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Cycle Time Optimization of a Multi-Product Production Line in a Multiple Objective Setting

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Abstract**—**Multi-objective optimization techniques have made inroad into various aspects of managerial decision making process such as investment, environmental analysis of major projects, etc. The field of operations management, in particular production management, has been somewhat behind in utilizing such techniques. In this paper, we consider a production line that produces several predetermined batches of products. Bi-objective optimization as a form of multi-objective optimization has been utilized in this paper. The aim is the optimization of two conflicting objectives by minimizing simultaneously the cycle time and the buffer sizes of the production line. We utilize the concept of satisfaction function for the purpose of aggregating these two incommensurable objectives. Numerical examples are presented to demonstrate the application of the model and the steps involved in the modeling of the problem.

Keywords**—**Production line, Capacity, Buffer size, Compromise programming, Satisfaction function

1. INTRODUCTION

The early optimization models on production lines consider only a single objective to optimize the line's capacity. Dallery and Gershwin (1992), De Koster (1989), and Abdul-Kader (1997) provide coverage of such issues and include adequate list of references on these subjects. The objective is throughput or efficiency maximization or cycle time minimization. Buffer contribution is also a major issue in such models. Due to the trade-off nature of buffer size and production line capacity, almost all these single-objective models consider buffer size as a parameter. As the trade-off between the buffer size and the production line capacity has an asymptotic trend, researchers use trial and error or experimental design methods to determine the best cycle time without increasing unduly the buffer size; see Abdul-Kader and Gharbi (2002).

Figure 1 presents the typical tradeoff between the cycle time and the buffer size; see Abdul-Kader and Gharbi (2002). The graph in Figure 1 is compatible with the non-concavity rule of Conway et al. (1988), which states that throughput increases in a non-concave manner when successive buffers are placed optimally. Very small buffer sizes (the left of line A, see Figure 1) result in unacceptably large cycle times; and very large buffer sizes (the right of line B) have insignificant amount of contribution in lowering the cycle time. The challenge for production managers is to decide on a point between lines A and B.

Table 1 presents a summary of the reviewed papers that tackled the production line capacity improvement problem. In column 3, the objective to be maximized or minimized is given. Column 4 shows if the model offers an analytic or approximation solution methodology. The last column indicates the role of buffer in the model. In addition of being single-objective optimization the above listed papers do not assist the decision-maker to make a choice on the cycle time-buffer size tradeoff line.

Through the establishment of a bi-objective model, this research aims to assist the manager to select the best compromise between these two conflicting objectives of minimizing the cycle time and the buffer size. It is good to mention that there is no optimal solution that can optimize simultaneously these two objectives. In our modeling approach, the decision-maker needs to make some trade-off regarding the achievement of his/her objectives, hence the decision-making process can be seen as an evolution towards the best recommendation and it is far away from the traditional single objective optimization approach. Without the loss of generality, the model presented for the case of two objectives can be applied to cases when more than two objectives are under consideration.

Multi-objective optimization techniques have made inroad into various aspects of managerial decision making process such as finance (Anvary Rostamy et al. (2003)), personnel management (Hoffman et al. (2004)), environmental protection (Mustajoki et al. (2004)), etc. The field of operations management in particular production

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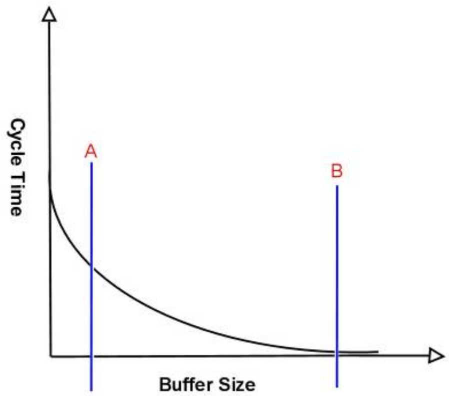

Figure 1. Cycle time−buffer size tradeoff.

