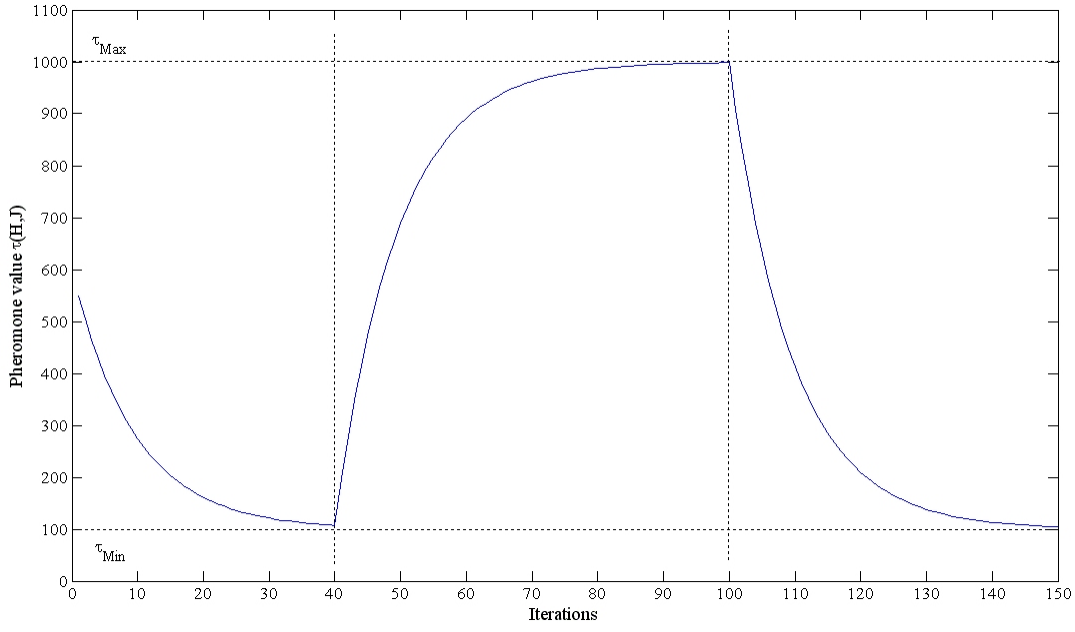
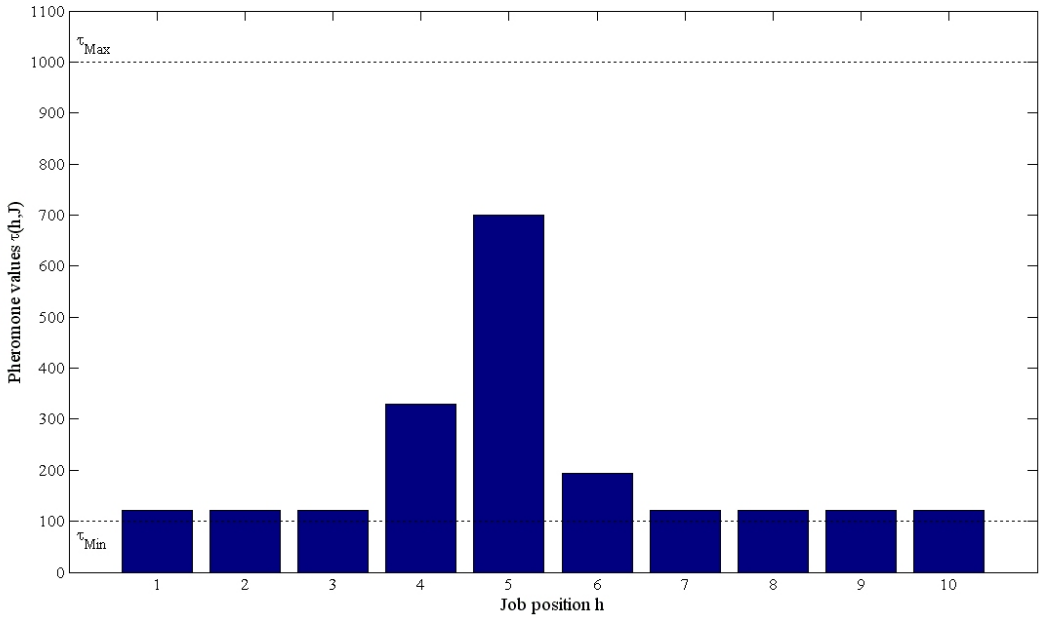
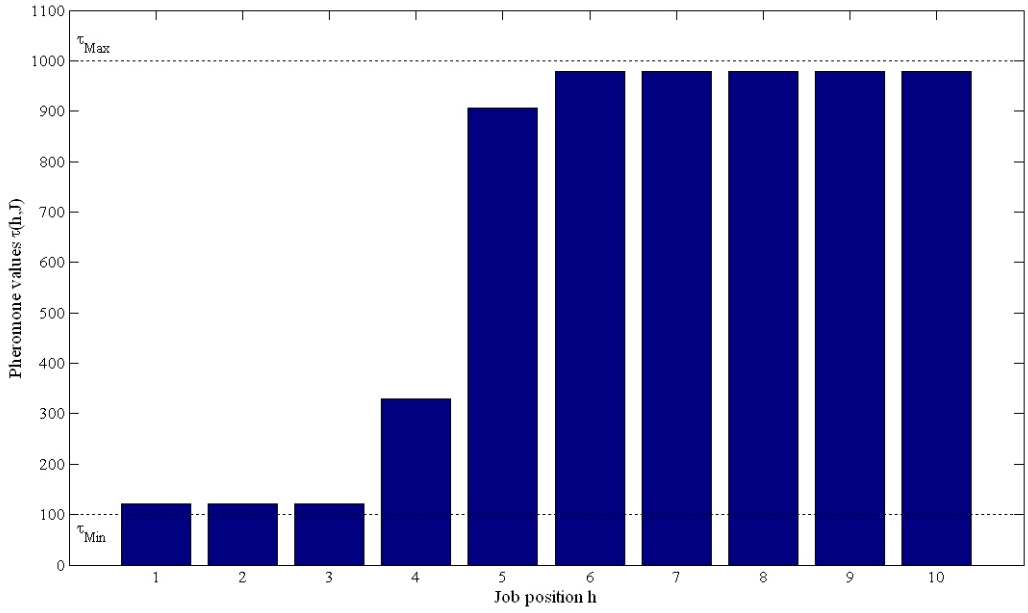


