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The Optimal Price and Period Control of Complete Pre-Ordered Merchandise Supply

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Abstract—This study addresses the problem of complete pre-ordered merchandise supply. We discuss the sale price and the supply period with the demand rate function of merchandise for customers and the ratio function of customers' willingness to wait for taking merchandise, wherein the supply period and the sale price at any point of time during the complete stock-out period are decision variables. The purpose of this study intends to seek the optimal supply period and the optimal price function at any point of time during the complete stock-out period for maximizing the unit time profit of complete pre-ordered store. The finding is for the periodical supply model of complete pre-ordered merchandise, the price which is a constant in the basic EOQ model is no longer an assumed condition, but a necessary condition for the optimal solution under the special assumptions of demand rate function and ratio function. Furthermore, for the members of decision-making team in a complete pre-ordered store, the outcomes of this study provide a guideline for the optimal supply opportunity, pricing strategies, promotion activities and sale practice of complete pre-ordered merchandise.

Keywords—Inventory management, Optimization, Pricing strategy, Complete pre-ordered, Price variability.

1. INTRODUCTION

The basic EOQ model was proposed by Harris and Wilson in 1915 to determine the optimal ordering period and quantity at a fixed demand and non stock-out allowance. Many different models on optimal supply quantity have been proposed ever since and they are derived from the basic EOQ model by loosening some of the assumptions (Montgomery, Bazaraa and Keswani (1973); Caine and Plaut (1976); Silver (1981); Tinarelli (1983); Maxwell and Muckstadt (1985) etc.). The model of emancipation from "non stock-out allowance" to "stock-out allowance" is the problem frequently found in many inventory systems. For various situations of stock-out allowance during the stock-out period, different supply models can be structured, but most solutions considered only one of the following assumptions when making such models in the past: **(1)** All demands during the stock-out period are backordered. **(2)** All demands during the stock-out period are lost forever. **(3)** A fraction b of demand is backordered and the remaining fraction $(1-b)$ is lost forever, where $0<b<1$.

Obviously, assumption **(3)** is an extension of assumption **(1)** and assumption **(2)**, aiming to examine the making of the optimal inventory policy when the customers are informed that the merchandise will be taken later and a fraction b of the customers who would wait for the merchandise. Although assumption **(3)** is more flexible for the procession of allowable stock-out inventory cycle problems, Chen and Wu (1995) considered that the procedure for processing the fraction of customers who are willing to defer delivery during a stock-out period in the model is still too simplified and the assumption must be improved. They suggest that customer's pre-order ratio in stock-out period be a function of the waiting time length for taking merchandise. That is, the longer the waiting time length for taking merchandise is; the lower the pre-order ratio of customer's willingness is. Therefore, they modified the fraction b into b_t in their model for obtaining the optimal supply opportunity and quantity of complete pre-ordered merchandise, where b_t is a function of the waiting time length for taking merchandise.

A complete stock-out refers to a situation where a store has no stock on hand to satisfy its customer's demand. Chen and Wu (1995) defined the kind of ordering behavior of customers during the complete stock-out period as complete pre-ordered, the merchandise as complete pre-ordered merchandise (CPM), and the store as a complete pre-ordered store (CPS). A CPS, as proposed in their model, is characterized by the following:

(1) When customers are willing to buy a product, the supplier will inform them that the merchandise can only be available later.

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(2) When customers express their willingness of purchasing merchandise that cannot be obtained immediately, they will inquire the waiting time length for obtaining the merchandise.

(3) Customers will make a purchase decision based on a basis of the waiting time length for the merchandise to be available.

Basically, the sale price and demand quantity at each point of time during the whole stock-out period are fixed according to the assumptions of aforementioned characteristics of a CPS. At this circumstance, a customer's willingness of pre-order at a CPS is relevant to the waiting time length for taking merchandise only. The earlier the pre-order is, the longer the waiting time length before obtaining merchandise is. Hence, the lower the willingness of a customer's pre-order is. In this study, a variable price function is considered, that is, sale price levels vary with a time factor. One cannot overlook the influence that the psychological factor of the expected price fluctuation will have on the customers when price levels may differ with time and the time of pre-order is far from the time for taking merchandise. The main difference of this study from those discussed above is that we take into account the customer's expectation of price variability to propose a mathematical model describing the optimal price at each point of time during a stock-out period. Wherein, the willingness of pre-ordering merchandise for customers is relevant to the price level and price variability at that time, and the waiting time length for taking merchandise is also a pivotal factor for making decisions on placing an order.