Table 1. Earlier single-objective focused models

Year	Author	Performance Measures or single objective	Model	Buffer
1967	Buzacott	Efficiency as a function of buffer size	Analytic	Parameter
1977	Ignall & Silver	Avg. hourly output	Analytic Approximation	Parameter
1987	Johri	Cycle time	Linear Programming	Small, Parameter
1987	Iyama & Ito	Max. production rate	Analytic Approximation	Parameter
1987	Ancelin & Semery	Production rate	Analytic Approximation	Parameter
1995	Burman	Throughput	Approximation based on CTMC	Parameter
2001	Patchong & Wilayes	Production rate	Analytic Approximation	Parameter
2002	Abdul-Kader & Gharbi	Cycle time	Simulation	Parameter
2005	Colledani et al.	Throughput Average buffer level	Analytic Approximation	Parameter
2006	Abdul-Kader	Cycle time	Analytic Approximation	Parameter

management has been somewhat behind in utilizing such techniques. In this paper, we consider a production line that produces several predetermined batches of products and apply the technique of Compromise Programming (CP) as a tool to improve the capacity of a multi-product production line that is described in next paragraphs. For a discussion on CP, the readers are referred to the original work by Zeleny (1974). The CP model attempts to minimize the deviation between the ideal points or targets of each objective and the proposed solution. There are several ways to establish a target level for an objective. In traditional GP models the target levels of objective are set by the decision maker. In CP such targets are determined through single objective optimization. These targets, in our opinion, are more suitable to aim for that is why we considered utilizing CP approach. Since the objectives are commonly incommensurable so are their individual deviations. There are different ways to aggregate incommensurable objectives (or their deviations) among them, the use of satisfaction functions as proposed by Martel and Aouni (1990) is a widely accepted method.

As shown in Table 1, Johri (1987) presented a linear programming model for optimizing the cycle time of a production setting to process several products on a multi-stage production system subject to random failures. The effect of failures was implicitly incorporated into the model through the reduction of the average capacity of various stages of the production system. The importance of Johri's model on our work rests on the fact that we utilize two of its constraints in our model. We shall elaborate on this point in section 2.

The production line under study is composed of a set of work stations connected in series. The different products are processed in batches of predetermined size and all products start at the first station and go through all the subsequent stations according to a predefined sequence, before exiting from the last station of the production line. A typical example of such setting is the processing of several batches of different computer chips in the same facility. The processing times as well as the set-up times are assumed to be deterministic. Another assumption is that the first station has always raw material to work on, so it is never starved, and the last station has always space to unload its finished products and consequently, it is never blocked. There are c product types that go through all stations in a predetermined sequence. One buffer space is assumed to fit one unit of any type of products. The conflicting objectives are, as indicated above, the cycle time and the size of the intermediate buffers.

Starvation and blocking of work stations are a concern as they do affect the production capacity. Variation in processing times of the same products on two consecutive work stations as well as the variation in processing times of two consecutive product types on the same station may contribute to blocking and starvation. Buffers of semi-finished products located between the stations are aimed at reducing the impact of such variation on the production line's capacity. However, since the introduction of such buffers is not free of cost, one would prefer to keep this buffer size at the lowest possible level. The two earlier mentioned constraints are functions of the buffer sizes and are aimed at reducing starvation and blocking.

The remainder of this paper is organized in 5 sections where section 2 presents the notation used in this paper as well as the two constraints. Section 3 presents the methodology of work and the bi-objective model. Section 4 provides examples to demonstrate how the model can be used. The paper concludes with comments about the relevance of the approach as well as suggestions for further research.

2. NOTATION AND TWO ADOPTED CONSTRAINTS

The indices, parameters and variables used in the model are:

i = an index representing a station, $i = 1$ to s

j = an index representing a product type, $j = 1$ to c

k = an index representing the number of product types (or batches) before batch j in the sequence where k may take any value from 0 to $c - 1$. This index is utilized in the two adopted constraints

α = a parameter in a satisfaction function indicating the size of deviation from an ideal point

b_i = the size of the buffer between stations i and ($i + 1$). It is expressed in terms of the number of units of products it may hold. If the level of b_i is not predetermined, it would be considered as a decision variable as it is the case in the proposed model