Figure 3. An example of the asymptotical variation of the pheromone value between its lower and upper bounds due to the UPU rule.



(a)



(b)

Figure 4. An example of the effect on the UPU rule of the two different options for defining Ω_k^*.

An example of the effects of the UPU rule with the two different options for defining the best component set is provided in Figure 4. Both diagrams in this figure represent the pheromone values $\tau_k(h,J)$, h = 1, ..., 10, associated with a specific job $j = J$, after 60 iterations (having fixed α = 0.05); in particular, it is assumed that for k = 1, ..., 20, $(6,J) \in \Omega_k^*$, for k = 21, ..., 40, $(4,J) \in \Omega_k^*$ and finally for k = 41, ..., 60, $(5,J) \in \Omega_k^*$. Therefore, it is reasonable to assume that the algorithm has learned that the "good" position for job J should be around h = 5. However, using a BS solution component set, the pheromone values depicted in diagram (a) of Figure 4 highlight the possibility that such knowledge could be almost completely disregarded: if, for example, the application of the exploration rule (3) fails to sequence J in position 5, this job could be dramatically delayed since its pheromone values for the subsequent positions could be much smaller than the one of competitor jobs. On the other hand, the CBS, incrementing also the pheromone values of the positions following the one of job J in the best so far solution x^*, produces the pheromone values shown in diagram (b) of Figure 4 that make very unlikely the delayed sequencing of J previously described. Note that a rationale similar to the CBS was used in the PS rule described in Merkle and Middendorf (2000 and 2003) and

in the RPE method in Merkle and Middendorf (2002); however, in the ACO_{AP} the CBS is an option of the UPU rule that alters the learning mechanism forcing the increase of pheromone values, whereas the mentioned previous methods are used to modify the evaluation of pheromone trails in the solution construction process.

Termination conditions. The algorithm is stopped when a maximum number of iterations, or a maximum number of iterations without improvements, is reached.

4. EXPERIMENTAL ANALYSIS OF THE PROPOSED ACO APPROACH

The ACO_{AP} algorithm was coded in C++ and an experimental campaign was executed on a Pentium IV, 2.8 GHz, 512 Mb PC, in order to analyze its performances. The adopted benchmark was the set of 120 problem instances with 60 jobs provided by Cicirello (2003), available at http://www.cs.drexel.edu/~cicirello/benchmarks.html. Note that the same benchmark was used for testing the ACO_{LJ} in Liao and Juan (2007). The benchmark was produced by generating 10 instances for each combination of three different factors usually referenced in the literature (for a definition and discussion see, e.g., Pinedo (1995)): the due date tightness δ, the due date range $\boldsymbol{R}$, and the setup time severity ξ, selected as follows: $\delta \in \{0.3, 0.6, 0.9\}$, $R \in \{0.25, 0.75\}$, $\xi \in \{0.25, 0.75\}$. For each test two possible reference results were considered: the best known solutions available from Cicirello (2006) (denoted in the following with BKC) and the best solutions provided by the ACO_{LJ} algorithm in Liao and Juan (2007). In addition, the best results obtained by the ACO_{AP} were finally compared to the up-to-date best known results among the ones presented in Cicirello and Smith (2005) and Cicirello (2006), the ones produced by ACO_{LJ} and by the SA, GA and TS algorithms proposed in Lin and Ying (2006), presenting a new set of best known results for this benchmark.

In order to make the comparison between the ACO_{AP} and ACO_{LJ} results more sound, a set of $m = 30$ ants was considered and the same pair of termination criteria used in Liao and Juan (2007) were adopted, i.e., the maximum number of iterations = 1000, and the maximum number of non improving iterations = 50. In addition, some preliminary experiments were conducted on a subset of instances to determine suitable values for the other parameters needed by the ACO_{AP}. In particular, they were set as follows: $\alpha = 0.1$, $\beta_0 = 1$, $\rho = 0.05$, $q_0 = 0.7$, and $\varphi = 0.9$; such values were respectively selected from the following sets, $\alpha \in \{0.05, 0.1, 0.3\}$, $\beta_0 \in \{0.5, 1, 1.5, 3\}$, $\rho \in \{0.05, 0.08, 0.1\}$, $q_0 \in \{0.5, 0.7, 0.9\}$, and always setting $\varphi = 1 - \alpha$. The selection of these parameter values may affect the algorithm performance, but the tests conducted denoted a low sensitivity to their changes, showing an average relative cost variation not greater than 2%. It should be mentioned for the sake of completeness that the upper bound of the relative pheromone value was fixed in the ACO_{AP} code as $\tau'_{Max} = 100$, so that an initial pheromone value $\tau'_0(h, j) = 50$ was associated with any solution component; however, it must be remarked once again that any positive value can be assigned to τ'_{Max} since this choice does not affect the algorithm behaviour.