Complete stock-out is a common marketing problem for products such as heavily promoted new products, seasonal goods, or those that are time-critical. In order to be able to control inventory levels or to reach sales goals, companies often use pre-order sales strategies. Quantity control is very important when the merchandise is specific and with an expiration date (vaccines, for example). It is because a too high or too low inventory (or production) level will cause losses. In addition, companies providing non-time critical merchandise may utilize a similar strategy to reach sales targets earlier or to control production quantity. These businesses may be a travel agency planning a tour, a cram school scheduling a new course, a construction company promoting housing sale, a newly established firm, a magazine publisher issuing a new issue, or sales of seasonal and holiday-related products.

For a CPS, it could have different pricing at different time in order to convert the customer's consumer surplus into corporate profits during a complete pre-ordered period. In the hopes of more customers to order during a stock-out period, businesses might adopt marketing strategies that offer the substantial pricing privilege or free gifts with purchasing to accumulate quantities or raise funds. The purpose of using such marketing strategies is to meet company's profit targets. These strategies generally set a critical point, which may be a point of time or a total pre-ordered quantity. Stores offer promotions to customers who pre-order prior to the critical point (of time or quantity) and rescind offers after that point. This kind of promotion strategy does help to increase customers' willingness to place an order. However, this pricing practice brings unnecessary disputes and the determination of the critical point is debatable. Here, we consider a continuous price function of time.

According to the assumptions of the static demand function, customers' willingness to buy will be stronger at a lower price level. However, when the price level is a function of time, the price level and the price variability at a given time will affect customers' needs of the goods simultaneously. Chen and Chen (1998, 1999) stated that price levels vary with time, and so does the internal reference price (IRP) of customer for merchandise. At a specified time, the price level may seem high. Because of price variability, if the price level is expected to rise, customer's IRP will go up. Consequently, customers will take action to make a purchase at the price level that seems acceptable at that time. On the other hand, customer's IRP will be low if the price is expected to reduce, customers will not purchase the goods at that time. The characteristic of CPM is that the consumers are not able to enjoy the satisfaction for taking merchandise immediately after pre-ordering and each customer's anticipation on price variability is not the same during the stock-out period. Hence, when the price level of a CPM is different at different time, the purchase behavior of customers is affected not only by the price level, but also by the price variability at that time. Herein, we consider the demand rate as a function of price level and price variability as well to describe the potential demand rate at that time for customers.

This study focuses on the decision making of a consumer during a stock-out period on pre-ordering of CPM at specific time that is influenced by both "price level and price variability at that time" and "the waiting time length for obtaining the desirable merchandise." Moreover, the time for supplying the merchandise in a CPS is cyclic and the length of the cycle is a decision variable in this study. The merchandise is supplied at a specified time and is pre-ordered during the stock-out period before supplying. However, the optimal supply period of merchandise has to be determined in advance. Therefore, the purpose of this study is to construct a mathematical model and to determine the optimal price function at any point of time and the optimal supply period of merchandise for optimizing the unit time profit of the CPS in the specified period.

2. NOTATIONS AND ASSUMPTIONS

2.1 Notations

We study the pricing strategy and period control of a CPM that is being pre-ordered all the time and supplied at a fixed time. The notations, definitions of parameters and decision variables are as follows.

$p(t)$: The price level at time t during the stock-out period for merchandise when customers place an order. This is a decision function.

$p'(t)$: The price variability of merchandise at time t .

T : The length of each supply period. This is a decision variable. A CPS supplying merchandise at time T , $2T$, $3T$,…, iT .

In this study, we consider one supply period $(0\,,T]$. That is, if a pre-order is placed at any point of time t during the period $(0\,,T]$, one must wait for a time length $T-t$ to receive the merchandise.

c : Unit cost of goods procured in lot.

k : Set-up cost, a fixed cost occurred when a CPS replenishes stocks in lot.

$D(p(t),p'(t))$: The potential demand rate function of merchandise for customer at time t during the period $(0,T]$. It is a function of both $p(t)$ and $p'(t)$.

$\theta(T-t)$: The ratio function of customers who are willing to pre-order at time t during the period $(0,T]$ after considering the waiting time length $T-t$.