δ = a variable indicating the size of deviation from an ideal point

I = a binary variable used to identify a selected segment in a satisfaction function as shown in Figure 2.

n_j = a predetermined number representing the batch size of product type j

$P_{i,j}$ = a deterministic processing time of product j in station i

$St_{i,j}$ = a deterministic set-up time of station i to produce product type j

$W_{i,j,k}$ = the time taken by station i while processing batch j and k previous batches to fill buffer i. W is a function of b_i. Smaller buffer sizes lead to smaller W. W is one of the elements in the first adopted constraint

$Y_{i,j,k}$ = the time taken by station i processing batch j and k previous batches to empty buffer ($i - 1$). Y is a function of b_i. Smaller buffer sizes lead to smaller Y. Y is one of the elements in the second adopted constraint

$d_{i,j}$ = a decision variable representing the time allotted to product j in station i. This time at least, equals the set up time plus the processing time

T = a decision variable representing the cycle time of the production line. T is the time spent to process all the batches on a work station that requires the longest time for their completion. It should be noted that some batches may take their shortest time on this work station in that sense this work station is not a bottleneck for that specific product. However, it may be considered to be the bottleneck for the entire run of all batches.

The two constraints are presented as expressions (1) and (2). We shall provide a brief review of these expressions. For detailed explanation, the readers are referred to Johri (1987). One should pay attention that expression (1) or (2) is not a single expression, it is rather a general form for a large number of expressions as determined by the three indices of W or Y. Expression (1) is called the input-side constraints. It states that the total time dedicated to batches ($j - k$) to (j) in station i should be greater than or equal the total time in the previous station minus the time required to filling the intermediate buffer plus the processing time for batch j and the set up time of batch ($j - k$) on station i.

$$\sum_{r=0}^{k} d_{i,j-r} \geq \sum_{r=0}^{k} d_{i-1,j-r} - W_{i-1,j,k} + P_{i,j} + St_{i,j-k},$$

for $i = 2$ to s; $j = 1$ to c; and $k = 0$ to $c - 1$ (1)

Expression (2) is called the output-side constraints. It states that the total time dedicated to batches ($j - k$) to (j) in station i should be greater than or equal the total time in the next station minus the time to empty the intermediate buffer plus the processing time for batch ($j - k$) on station i minus the set up time of batch ($j - k$) on the downstream station.

$$\sum_{r=0}^{k} d_{i,j-r} \geq \sum_{r=0}^{k} d_{i+1,j-r} - Y_{i+1,j,k} + P_{i,j-k} - St_{i+1,j-k},$$

for $i = 1$ to $s - 1$; $j = 1$ to c; and $k = 0$ to $c - 1$ (2)

Considering the variables $W_{i,j,k}$ and $Y_{i,j,k}$, if the intermediate buffer has a size smaller than or equal to the batch size n_j of product j, then only batch j or a fraction of it is adequate to fill or empty the buffer.

In order to show the relation between $W_{i,j,k}$ and/or $Y_{i,j,k}$ and b_i, we present the development of new expressions for these two quantities. By manipulating expression (1) we find the expression for W to be:

$$W_{i-1,j,k} = \sum_{r=j-k}^{j-k+\eta-1} (d_{i-1,r} + \gamma_{i-1,j,k}(d_{i-1,j-k+\eta} - St_{i-1,j-k+\eta}) + St_{i-1,j-k+\eta})$$

where $\gamma_{i-1,j,k} = b_{i-1} - \sum_{r=j-k}^{j-k+\eta-1} n_r \Big/ n_{j-k+\eta}$ is the fraction of the buffer space that is unfilled by products $(j - k + \eta - 1)$. The integer η is determined such that: $\sum_{r=j-k}^{j-k-\eta} n_r \geq b_i$ and $\sum_{r=j-k}^{j-k-\eta-1} n_r < b_i$.