The experimental campaign performed consisted of five tests described in the rest of this section.

4.1 Determination of the ACO_{AP} best configuration (Test 1)

The purpose of this test is to evaluate which of the following ACO_{AP} features can improve the algorithm performance for the considered benchmark: progressive decrease of the importance of the heuristic value (β*dec*); use of the *cumulative* BS solution components in the global pheromone update (*CBS*); reset of the LPU at the end of each iteration (*RLPU*).

The ISWLS timing rule was used for the LS, and, in order to compare the results from this test with the ones of ACO_{LJ}, the same experimental scheme in Liao and Juan (2007) was adopted, i.e., 10 algorithm runs were executed taking for each benchmark instance the best result. Table 1 reports in the first three columns the type of ACO_{AP} configuration for the three features (β*dec*, *CBS*, *RLPU*) tested, denoting with a binary value the presence ("1") or absence ("0") of the relevant feature. The produced results for each configuration are compared with the reference ones in Liao and Juan (2007) both reporting the average percentage deviation (*Avg % Deviation*) (computed as $100 \cdot (result - reference)/reference$) with both *reference* and *result* greater than zero) and the average percentage number of instances whose best result found by ACO_{LJ} was improved by the ACO_{AP} algorithm (*Avg % Number of Improved Instances*); note that the latter column also reports in brackets respectively the average percentage number of instances for which the ACO_{AP} got worse and equal results than ACO_{LJ}. From Table 1 it should be apparent that only the feature corresponding to the reset of the LPU after the completion of any iteration is actually important for producing improved performance with the considered benchmark. This fact is confirmed by the diagram in Figure 5, showing the average percentage deviations with their 95% confidence intervals: in this diagram all the intervals of the configurations with the *RLPU* are not overlapping and lower than the ones without it. A further analysis was conducted in order to evaluate if the comparisons with the ACO_{LJ} results can be considered appropriate or they are biased by the presence of outliers: in fact, since the objective values in the benchmark (see Table 7) differ for several orders of magnitude, such a correction would mitigate the possible influence of quite reduced absolute differences in the objectives for instances with small reference values. Thus, Table 2 shows the results obtained after a correction eliminating from the computation of the averages the instances with a percentage deviation not in the interval (−40%, 40%); in this table the *Avg % Number of Improved*

Instances, as well as the worse and equal ones in brackets, are computed with respect to the remaining number of instances after having eliminated the outliers. The values in Table 2 are quite similar to the ones in Table 1 (note that in the two tables the average % number of instances with result equal to the ACO_{LJ} one is mainly due to zero cost solutions); in addition, Figure 6 confirms also in this case the relevance of the *RLPU* feature. Thus, Test 1 underlined that it is fundamental to associate this feature with the used LPU in the new pheromone model adopted in the ACO_{AP} algorithm. The average CPU time required for this test was 4.30s (with a minimum of 0.59s and a maximum of 1970s), which is comparable with the computation time indicated in Liao and Juan (2007). Then, Test 1 seems to highlight the good quality of ACO_{AP} with respect to ACO_{LJ}, since the ACO_{AP} configurations with the *RLPU* feature improved on the average the best known results of ACO_{LJ}; in addition, note that with such configurations ACO_{AP} was also able to find one zero cost solution more than ACO_{LJ}.

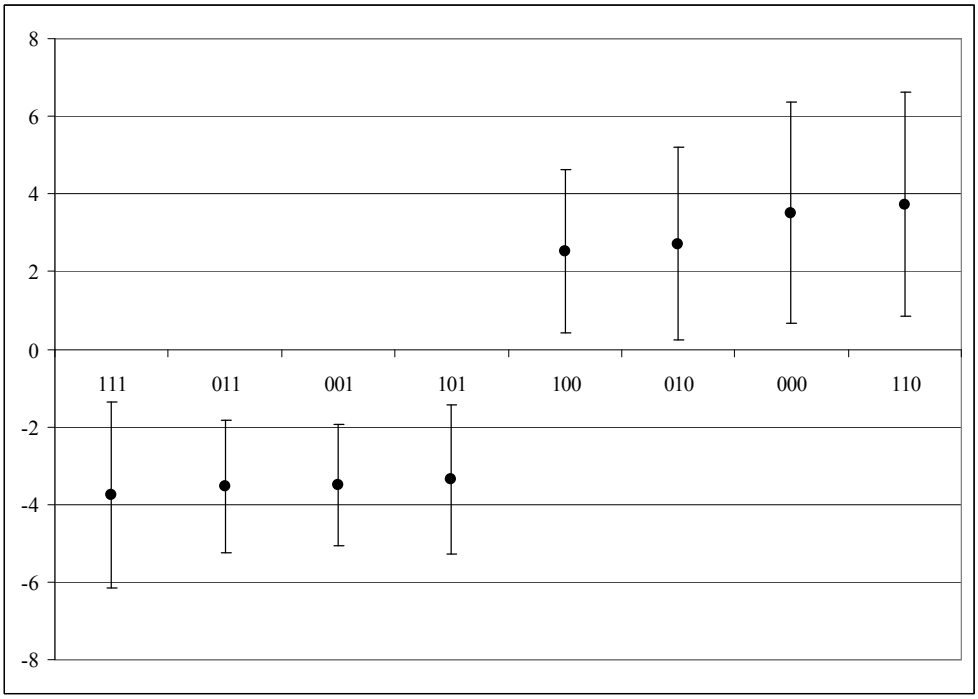


Figure 5. The average percentage deviations of the different configurations in Table 1 with the 95% confidence intervals.