$Q(t)$: The pre-ordered quantity of the CPM at time t during the period $(0,T]$.

2.2 Basic assumptions

1. The potential demand rate function D at time t is assumed to be a linear function of the $p(t)$ and $p'(t)$ as follows.

$$D(p(t),p'(t)) = A_1 - A_2 p(t) + A_3 p'(t)\,, \tag{2.1}$$

where A_1, A_2, A_3 are positive numbers and $A_1 - cA_2 = D(c,0) > 0$. The coefficient A_1 is the upper limit of demand, the coefficient A_2 is the impact rate of price upon demand, and the coefficient A_3 is the impact rate of price variability upon demand. The values of these parameters depend on the degree of customer's preference for merchandise.

The higher the price level $p(t)$ of merchandise at time t is, the lower the demand rate of merchandise for customer will be. Moreover, the positive or negative value of $p'(t)$ at time t for merchandise will affect customer's expectation of price rising or falling and then lead to a high or low demand rate (Chen and Chen (1998, 1999)). Therefore, the potential demand rate function has the following properties: $\dfrac{\partial D(p(t),p'(t))}{\partial p(t)} < 0$ and $\dfrac{\partial D(p(t),p'(t))}{\partial p'(t)} > 0$. When $p(t) \equiv c$, we assume an inequality $D(c,0) > 0$ (i.e., as long as the sale price of goods is set at unit cost c at all points of time, the demand rate is positive).

2. The ratio function θ at time t is assumed to be an exponential function of waiting time length $T-t$, i.e.,

$$\theta(T-t) = e^{-\lambda\cdot(T-t)}, \quad \forall 0 \le t \le T\,, \tag{2.2}$$

where λ is a constant, and $\lambda > 0$ represents the tolerance of potential customers who are willing to receive merchandise later (the greater the λ value is, the lower the tolerance is).

Customers contact a CPS at time t during the period $(0\,,T]$ showing their interest in purchasing, and they have to wait for a time length $T-t$ to obtain the merchandise at time T . Some customers may not pre-order because of the long waiting time length; others may not mind waiting for the merchandise. In general, if the waiting time length for taking merchandise increases, the ratio of customers' willingness to pre-order will decrease. That is, $\theta(T-t)$ is a decreasing function of $T-t$ and satisfies $0 \le \theta(T-t) \le 1$, $\forall\, t \in [0,T]$, $\theta(0) = 1$.

3. The pre-ordered quantity of the CPM at time t is $Q(t) = \theta(T-t)\cdot D(p(t),p'(t))$.

Based on the two assumptions above, when a customer enters a CPS at time t during the period $(0,T]$, the quantity of pre-ordering is influenced both by the demand rate function $D(p(t),p'(t))$ and the ratio function $\theta(T-t)$.

3. MODEL CONSTRUCTION AND SOLUTION

3.1 Model construction

Based on the aforementioned notations and basic assumptions, a customer pre-orders merchandise at time t and does not cancel the order, the total income and the total cost (including set-up cost) of a CPS during the period $[0\,,T]$ can be written:
$\int_0^T p(t)\cdot\theta(T-t)\cdot D(p(t),p'(t))\,dt$ and $\int_0^T c\cdot\theta(T-t)\cdot D(p(t),p'(t))\,dt+k$, respectively. The total profit is $\int_0^T \big(p(t)-c\big)\cdot\theta(T-t)\cdot D(p(t),p'(t))\,dt-k$.

The goal of a CPS is to determine the optimal price function $p^*(t)$ and the optimal supply period T^* for maximizing the unit time profit during the period $[0\,,T]$. The mathematical model is

$$\underset{T,p(t)}{Max}\ \frac{1}{T}\cdot\left\{\int_0^T \big(p(t)-c\big)\cdot\theta(T-t)\cdot D(p(t),p'(t))\,dt-k\right\}, \tag{3.1}$$

where T is a decision variable, and $p(t)$ is a decision function of t.

3.2 Model solution

In this study, we use two stages to explore the optimal solution of **Model (3.1)**. First, for the fixed T value, we find the price function $p_T^*(t)$ that is a function of T. Next, by inserting $p_T^*(t)$ into the target function of **Model (3.1)** to find the optimal solution T^* and the optimal solution $p^*(t)$.