In general, the expression for $Y_{i+1,j,k}$ is determined in a similar manner as to W. The net result of such manipulation converts expressions (1) and (2) to expressions (3) and (4) respectively:

$$\sum_{r=0}^{k} d_{i,j-r} \geq \sum_{r=0}^{k} d_{i-1,j-r} + P_{i,j} + St_{i,j-k} - \sum_{r=j-k}^{j-k-\eta-1} d_{i-1,r} - \left(\frac{b_{i-1} - \sum_{r=j-k}^{j-k-\eta-1} n_r}{n_{j-k+\eta}} \right) \times (d_{i-1,j-k+\eta} - St_{i-1,j-k+\eta}) - St_{i-1,j-k+\eta}$$

for $i = 2, 3, ..., s$; $j = 1, 2, ..., c$; and $k = 0, 1, ..., c - 1$ (3)

$$\sum_{r=0}^{k} d_{i,j-r} \geq \sum_{r=0}^{k} d_{i+1,j-r} - \sum_{r=j-\eta+1}^{j} d_{i+1,r} - \left(\frac{b_i - \sum_{r=j-\eta+1}^{j} n_r}{n_{j-\eta}} \right) \times (d_{i+1,j-\eta} - St_{i+1,j-\eta}) + P_{i,j-k} - St_{i+1,j-k}$$

for i =1, 2, ..., $s - 1$; $j = 1, 2, ..., c$; and $k = 0, 1,... , c - 1$ (4)

The above expressions (3) and (4) will be used in our proposed bi-objective model. The following section presents the methodology and the steps involved in modeling the proposed bi-objective model.

3. PROPOSED BI-OBJECTIVE MODEL

As stated earlier, the conflicting objective functions are the cycle time and the buffer size. The aim of the model is to minimize the deviations from the predefined targets for each objective function. As indicated earlier and in order to overcome the issue of incommensurability of multiple objectives, the concept of satisfaction function developed by Martel and Aouni (1990) and Martel and Aouni (1996) is used. To summarize the approach in formulating the model, we give all the steps involved as follows: 1. Setting targets for single objectives, 2. Formulation of deviation constraints, 3. Selecting satisfaction functions, 4. Using satisfaction functions, and 5. Formulating the complete model.

3.1 Setting targets for the single objectives

Although in some cases the target value for the objectives may be set by the decision-maker, the most common method of setting target values is done by individual optimization of each objective function. The single objective functions to be optimized (minimized) in the proposed model are the cycle time T and the sum of buffer sizes b_i, for $i = 1$ to $(s - 1)$.

It is intuitive to set the minimum value of the buffer size at zero. The minimization of the cycle time would lead to the smallest possible cycle time and a very large buffer size. We shall utilize notations T^* and b^* to refer to the target values of the cycle time and buffer size respectively.

3.2 Formulation of the deviation constraints

Positive and negative deviations from T^* are represented with δ_T^+ and δ_T^- respectively. Similarly, positive and negative deviations from b^* are represented with δ_b^+ and δ_b^- respectively. Hence, the following restrictions must be maintained.

$$T = T^* + \delta_T^+ - \delta_T^- \quad (5)$$

$$\sum_{i=1}^{s-1} b_i = b^* + \delta_b^+ - \delta_b^- \quad (6)$$

It should be noted that in the final solution at least one of the two deviations δ^+ and δ^- would be zero, and if we are lucky both become zero. In the case of CP, unlike the general GP model, the values of T^* and b^* are determined by single-objective optimization. Since T^* and b^* are the minimal values of the single objectives, the negative deviations δ_T^- and δ_b^- must be zero or the solution would be infeasible.

3.3 Satisfaction functions' selection

The satisfaction function is a relation that converts the size of the deviations to a number between zero and one. This number reflects the decision-maker's satisfaction with such deviation. While the concept was originally developed for Goal Programming (GP), it may equally be utilized in

CP. The literature discusses six different types of satisfaction functions that cover the majority of decision scenarios and personalities (Martel and Aouni (1990)). The most appealing function to us is type V where the small deviations do not result in any loss of satisfaction, however beyond a certain point, the satisfaction starts decreasing linearly with the increasing deviation until it becomes zero. Very large values of deviations are considered completely unacceptable and hard constraints are used to eliminate the possibility of very large deviations. Such deviation level is known as the veto level. Figure 2 depicts the general shape of this type of satisfaction function. Parameters α_1, α_2, and α_3 which define the location and slope of the decreasing function would be provided by the decision-maker.

