

## Sequence-Dependent Setup Times in a Two-Machine Job-Shop with Minimizing the Schedule Length

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**Abstract**—This article addresses the job-shop problem of minimizing the schedule length (makespan) for processing  $n$  jobs on two machines with sequence-dependent setup times and removal times. The processing of each job includes at most two operations that have to be non-preemptive. Machine routes may differ from job to job. If all setup and removal times are equal to zero, this problem is polynomially solvable via Jackson's permutations, otherwise it is *NP*-hard even if each of  $n$  jobs consists of one operation on the same machine. We present sufficient conditions when Jackson's permutations may be used for solving the two-machine job-shop problem with sequence-dependent setup times and removal times. For the general case of this problem, the results obtained provide polynomial lower and upper bounds for the makespan which are used in a branch-and-bound algorithm. Computational experiments show that an exact solution for this problem may be obtained in a suitable time for  $n \leq 280$ . We also develop a heuristic algorithm and present a worst case analysis.

**Keywords**—Scheduling theory, Setup, Job-shop

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