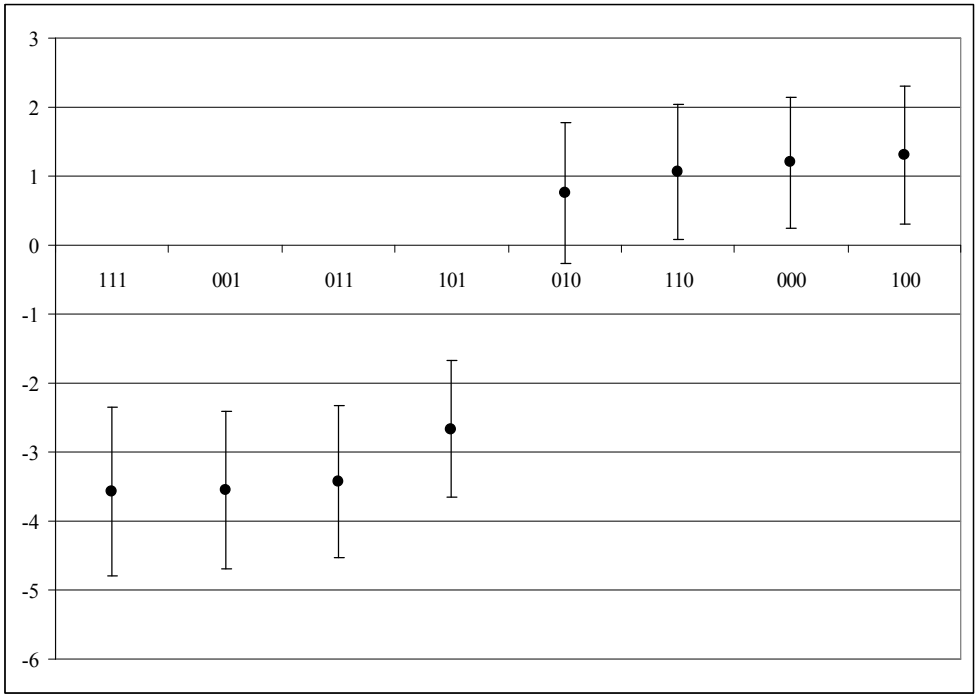


Figure 6. The average percentage deviations of the different configurations in Table 2 with the 95% confidence intervals.

Table 1. Comparison of the different configurations of the ACO_{AP} tested (Test 1)

Configuration			Comparison with ACO_{LJ}	
βdec	*CBS*	*RLPU*	Avg % Deviation	Avg % Number of Improved Instances
1	1	1	−3.75%	67.5% (19.2%, 13.3%)
0	1	1	−3.54%	70.0% (16.7%, 13.3%)
0	0	1	−3.50%	65.8% (20.8%, 13.3%)
1	0	1	−3.34%	63.3% (23.3%, 13.3%)
1	0	0	2.51%	28.3% (58.3%, 13.3%)
0	1	0	2.71%	34.2% (52.5%, 13.3%)
0	0	0	3.52%	26.7% (59.2%, 14.2%)
1	1	0	3.74%	34.2% (52.5%, 13.3%)

Table 2. Comparison of the different configurations of the ACO_{AP} tested without the (−40%, 40%) outliers (Test 1)

Configuration			Comparison with ACO_{LJ}	
βdec	*CBS*	*RLPU*	Avg % Deviation	Avg % Number of Improved Instances
1	1	1	−3.57%	63.3% (23.3%, 13.3%)
0	0	1	−3.55%	61.7% (25.0%, 13.3%)
0	1	1	−3.44%	65.8% (20.8%, 13.3%)
1	0	1	−2.67%	58.3% (28.3%, 13.3%)
0	1	0	0.75%	31.7% (55.0%, 13.3%)
1	1	0	1.06%	31.7% (55.0%, 13.3%)
0	0	0	1.20%	24.2% (61.7%, 14.2%)
1	0	0	1.31%	25.8% (60.8%, 13.3%)