Since the main objective is to minimize T and sum of b_i, the focus of the model would then be on reducing the positive deviations from the target values, or maximizing the satisfaction drawn from making these positive deviations as small as possible.

3.4 Using satisfaction functions

This step involves the creation of constraints related to the satisfaction functions as well as the creation of the objective function in terms of the satisfaction. To incorporate the satisfaction for each positive deviation δ_T^+ and δ_b^+, we have to introduce three binary variables, I_{1T}, I_{2T}, I_{3T}, and I_{1b}, I_{2b}, I_{3b}, and introduce four additional constraints. Since only one of the three binary variables would be 1, then the other two are zero. These constraints determine one of the three ranges where the positive deviation would be located in. The expressions (7) and (8) present these constraints for the cycle time and for the buffer size:

$$\begin{cases} I_{1T} + I_{2T} + I_{3T} = 1 \\ \alpha_{1T} \times I_{2T} + \alpha_{2T} \times I_{3T} \le \delta_T^+ \\ \alpha_{1T} \times I_{1T} + \alpha_{2T} \times I_{2T} + \alpha_{3T} \times I_{3T} \ge \delta_T^+ \\ \delta_T^+ \le \alpha_{3T} \end{cases} \tag{7}$$

$$\begin{cases} I_{1b} + I_{2b} + I_{3b} = 1 \\ \alpha_{1b} \times I_{2b} + \alpha_{2b} \times I_{3b} \le \delta_b^+ \\ \alpha_{1b} \times I_{1b} + \alpha_{2b} \times I_{2b} + \alpha_{3b} \times I_{3b} \ge \delta_b^+ \\ \delta_b^+ \le \alpha_{3b} \end{cases} \tag{8}$$

The amount of the satisfaction drawn from the cycle time and buffer deviations would be given in (9) and (10):

$$I_{1T} + \frac{I_{2T} \times (\alpha_{2T} - \delta_T^+)}{(\alpha_{2T} - \alpha_{1T})} \tag{9}$$

$$I_{1b} + \frac{I_{2b} \times (\alpha_{2b} - \delta_b^+)}{(\alpha_{2b} - \alpha_{1b})} \tag{10}$$

Hence, the objective function would be the maximization of the sum of satisfactions as shown in Eq. (11).

$$Max\ Z = I_{1T} + \frac{I_{2T} \times (\alpha_{2T} - \delta_T^+)}{(\alpha_{2T} - \alpha_{1T})} + I_{1b} + \frac{I_{2b} \times (\alpha_{2b} - \delta_b^+)}{(\alpha_{2b} - \alpha_{1b})} \tag{11}$$

3.5 Formulation of the complete model

The complete model will have an objective function as described in Eq. (11). The set of constraints is composed of the constraints that describe the physical system (production line under study) and the constraints that are related to the satisfaction functions as described in section 3.4 above.

In addition to the physical constraints (3) and (4) presented in section 2, two more sets of constraints are added to account for the duration of the time it takes to process product j in station i, and for the determination of the cycle time. These two sets of constraints are given below:

Production Time Constraints: $d_{i,j} \ge P_{i,j} n_j + St_{i,j}$;
for $i = 1$ to s, and $j = 1$ to c (12)

Cycle Time Constraints: $T \ge \sum_{j=1}^{c} d_{i,j}$; for $i = 1$ to s (13)

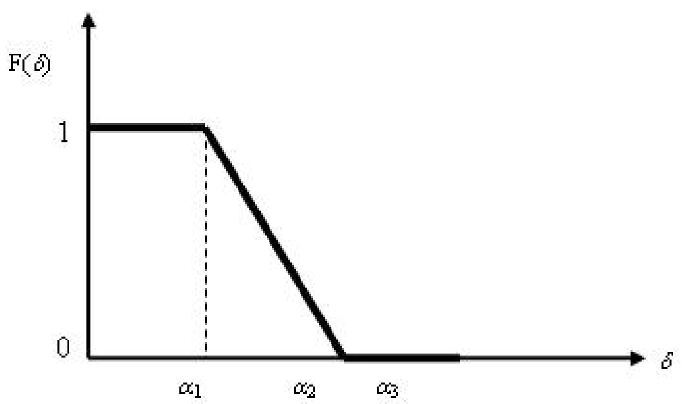

Figure 2. Type V satisfaction function.