Stage I:

For any given fixed T value, let $p_T^*(t)$ be the optimal solution of the following model:

$$\underset{p(t)}{Max}\ \frac{1}{T}\cdot\left\{\int_0^T \big(p(t)-c\big)\cdot\theta(T-t)\cdot D(p(t),p'(t))\,dt-k\right\}. \tag{3.2}$$

This is a problem of calculus of variations and $p_T^*(t)$ has to satisfy Euler equation (Kamien and Schwartz (1991)):

$$\frac{d}{dt}\left\{\big(p(t)-c\big)\cdot\theta(T-t)\cdot\frac{\partial}{\partial\, p'(t)}D(p(t),p'(t))\right\}$$
$$=\theta(T-t)\cdot D(p(t),p'(t))+\big(p(t)-c\big)\cdot\theta(T-t)\cdot\frac{\partial}{\partial\, p(t)}D(p(t),p'(t)). \tag{3.3}$$

By using Eq. (2.1), we simplify Eq. (3.3):

$$\frac{d}{dt}\left\{\ \big(A_3p(t)-cA_3\big)\cdot\theta(T-t)\ \right\}=\theta(T-t)\cdot\big[A_1+cA_2-2A_2p(t)+A_3p'(t)\big]. \tag{3.4}$$

From Eq. (3.4), by using Eq. (2.2), the optimal solution $p_T^*(t)$ is obtained:

$$p_T^*(t)\ =c+\frac{A_1-cA_2}{2A_2+\lambda A_3}\ ,\ \ \forall\ 0\le t\le T\ . \tag{3.5}$$

From Eq. (3.5), the optimal solution $p_T^*(t)$ is a constant function irrelevant to time t and period length T when the ratio function is an exponential function.

Stage II

Let $L(T)=\underset{p(t)}{Max}\ \frac{1}{T}\cdot\left\{\int_0^T \big(p(t)-c\big)\cdot\theta(T-t)\cdot D(p(t),p'(t))\,dt-k\right\}$ be a function of T , from Eq. (2.1), (2.2) and (3.5), we obtain

$$L(T) = \frac{(A_1 - cA_2)^2 \cdot (A_2 + \lambda A_3)}{(2A_2 + \lambda A_3)^2} \cdot \frac{1 - e^{-\lambda T}}{\lambda T} - \frac{k}{T}$$

and $$\frac{dL(T)}{dT} = \frac{(A_1 - cA_2)^2 \cdot (A_2 + \lambda A_3)}{\lambda T^2 \cdot (2A_2 + \lambda A_3)^2} \cdot \left\{ \frac{\lambda T + 1}{e^{\lambda T}} - \left[1 - \frac{\lambda k \cdot (2A_2 + \lambda A_3)^2}{(A_1 - cA_2)^2 \cdot (A_2 + \lambda A_3)} \right] \right\},$$

where

$$\frac{d}{dT}\left(\frac{\lambda T + 1}{e^{\lambda T}} \right) < 0, \ \forall\, T \geq 0; \ \lim_{T \to 0} \frac{\lambda T + 1}{e^{\lambda T}} = 1 \text{ and } \lim_{T \to \infty} \frac{\lambda T + 1}{e^{\lambda T}} = 0. \tag{3.6}$$

It is known from Eq. (3.6), the sufficient and necessary condition for the existence of the optimal solution T^* is

$$0 < \frac{\lambda k \cdot (2A_2 + \lambda A_3)^2}{(A_1 - cA_2)^2 \cdot (A_2 + \lambda A_3)} < 1. \tag{3.7}$$

And the optimal solution T^* satisfies the following equality

$$\frac{\lambda T^* + 1}{e^{\lambda T^*}} = 1 - \frac{\lambda k \cdot (2A_2 + \lambda A_3)^2}{(A_1 - cA_2)^2 \cdot (A_2 + \lambda A_3)}. \tag{3.8}$$

Hence, after the parameters ascertained and satisfy Eq. (3.7), the optimal supply period T^* of merchandise can be determined from Eq. (3.8), and the optimal price function is

$$p^*(t) = c + \frac{A_1 - cA_2}{2A_2 + \lambda A_3}, \ \forall\, 0 \leq t \leq T^*. \tag{3.9}$$

Now, the potential demand rate function under the optimal price at time t is

$$D(p^*(t), p^{*\prime}(t)) = \frac{(A_1 - cA_2) \cdot (A_2 + \lambda A_3)}{2A_2 + \lambda A_3}. \tag{3.10}$$

The optimal total quantity of pre-ordering is

$$Q^* = \frac{(A_1 - cA_2) \cdot (A_2 + \lambda A_3)}{2A_2 + \lambda A_3} \cdot \left\{ \frac{1}{\lambda} \cdot \left(1 - e^{-\lambda T^*} \right) \right\}, \tag{3.11}$$

where T^* is determined by Eq. (3.8) solely.