4.2 Evaluation of the ACO_{AP} average results (Test 2)

In spite of the encouraging results from Test 1, it was considered sensible analysing the ACO_{AP} performance with a different experimental scheme. In fact, according to Birattari and Dorigo (2005), it seems questionable to evaluate the performance of a stochastic algorithm on the basis of its best result over *M* runs, but an average result is instead considered a more appropriate performance index. As pointed out in Birattari and Dorigo (2005), taking the best result over *M* runs corresponds to a sort of trivial "*null-metaheuristic*" which is based on the random restart of the algorithm; in addition, the actual computation time of such *null-metaheuristic* is *M* times greater than the computed average CPU time for a single run. Test 2 was then designed in order to evaluate the average performance over 10 runs of the proposed algorithm with a different LS timing rule, the BI one, which usually showed longer computation times. This choice seemed appropriate because the resulting CPU times were approximately *M* times greater than the ones for Test 1. On the other hand, the use of the LS with BI rule appeared a suitable way to exploit the extended time that in this test becomes available for each run, making the average results comparable with the best ACO_{LJ} ones. Test 2 was performed for only one ACO_{AP} configuration selected on the basis of the outcome of Test 1, i.e., (*βdec*, *CBS*, *RLPU*) = (1, 1, 1). The results obtained are shown in Table 3, which reports in the columns the average percentage deviation and the average number of improved (worse and equal) instances of both the average and the worst ACO_{AP} results over 10 runs with respect to BKC, ACO_{LJ}, and ACO_{LJ} without the (−40%, 40%) outliers. Again, the comparison with ACO_{LJ} puts into evidence the quality of the proposed algorithm; the relevant role of the outliers (in this case favouring ACO_{AP}) can be observed considering the difference in the comparison with ACO_{LJ} including or excluding them in the computation of the averages. In addition, even for this test the ACO_{AP} was able to find one zero cost solution more than ACO_{LJ}. It seems quite important to underline the robustness of the proposed algorithm with the BI timing rule by observing that even the ACO_{AP} worst results over 10 runs outperformed on the average the BKC and ACO_{LJ} best known ones; this suggests that the results obtained in each single run in Test 2 were quite stable and the average ACO_{AP} performance could be considered a representative index of the algorithm behaviour in each run. The observed average CPU time for Test 2 was 65.90s (with a minimum of 1.00s and a maximum of 265.59s). Such a greater computation time is due to the different behaviour of the BI timing rule, which executes one LS per iteration, compared to the ISWLS one. To better understand the difference between the two LS timing rules, both the best and average results obtained for the (*βdec*, *CBS*, *RLPU*) = (1, 1, 1) configuration with the ISWLS rule and with the BI one were compared, as well as the relevant computation times. It was first observed that the best and average results with the ISWLS rule were respectively 9.84% and 40.25% worse than the ones produced with the BI rule; on the other hand, the superiority of the BI rule seems compensated by the longer average CPU time, 65.90s, compared to 4.30s of the ISWLS. The reason of such a large difference can be understood by observing that the ratio between the number of LSs and the number of iterations executed by the ACO_{AP} with the BI rule is

obviously 100%, whereas with the ISWLS is on the average only 8.2% (note that in the worst, but very unlikely, case the ISWLS could execute a LS for all the ants in every iteration). Besides, in Test 2 with the BI rule the average percentage of the total CPU time spent by the ACO_{AP} algorithm in executing LSs was 92.8% with an average number of LSs equal to 127.4, whereas in Test 1 with the ISWLS rule it was 53.6% with an average of 9.5 LSs. However, in accordance with the mentioned observation of Birattari and Dorigo (2005), the fair average CPU time for the algorithm with the ISWLS rule should be about 43.0s since its best results were obtained with 10 restarts; this fact reduces the gap between the average times to the same order of magnitude, so that the BI rule can be again considered superior.

Table 3. Comparison of the results of ACO_{AP} over 10 runs (Test 2)

		Comparison with		
		BKC	ACO_{LJ}	ACO_{LJ} without outliers
Average results over 10 runs	Avg % Deviation	−7.01%	−6.58%	−3.65%
	Avg % Number of Improved Instances	74.2% (11.7%, 14.2%)	65.0% (21.7%, 13.3%)	63.2% (22.8%, 14.0%)
Worst results over 10 runs	Avg % Deviation	−2.80%	−2.15%	−1.64%
	Avg % Number of Improved Instances	48.3% (35.5%, 14.2%)	45.0% (41.7%, 13.3%)	42.1% (43.9%, 14.0%)

Table 4. The performance of ACO_{AP} with the ACO_{LJ} LS algorithm (Test 3)

	Comparison with		
	BKC	ACO_{LJ}	ACO_{LJ} without outliers
Avg % Deviation	−4.96%	−4.16% [−3.75%]	−3.14% [−3.57%]
Avg % Number of Improved Instances	65.0% (20.0%, 15.0%)	62.5% [67.5%] (24.2%, 13.3%)	61.9% [63.3%] (24.6%, 13.6%)

Table 5. The performance of ACO_{AP} without learning mechanism (Test 4)

	Comparison with		
	BKC	ACO_{LJ}	ACO_{LJ} without outliers
Avg % Deviation	0.76% [−7.01%]	1.28% [−6.58%]	0.60% [−3.65%]
Avg % Number of Improved Instances	38.3% [74.2%] (45.5%, 14.2%)	35.0% [65.0%] (51.7%, 13.3%)	35.0% [63.2%] (51.3%, 13.7%)

Table 6. Analysis of the statistical significance of the results of Test 2 (Test 5)

		Avg % deviation from ACO_{LJ}	Statistical significance
δ	0.3	−14.73%	yes
	0.6	−5.38%	yes
	0.9	0.18%	no
R	0.75	−6.25%	yes
	0.25	−6.90%	yes
ξ	0.25	−5.55%	yes
	0.75	−7.58%	yes
Global		−6.58%	yes