The "greater than" sign in Eq. (12) assures that the time dedicated to product *j* in station *i* is not only enough for set-up and processing but also for any potential blocking and starvation that may occur while product *j* is processed in station *i*.

The complete form of the bi-objective model would then be composed of Eq. (11) as Objective Function and the constraints are the following Eqs. (3) to (8), and Eqs. (12) and (13).

In order to demonstrate the application of this model and its relevance in terms of the satisfaction of the decision-maker, numerical examples using data taken from a published work is presented in the following section.

4. NUMERICAL EXAMPLES

In this section we present two numerical examples. The first example is small in size and is used to demonstrate development of different parts of the model. The larger example demonstrates the utility of the model without taking too much space to develop the large number of constraints that are associated with this example.

Example 1. 2-machine one buffer production line

To demonstrate how the above model is used, we present its use on a numerical example taken from Johri's (1987) example #1 (2-machine 2-product production line) where the parameters of the system are:

n_1 = 60 units; $St_{1,1}$ = 300 seconds; $p_{1,1}$ = 80 seconds; $St_{2,1}$ = 200 seconds; $p_{2,1}$ = 20 seconds; n_2 = 75 units; $St_{1,2}$ = 300 seconds; $p_{1,2}$ = 40 seconds; $St_{2,2}$ = 200 seconds; $p_{2,2}$ = 100 seconds.

4.1 Single-objective function targets' setting

The single-objective minimization problem of the cycle time produces a T^* = 9100 seconds. As expected, the corresponding value of the buffer size was extremely high (277,480 units). The target value of the buffer was intuitively set at zero (b^* = 0). This value of b^* = 0, corresponds to a cycle time T = 12,800 seconds. The next section explains how deviations from target values are formulated.

4.2 Formulation of deviation constraints

As indicated earlier, the positive deviations are denoted with δ^+. Using Eq. (5) and (6) and as per the results given in section 4.1, the two constraints expressing our objective variables in terms of their target values and deviations from the target values are respectively:

$$T = 9100 + \delta_T^+$$
$$b_1 = 0 + \delta_b^+$$

The next step in our methodology is the selection of the satisfaction function, which is described below.

4.3 Selecting satisfaction functions

Earlier, we explained the reasons for choosing type V satisfaction function. At this point, we need to obtain three parameters for each satisfaction function in a way to represent the decision-maker's attitude towards the size of deviations. One of the co-authors played the role of the decision maker and provided the parameters that suited his attitude towards deviations. Table 2 below presents the three parameters of the satisfaction function for the deviations δ_T^+ and δ_b^+ as presented by our decision-maker. One must take notice that the values presented in Table 2 are the reflection of one person's attitude towards deviations of a given magnitude. One may expect different values for these parameters if a different decision-maker is involved.

4.4 Using satisfaction functions

Since type V satisfaction function has three distinct segments, we need to introduce three binary (0−1) variables for each deviation. For the first deviation, δ_T^+, we use the constraints as per Eq. (7), and for the second deviation, δ_b^+, constraints are added as per Eq. (8). These are presented respectively below:

$$\begin{cases} I_{1T} + I_{2T} + I_{3T} = 1 \\ 600 \times I_{2T} + 2400 \times I_{3T} \le \delta_T^+ \\ 600 \times I_{1T} + 2400 \times I_{2T} + 3000 \times I_{3T} \ge \delta_T^+ \\ \delta_T^+ \le 3000 \end{cases}$$

$$\begin{cases} I_{1b} + I_{2b} + I_{3b} = 1 \\ 10 \times I_{2b} + 25 \times I_{3b} \le \delta_b^+ \\ 10 \times I_{1b} + 25 \times I_{2b} + 30 \times I_{3b} \ge \delta_b^+ \\ \delta_b^+ \le 30 \end{cases}$$