4. SENSITIVITY ANALYSIS AND IMPLICATIONS

The characteristic of supplying merchandise for a CPS is that it has no stock on hand to satisfy its customers at the time customers show their willingness to purchase during a time period $(0, T]$, and customers have to take the course of pre-ordering and wait for the CPS to supply the merchandise at time T. As known from Eq. (3.7), the optimal supply opportunity for a CPS depends on the inequality. For convenience, we let a criterion function CF as follows:

$$CF(c, k, A_1, A_2, A_3, \lambda) = \frac{\lambda k \cdot (2A_2 + \lambda A_3)^2}{(A_1 - cA_2)^2 \cdot (A_2 + \lambda A_3)}. \tag{4.1}$$

After the value of parameters is assessed, the function value of criterion function is obtained. When $CF \geq 1$ is established, it means that the optimal solution for the situation that decision makers faced is non-existent. Therefore, our model can only be applied to an inequality $0 < CF < 1$, so the optimal supply period T^* is determined by Eq. (3.8) solely. And then the optimal price $p^*(t)$ at each point of time is based on Eq. (3.9).

The criterion function CF, the optimal price function $p^*(t)$ and the optimal supply period T^* are all related to the values of the parameters c, k, A_1, A_2, A_3 and λ. Among these parameters, the parameters c and k are internal factors in CPS and the values can be precisely estimated according to the purchase practice; whereas the other parameters are external factors of CPS, and they are contingent upon the degree of customer's preference over the types of merchandise. We can estimate the values by doing marketing research or analyzing historical data. In this section, the criterion function CF is first analyzed with respect to the parameters. Next, a sensitivity analysis for the optimal solutions $p^*(t)$ and T^* is conducted and the related implications are specified, given the existence of the optimal solution. Finally, the demand rate function and the ratio function that assumed in this study are discussed with parameters.

4.1 Parameter analysis of the criterion function

To a CPS, the existence of the optimal supply opportunity depends on the value of the criterion function CF. However, the value of the criterion function is related to the values of parameters. We have the sensitivity analysis of criterion function to parameters under an inequality $A_1 - cA_2 > 0$, shown in Table 1.

Table 1. Sensitivity analysis of criterion function CF to parameters.

Criterion function \ Parameters	c	k	A_1	A_2	A_3	λ
CF	+	+	−	+	+	+

Note: "+" represents that the criterion function is an increasing function of parameters.
"−" represents that the criterion function is a decreasing function of parameters.

Table 1 shows that the value of criterion function decreases with the increment of the value of parameter A_1 and increases with the increment of the value of other parameters. The outcomes provide information for decision makers of the CPS to form strategies before starting pre-ordering. When parameters' information is assessed and known, one can find the value from Eq. (4.1). If $CF < 1$, then the optimal solutions exist, and if $CF \geq 1$, the decision maker should reassess immediately and try to lower the values of the internal parameters c or k. Then, using Eq. (3.8) and (3.9) to find the optimal supply period T^* and the optimal price function $p^*(t)$. After all these, a CPS can carry out pre-order.

4.2 Sensitivity analysis of the optimal solutions

Under the existence of the optimal solutions, the value of the optimal supply period T^* and the optimal price function $p^*(t)$ at time t during the period $(0, T^*]$ vary with the value variations of parameters c, k, A_1, A_2, A_3 and λ. For a fixed t, the sensitivity analyses of $p^*(t)$ and T^* to various parameters under the inequality $A_1 - cA_2 > 0$ are shown in Table 2.

Table 2. Sensitivity analyses of $p^*(t)$ and T^* to parameters.

Decision function or variable \ Parameters		c	k	A_1	A_2	A_3	λ
$p^*(t)$	1st order partial derivative	+	0	+	−	−	−
	2nd order partial derivative	0	0	0	+	+	+
T^*	1st order partial derivative	+	+	−	+	+	#

Note: "#" represents derivative contingency on the relative value of other parameters.