4.3 Evaluation of the importance of the LS algorithm (Test 3)

Test 1 and Test 2 could raise the doubt about how much the role of the LS is critical. Test 3 and the successive Test 4 try to make this aspect clearer. Test 3 was not performed using the LS described in Figure 2, but with an alternative LS algorithm similar to the one adopted in Liao and Juan (2007) with the ISWLS timing rule; in addition, as for Test 2, only the (β*dec*, *CBS*, *RLPU*) = (1, 1, 1) configuration was analysed for the ACO_{AP} and, as for Test 1, the best result over 10 runs was taken. Hence, the purpose of this test was to evaluate if the goodness of ACO_{AP} compared to the ACO_{LJ} results was only due to a better effectiveness of the LS algorithm utilized in the previous tests. The outcomes from Test 3 are presented in Table 4 that reports the average percentage deviations and the average number of improved (worse and equal) instances with respect to the BKC, ACO_{LJ}, and ACO_{LJ} without the (−40%, 40%) outliers, including for the two latter cases in square brackets the associated results previously shown for Test 1. The required average CPU time for this test was 16.16s (with a minimum of 0.59s and a maximum of 38.95s), whose 62.9% was devoted to LS explorations with an average number of LS executions equal to 75.7. Table 4 underlines the good behaviour of the ACO_{AP} algorithm even with a simpler LS procedure; in particular, the column excluding the outliers puts into evidence the overall robustness of the ACO_{AP} results for the considered benchmark. Finally, note that in this test, based on the best result over 10 runs, the ACO_{AP} produced an average percentage deviation from the ACO_{LJ} better than the corresponding one in Test 1 and was also able to find two zero cost solutions more than ACO_{LJ}: this fact seems to suggest further the appropriateness of an average performance index for stochastic algorithm, as this better percentage deviation was due to particularly good results in some of the runs for a few instances which belong also to the outliers.

4.4 Evaluation of the importance of the ACO learning mechanism (Test 4)

As a counterpart of the previous Test 3, Test 4 aims at evaluating how important is the ACO core algorithm implemented in the ACO_{AP}, i.e., the pheromone trail based learning mechanism. Thus, a no-learning configuration was forced for the ACO_{AP}, imposing no pheromone update ($\alpha = \rho = 0$) and (β*dec*, *CBS*, *RLPU*) = (0, 0, 0). On the other hand, as for Test 2, the more powerful LS with the BI rule was used, and the average result over 10 runs was considered. The percentages in Table 5 clearly show a worsening with respect to the previous results for Test 2, which are here reported in square brackets. As for Test 1 and Test 2, even in this case the algorithm was able to find one zero cost solution more than ACO_{LJ}. The average CPU time for this test was 41.74s (with a minimum of 0.98s and a maximum of 135.43s), devoted for 91.5% to LS executions whose average number, corresponding to the average number of iterations, was 79.5.

4.5 Statistical significance of the results (Test 5)

A final analysis was executed whose purpose was twofold: to verify the statistical consistency of the results obtained (i.e., to determine if the differences in the ACO_{AP} results with respect to the ACO_{LJ} ones were produced by chance or if they are sufficient to consider the ACO_{AP} better on the average than the ACO_{LJ} for the considered benchmark); to deeply analyse the performance of the ACO_{AP} for the different classes of problem instances included in the benchmark set. The results previously obtained for Test 2, detailed in Table 4, were here considered representative of behaviour of the ACO_{AP} (note that, for not reducing too much the number of available samples, no outlier was removed); then, two well-known non parametric statistical tests, the *Friedman's test* and the *Wilcoxon ranksum* test Devore (1991), were used to compare the best ACO_{LJ} results with the average ACO_{AP} ones. Both statistical tests produced the same responses, which are reported in Table 7 in the *Statistical significance* column: here, "yes" denotes that the results from ACO_{AP} and ACO_{LJ} are significantly different (i.e., the null hypothesis that the differences in the outcomes of the two algorithms are caused by randomness can be rejected), "no" otherwise. The column *Avg % deviation from* ACO_{LJ} reports, as in the previous tables, the average percentage deviations from ACO_{LJ}. The *Global* row shows that the whole result of Table 2 is actually representative of a better behaviour of the ACO_{AP}. The other rows in Table 7 are grouped according to the due date tightness δ, due date range R, and setup time severity ξ factors. According to the results in Table 7, the R and ξ parameters do not seem to greatly affect the improved effectiveness of the proposed algorithm when it is compare with ACO_{LJ}. However, the improvement provided by the ACO_{AP} algorithm increases as the factor δ decreases. It can be observed that the ACO_{AP} produced better average results than ACO_{LJ}, which are also significantly different, for all the sub-classes of benchmark instances but one: for δ = 0.9, when the due dates are the tightest, the ACO_{AP} was not able to improve the results of the ACO_{LJ}, but for this case the two algorithms showed a comparable behaviour.