The objective is to maximize the sum of the satisfaction derived from δ_T^+ and δ_b^+. For the type V satisfaction function, the general expression for the satisfaction derived from any deviation of size δ^+ would be:

$$I_1 + \frac{I_2 \times (\alpha_2 - \delta^+)}{(\alpha_2 - \alpha_1)}$$

Hence, the satisfaction from δ_T^+ is (see Eq. (9)):

$$I_{1T} + \frac{I_{2T} \times (2400 - \delta_T^+)}{(2400 - 600)},$$

and the satisfaction from δ_b^+ is (see Eq. (10)):

$$I_{1b} + \frac{I_{2b} \times (25 - \delta_b^+)}{(25 - 10)}$$

Table 2. Parameters of two satisfaction functions

	α_1	α_2	α_3
Parameters of δ_T^+	600	2400	3000
Parameters of δ_b^+	10	25	30

Consequently, the objective function would be (as per Eq. (11)):

$$Max\ Z = I_{1T} + \frac{I_{2T} \times (2400 - \delta_T^+)}{1800} + I_{1b} + \frac{I_{2b} \times (25 - \delta_b^+)}{15}$$

The last step of the methodology is to create the complete model as outlined in (14) below.

4.5 Formulating the complete model

Once we add the constraints describing the physical aspect of the production line, the model for our numerical example would be completed. This model is presented in (14).

By solving the above model, we obtain the following solution: δ_T^+ = 2820 seconds (i.e., T = 9100 + 2820 or 11920 seconds) with zero satisfaction and δ_b^+ = 10 (i.e. b_1 = 0 + 10 units) with a satisfaction of 1. Hence, the total satisfaction at the optimal level is 1 (0 + 1 = 1) for a cycle time, T = 11,920 seconds and a buffer size = 10 units.

Comparing this solution to the single-objective solution presented by Johri, we can make the following comments. The choice of a buffer of size 6 in Johri's (1987) model was quite arbitrary. There was no discussion as to "how satisfied" the decision maker would be with regard to the level of this variable and hence there could no question be raised regarding the value of T = 12240 seconds. Based on the parameters of satisfaction functions presented in Table 2, we are assured that the decision maker prefers the solution of T = 11920 and b_1 = 10 to the solution of T = 12240 and b_1 = 6.

In the above example because we had only one buffer location, the sum of b_i, in effect, was b_1. The generality of the formulation is maintained for longer production lines with more than one buffer locations; one may attempt to maximize the satisfaction related to the deviation from the target level for the sum of the buffers' sizes. Example 2 below presents such a case.

Example 2. 5-machine 4-buffer production line

To extend the applicability of the method to longer production lines, this example presents the case of a production line composed of 5 machines and 4 buffers and processing three different products. The three parameters of the satisfaction function for the deviations δ_T^+ and δ_b^+ are those presented in Table 2 earlier. Tables 3 and 4 below, present the processing times of the three products and the set-up times of the five machines that compose the production line. The batch sizes of products 1, 2 and 3 are 70, 85, and 100 units respectively.