We conclude two properties from Table 2. First, $p^*(t)$ is irrelevant to the value of k and increases linearly with the increasing values of c and A_1 with slopes of $(A_2 + \lambda A_3)/(2A_2 + \lambda A_3)$ and $1/(2A_2 + \lambda A_3)$, respectively. $p^*(t)$ decreases concave up with the increasing values of A_2, A_3 and λ. Second, T^* increases with the increasing values of c, k, A_2 and A_3, respectively and decreases with the increasing value of A_1. Its decrease or increase with the increasing value of λ is decided by the relative value of other parameters.

The implications in management of the aforementioned properties are:

(1) Neither a too long or too short supply period of a CPS is acceptable in real world's practice; therefore, decision makers must reassess the values of the internal factors c or k within the organization for adjustment. In Table 2, the variation of T^* to parameters c or k provides the information for decision makers to make adjustments on the store's strategies. Taking into concern of consumers' perception toward the price, if the value of T^* is too large, the unit cost c of the merchandise should be lowered; on the other hand, the value of T^* is too small, the set-up cost k should be raised.

(2) The optimal price increases linearly with the unit cost. For every incremental unit, the optimal price should be raised $(A_2 + \lambda A_3)/(2A_2 + \lambda A_3)$.

(3) The external parameters for a CPS that we focus on are the influence of parameters A_3 and λ upon the decision function $p^*(t)$. When the value of A_3 or λ is getting greater, the optimal price level will go the opposite direction. From Eq. (3.10), the greater values of A_3 and λ help boost customer's potential demand rate for merchandise. Meanwhile, the greater value of A_3 will also promote the total quantity from Eq. (3.11). We also conclude that the ratio value of the change rate of $p^*(t)$ to A_3 and the change rate of $p^*(t)$ to λ is λ/A_3. For consumers, the question of which factor is important: "the impact of the influence degree of merchandise price variability upon the price" or "the impact of tolerance to wait for taking merchandise upon the price" lies on the relative ratio value of λ to A_3.

4.3 Parameter analysis of demand rate function and ratio function

The pricing strategies formed by CPS' decision makers are based on the demand rate function for merchandise and the ratio function for customers' willingness to wait for taking merchandise. The demand rate function that we considered is a linear function of the price level and price variability, and the ratio function is an exponential function of the waiting time length. In this section, we will analyze and discuss the optimal price function, the optimal potential demand rate function and the optimal total quantity when the values of A_2 and A_3 are extremely small. The ratio function of exponential form according to the value of λ will be discussed as well.

4.3.1 Parameter analysis of demand rate function

(1) When $A_2 \to 0^+$:

The potential demand rate for merchandise of customer at time t is nearly only affected by the price variability, not by the price level at that time. When the ratio function is assumed to be an exponential function, the optimal price function at time t is $p^*(t) = c + (1/\lambda)\cdot(A_1/A_3)$. This means that the optimal price function is a constant function irrelevant to time t of customer pre-ordering. Now, the optimal potential demand rate function is A_1, the upper limit of demand; the optimal total pre-ordered quantity is $Q^* = A_1\cdot(1-e^{-\lambda T^*})/\lambda$.

(2) When $A_3 \to 0^+$:

The potential demand rate for merchandise of customer at time t is nearly only affected by the price level, not by the price variability at that time. From Eq. (3.9), regardless that the form of the ratio function is, the optimal price function is $p^*(t) = (1/2)\cdot\left(c + A_1/A_2\right)$. This means that the optimal price function is the average of the unit cost and the highest sale price, and it is a constant function irrelevant to time t of customer pre-ordering. Now, the optimal potential demand rate function is $(1/2)\cdot(A_1 - cA_2)$, it is a half of the demand rate when the sale price is at unit cost. However, the optimal total pre-ordered quantity varies with the form of the ratio function. When the ratio function is assumed to be an exponential function, then the optimal total pre-ordered quantity $Q^* = (1/2)(A_1 - cA_2)\cdot(1-e^{-\lambda T^*})/\lambda$ is obtained.