4.6 An updated set of best known result for the Cicirello's benchmark

A final complete report of the best known results produced by the ACO_{AP} during the whole experimental campaign on the Cicirello's benchmark is shown in Table 7, where such results are compared with the up-to-date best known results available from the literature. In detail, Table 7 reports the best known results from the proposed algorithm in the column ACO_{AP} *best*, whereas the previous up-to-date best known results in the column *Previous BK*. In addition, the type of algorithm that produced the previous best known result is reported in the

Table 7. The best-know results for the Cicirello's (2003) benchmark including the new ACO_{AP} ones

Inst.	ACO_{AP} Best	Previous BK	Previous best algorithm	Inst.	ACO_{AP} Best	Previous BK	Previous best algorithm
1	**513**	684	GA[3]	61	**75916**	76396	SA[3]
2	**5082**	**5082**	TS[3]	62	44869	**44769**	TS[3]
3	**1769**	1792	SA[3]	63	**75317**	**75317**	SA[3]
4	**6286**	6526	GA[3]	64	**92572**	**92572**	SA[3]
5	**4263**	4662	GA[3]	65	**126696**	127912	SA[3]
6	7027	**5788**	ACO_{LJ}[2]	66	**59685**	59832	SA[3]
7	**3598**	3693	GA[3]	67	**29390**	**29390**	GA[3]
8	**129**	142	GA[3]	68	**22120**	22148	TS[3]
9	**6094**	6349	GA[3]	69	71118	**64632**	ACO_{LJ}[2]
10	**1931**	2021	SA[3]	70	**75102**	**75102**	GA[3]
11	**3853**	3867	GA[3]	71	**145825**	150709	GA[3]
12	**0**	**0**	ALL	72	**45810**	46903	TS[3]
13	**4597**	5685	GA[3]	73	**28909**	29408	SA[3]
14	**2901**	3045	GA[3]	74	**32406**	33375	TS[3]
15	**1245**	1458	GA[3]	75	22728	**21863**	TS[3]
16	**4482**	4940	GA[3]	76	55296	**55055**	SA[3]
17	**128**	204	SA[3]	77	**32742**	34732	SA[3]
18	**1237**	1610	GA[3]	78	**20520**	21493	TS[3]
19	**0**	208	GA[3]	79	**117908**	121118	GA[3]
20	**2545**	2967	GA[3]	80	**18826**	20335	GA[3]
21	**0**	**0**	ALL	81	**383485**	384996	GA[3]
22	**0**	**0**	ALL	82	**409982**	410979	SA[3]
23	**0**	**0**	ALL	83	**458879**	460978	TS[3]
24	**1047**	1063	GA[3]	84	**329670**	330384	SA[3]
25	**0**	**0**	ALL	85	**554766**	555106	SA[3]
26	**0**	**0**	ALL	86	**361685**	364381	SA[3]
27	**0**	**0**	SA[3], GA[3], TS[3]	87	**398670**	399439	GA[3]
28	**0**	**0**	GA[4], SA[3], GA[3], TS[3]	88	**434410**	434948	GA[3]
29	**0**	**0**	ALL	89	**410102**	410966	SA[3]
30	**130**	165	SA[3]	90	**401959**	402233	GA[3]
31	**0**	**0**	ALL	91	**340030**	344988	TS[3]
32	**0**	**0**	ALL	92	**361407**	365129	GA[3]
33	**0**	**0**	ALL	93	**408560**	410462	VBSS[1]
34	**0**	**0**	ALL	94	**333047**	335550	ACO_{LJ}[2]
35	**0**	**0**	ALL	95	**517170**	521512	GA[3]
36	**0**	**0**	ALL	96	**461479**	461484	ACO_{LJ}[2]
37	**400**	755	SA[3]	97	**411291**	413109	TS[3]
38	**0**	**0**	ALL	98	**526856**	532519	VBSS[1]
39	**0**	**0**	ALL	99	**368415**	370080	ACO_{LJ}[2]
40	**0**	**0**	ALL	100	**436933**	439944	GA[3]
41	**70253**	71186	TS[3]	101	**352990**	353408	TS[3]
42	**57847**	58199	TS[3]	102	493936	**493889**	TS[3]
43	**146697**	147211	SA[3]	103	**378602**	379913	ACO_{LJ}[2]
44	**35331**	35648	SA[3]	104	**358033**	358222	TS[3]
45	**58935**	59307	GA[3]	105	**450806**	450808	SA[3]
46	**35317**	35320	TS[3]	106	**455093**	455849	GA[3]
47	**73787**	73984	SA[3]	107	**353368**	353371	SA[3]
48	65261	**65164**	SA[3]	108	**461452**	462737	TS[3]
49	**78424**	79055	TS[3]	109	413408	**413205**	SA[3]
50	**31826**	32797	TS[3]	110	**418769**	419481	TS[3]
51	**50770**	52639	GA[3]	111	**346763**	347233	ACO_{LJ}[2]
52	**95951**	99200	GA[3]	112	**373140**	373238	ACO_{LJ}[2]
53	**87317**	91302	SA[3]	113	**260400**	261239	GA[3]
54	**120782**	123558	VBSS[1]	114	**464734**	470327	ACO_{LJ}[2]
55	**68843**	69776	GA[3]	115	**457782**	459194	ACO_{LJ}[2]
56	**76503**	78960	GA[3]	116	532840	**527459**	ACO_{LJ}[2]
57	**66534**	67447	ACO_{LJ}[2]	117	**506724**	512286	ACO_{LJ}[2]
58	**47038**	48081	TS[3]	118	355922	**352118**	ACO_{LJ}[2]
59	**54037**	55396	SA[3]	119	**573910**	579462	TS[3]
60	**62828**	68851	GA[3]	120	**397520**	398590	ACO_{LJ}[2]

Previous best algorithm column; in such column the superscript to the algorithm acronym denotes the reference where the associated result was presented, i.e., (1) Cicirello and Smith (2005), (2) Liao and Juan (2007), (3) Lin and Ying (2006), and (4) Cicirello (2006). Note that in that column "ALL" is used to denote the zero cost instances for which all the referred algorithms produced the same zero cost result. The results reported in bold are a new set of best known results for the Cicirello's benchmark. From Table 7 it can be observed that the best results provided by the ACO_{AP} are able to improve the previous best known ones for 72.50% of the instances, whereas they are worse for 8.33% and equal for 19.17% of the instances.