$$\begin{aligned}
& Max\ \ Z = I_{1T} + I_{2T} \times \frac{(2400-\delta_T^+)}{1800} \\
& \qquad + I_{1b} + I_{2b} \times \frac{(25-\delta_b^+)}{15} \\
& \text{Subject to:} \\
& d_{1,1} \ge n_1 p_{1,1} + St_{1,1} \\
& d_{1,2} \ge n_2 p_{1,2} + St_{1,2} \\
& d_{2,1} \ge n_1 p_{2,1} + St_{2,1} \\
& d_{2,2} \ge n_2 p_{2,2} + St_{2,2} \\
& T \ge d_{1,1} + d_{1,2} \\
& T \ge d_{2,1} + d_{2,2} \\
& d_{2,1} + \frac{b_1(d_{1,1} - St_{1,1})}{n_1} + St_{1,1} \ge d_{1,1} + p_{2,1} + St_{2,1} \\
& d_{2,2} + \frac{b_1(d_{1,2} - St_{1,2})}{n_2} + St_{1,2} \ge d_{1,2} + p_{2,2} + St_{2,2} \\
& d_{2,1} + d_{2,2} + \frac{b_1(d_{1,1} - St_{1,1})}{n_1} + St_{1,1} \\
& \ge d_{1,1} + d_{1,2} + p_{2,2} + St_{2,1} \\
& d_{1,1} \ge d_{2,1} + p_{1,1} - \frac{b_1(d_{2,1} - St_{2,1})}{n_1} - St_{2,1} \\
& d_{1,2} \ge d_{2,2} + p_{1,2} - \frac{b_1(d_{2,2} - St_{2,2})}{n_2} - St_{2,2} \\
& d_{1,1} + d_{1,2} \ge d_{2,1} + d_{2,2} + p_{1,1} \\
& \qquad - St_{2,1} - \frac{b_1(d_{2,2} - St_{2,2})}{n_2} \\
& T = 9100 + \delta_T^+ \\
& I_{1T} + I_{2T} + I_{3T} = 1 \\
& 600 I_{2T} + 2400 I_{3T} \le \delta_T^+ \\
& 600 I_{1T} + 2400 I_{2T} + 3000 I_{3T} \ge \delta_T^+ \\
& \delta_T^+ \le 3000 \\
& b_1 = 0 + \delta_b^+ \\
& I_{1b} + I_{2b} + I_{3b} = 1 \\
& 10 I_{2b} + 25 I_{3b} \le \delta_b^+ \\
& 10 I_{1b} + 25 I_{2b} + 30 I_{3b} \ge \delta_b^+ \\
& \delta_b^+ \le 30
\end{aligned} \qquad (14)$$

The formulation of five machine problem is very much like the one with two machines with two exceptions. First, the minimization of buffer includes the sum of four buffers. Second, expansion of constraints (3) and (4) takes

Table 3. Processing times in seconds, Abdul-Kader and Gharbi (2002)

MACHINE	PRODUCT		
	1	2	3
1	70	55	69
2	50	64	52
3	55	75	84
4	78	69	53
5	80	68	87

Table 4. Set-up times in seconds, Abdul-Kader and Gharbi (2002)

MACHINE	PRODUCT		
	1	2	3
1	280	280	280
2	210	210	210
3	230	230	230
4	310	310	310
5	320	320	320

much larger number of constraints than the one shown in model (14) for the case of two machines. The target cycle time value is T = 20,800 seconds for a total buffer sizes of more than 20 million units. The target buffer sizes are zero for a cycle time of 25×10^9 seconds. Solving the bi-objective mixed linear integer model by following the same steps as shown in section 3, we get a satisfaction value of 1.995 (maximum is 2.0), T = 21,612.10 seconds (or 20,800 + 812.10) and the total value for the 4 buffers is 15 units, i.e., 0 + 15 (or $b_1 = 3$, $b_2 = 4$, $b_3 = 4$, and $b_4 = 4$ units).

5. CONCLUSION

Whereas the techniques of multi-objective optimization have made substantial inroad to various decision-making areas, the progress in the field of operations and production management has not been that deep. In this paper, we presented a bi-criteria optimization model for a production line composed of several work stations and a variety of products. The conflicting objectives were the cycle time and the size of buffers between the work stations composing the production line. The application of the model was demonstrated by adopting a published problem and utilized its input data in our first numerical example. In this example, we formulated the model in the context of the given parameters, and then solved it.

As the decision situations are rarely single objective, the contribution of this work is the presentation of a more realistic (i.e. multi-objective) formulation of production systems' problems in general and the specific production line's problem, described in this paper, in particular. In reference to Figure 1, the decision-maker can, in a systematic manner, indicate his/her preferences and the model generates the optimal combination of cycle time and buffer size.

We conclude this paper by stating that there is an excellent potential to expand the use of multi-objective modeling techniques in other areas traditionally under the control of an operations manager (goods or services). For example, the number of servers in queueing models affects the cost of providing service as well as the level of service. Since the customer satisfaction and the cost of service are incommensurable, satisfaction functions would be a potential tool to deal with the problem of incommensurability.

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