To summarize the analyses above and to use the change rate of optimal potential demand rate function with respect to A_2 or A_3 from Eq. (3.10), we know that the optimal potential demand rate function must satisfy the inequality

$$(1/2)\cdot(A_1 - cA_2) \le D(p^*(t), p^{*\prime}(t)) \le A_1 . \tag{4.2}$$

This means that the optimal potential demand rate function is between a half of the demand rate when the sale price is at unit cost and the upper limit of demand. Moreover, the optimal total pre-ordered quantity must satisfy the inequality

$$(1/2)(A_1 - cA_2)\cdot(1-e^{-\lambda T^*})/\lambda \le Q^* \le A_1\cdot(1-e^{-\lambda T^*})/\lambda . \tag{4.3}$$

All of those provide the quantity control reference for decision maker in CPS.

4.3.2 Parameter Analysis of Ratio Function

We assume that the ratio function θ is an exponential function of the waiting time length $T-t$. $\theta(T-t)$ is a concave up decreasing function of $T-t$; that is, the ratio function decreases fast at first and then decreases steadily with the increasing waiting time length. That is a normal reaction for customers. Customers will purchase the desired merchandise immediately if they do not have to wait, but hesitate, otherwise. When the waiting time length is long, the impact is smaller for more or less a unit time to wait. Therefore, it is reasonable to describe the ratio of a customer's willingness to wait for taking merchandise with an exponential function. Besides, at a fixed time, the larger the value of λ

is; the smaller the value of the ratio function θ will be. That is, the greater the value λ becomes; the lower the customer tolerance of waiting for taking merchandise is. Consequently, the ratio of customers' willingness to pre-order is even lower. Meanwhile, it is known from Table 2 that the CPS shall offer low prices in order to increase the pre-ordered ratio when the value λ becomes large.

5. CONCLUSION

Under the assumptions in Section 2, the study provides the optimal supply period, the optimal price function, and the optimal total pre-ordered quantity for the merchandise to a CPS as shown in Eq. (3.8), (3.9), and (3.11). Besides, we conclude the results and the management implications as follows:

1. The optimal price function is a constant function irrelevant to the time of customer pre-ordering merchandise. This finding reveals, to maximize unit time profit, the optimal pricing strategy must eliminate customer's expectation toward the price variability. That is to maintain the optimal price as a constant. This result illustrates, under some assumed conditions, the assumption that prices of the basic EOQ model is a constant is no longer an assumed condition, but a necessary condition for the optimal solution.
2. The optimal price function is irrelevant to the supply period and the set-up cost. However, it varies with the value variations of the other parameters that a CPS confronts. This outcome illustrates that the pricing strategies of the optimal price of CPM shall focus internally on the unit cost of merchandise replenished in lot and externally on the customer's potential demand rate for merchandise and the tolerance for taking the merchandise.
3. The optimal pricing strategy for a CPS shall be "maintaining a consistent price all the time during the pre-ordered period for merchandise." This gives a theoretical basis to the reality that for many pre-ordered merchandise, the promotion strategies do not adopt the substantial pricing privilege strategy.
4. When the optimal supply period exists, it will be determined by Eq. (3.8) solely. When the optimal supply period does not exist or the optimal supply period is too long or too short, we may reassess and readjust the internal parameter values of a CPS for obtaining the optimal supply period.
5. Under the assumption that the demand rate function is a linear function of the price level and price variability, if the impact of the price level is insignificant, the optimal price function is related to the form of ratio function. If the impact of the price variability is very minor, the optimal price function is always irrelevant to the form of ratio function. Moreover, the range of the optimal total pre-ordered quantity is obtained from the analysis of the demand rate function with respect to parameters A_1 and A_2, and this information serves as a quantity control reference for a CPS buying in lot.

Altogether, the outcomes of this study provide the decision-making team of a CPS a guideline for the optimal supply opportunity, pricing strategy, promotion activities and sale practice of pre-ordered merchandise. Firstly, we suggest decision makers the optimal supply opportunity by assessing internal and external parameters. Secondly, the pricing decision makers can determine the optimal price function after relevant internal and external parameters are assessed and finalized. Thirdly, the pricing privilege strategy is not the best policy for marketing decision makers. Fourthly, under the control of the optimal price, those who make decision on when to replenish can find an optimal total pre-ordered quantity from a demand rate function. Lastly, once sale prices are set by using the optimal price function, sales associates at a CPS are able to quote the price immediately to the customers who express their needs to purchase goods to avoid confusing the customers.

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