4.7 A comparison with the ORLIB benchmark

To further evaluate its effectiveness and robustness, the ACO_{AP} algorithm was tested on a slightly modified problem disregarding the setup times, i.e., the single machine total weighted tardiness (STWT) scheduling. A benchmark for the STWT problem available via ORLIB (http://people.brunel.ac.uk/~mastjjb/jeb/orlib/wtinfo.html), consisting of three sets of 125 randomly generated instances with 40, 50, and 100 jobs, has been considered. This benchmark was used to analyse the performance of the ACS algorithm proposed for the STWT problem by den Besten et al. (2000). Optimal solutions are known for the 40 and 50 job instances, whereas for the 100 job instances only the best known ones are available; note that these latter best known solutions have been presented in Crauwels et al. (1998) and in Congram et al. (2002) and not modified anymore since then. This final test was performed by executing 10 runs of the ACO_{AP} algorithm with the same setting used for Test 2 (i.e., $\alpha = 0.1$, $\beta_0 = 1$, $\rho = 0.05$, $q_0 = 0.7$, $\varphi = 0.9$, and with the configuration (βdec, *CBS*, *RLPU*) = (1, 1, 1)), without performing any kind of tuning specific for this different benchmark and considering only the most challenging set of 100 job instances. The results obtained showed that the ACO_{AP} algorithm was able to find the best known solutions for all the 100 job instances on every run of the algorithm in acceptable computation times (9.60s on the average, with 0.046s minimum and 112.70s maximum) that could be considered comparable with the ones reported in den Besten et al. (2000).

5. CONCLUSIONS

In this paper the *NP*-hard single machine total weighted tardiness scheduling problem with sequence-dependent setups has been faced by means of a new ACO approach. This problem is particularly relevant since it aims at minimizing the costs caused by violations of due dates and it takes into account the time possibly wasted for changing the type of production, which are both important aspects in modern manufacturing. Therefore, such problem represents also a challenging combinatorial optimization problem to experiment the effectiveness of the proposed ACO approach.

The algorithm presented in the paper includes several new features: the most relevant one corresponds to the pheromone learning model based on a new type of asymptotic pheromone trails and a new global pheromone update mechanism (UPU).

The main novelties in ACO_{AP} algorithm are to make the pheromone trails, which can be thought of as a sort of proxy attributes measuring the utility of including a component in high quality solutions, independent of the objective (or quality) function of the specific problem or instance considered, and the introduction of a new UPU rule for the global pheromone update step, which makes the pheromone trails smoothly range between a lower and an upper bound only asymptotically reached. Differently from previous GPU rules, the UPU one does not increase the pheromone values of components on the basis of the absolute or relative objective function values associated with the (best) explored solutions, but on the basis of the persistence, iteration after iteration, of such components in the best solutions. In addition, ACO_{AP} includes an intra-iteration diversification mechanism based on a stronger LPU rule than in standard ACS approaches, and a RLPU feature allowing to reset the perturbation in pheromone trails induced by the LPU. Other ACO_{AP} additional features that seemed sensible to experiment in order to face the STWTSDS problem were (a) the progressive decrease of the importance of the heuristic value β already introduced in Merkle et al. (2002) to reduce a possible bias in the system learning mechanism, and (b) the use of the UPU rule with a CBS component solution set, since in a weighted tardiness scheduling context, if the algorithm learns that a certain sequence position could be the right one for a job (due to its urgency), it appears appropriate to reinforce its attitude to consider that job even more urgent for successive positions.

The effectiveness of the new ACO approach has been analysed through an extended experimental campaign on the benchmark instance set generated by Cicirello (2003), and it has been highlighted by the comparison with the recent ACO algorithm presented in Liao and Juan (2007) as well as the set of up-to-date best known results. In addition, the robustness of the proposed algorithm has been verified even testing it on a different STWT problem benchmark available from ORLIB. However, the results collected showed that the RLPU feature appears fundamental, whereas the progressive reduction of β as well as the use of CBS component solution set did not appreciably affect the algorithm performance for the considered benchmark. Particular attention has been paid to the importance of the algorithm intensification phase, implemented by a LS procedure, with respect to the ACO learning mechanism. A comparison of the results produced in Test 4 with the ones of Test 2, even taking into account the outcomes of Test 3, can suggest some remarks. LS or iterated LS algorithms certainly have an important role as intensification mechanisms in metaheuristics like ACO for combinatorial optimization, but they must be considered only a component of these ones. From an opposite standpoint, learning mechanisms, as the one present in ACO, can drive powerful LS

algorithms to deeply explore particular promising areas in the solution space. The cooperation between learning and intensification finally appears a decisive factor to design algorithms able to provide high quality results in an acceptable computation time. The improvement of such cooperation, as well as the study of more effective pheromone models to reduce the need of extended intensification phases, should represent possible future theoretical developments of the proposed approach.

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