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Deteriorating Inventory Model When Demand Depends on Advertisement and Stock Display

Nita H. Shah[1,*] **and Poonam Pandey**[2]

[1]Department of Mathematics, Gujarat University Ahmedabad Gujarat, India Corresponding Author: Dr. Nita H. Shah

[2]Globsyn Business School Campus 907/A, Uvarsad, Off NH-8, Dist:Gandhinagar, Gujarat Pin - 382422 India.

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Abstract—In this study, a mathematical model is developed to obtain optimal ordering policy of time dependent deteriorating item when demand rate is dependent on displayed stock level and frequency of advertisement through media. Shortages are not allowed. The objective is to minimize total cost. The significant features and the results are studied with the help of a numerical example. The effect of changes in the demand parameter, deterioration rate (α -constant deterioration, β- time dependent deterioration), rate of frequency of advertisements, stock dependent parameter and salvage parameter for deteriorated items on total cycle time, total cost and on procurement quantity is studied numerically.

Keyword—Lot–size, Time dependent deterioration, Advertisement frequency, Procurement quantity and total cost.

1. INTRODUCTION

Classical inventory Economic order quantity (EOQ) model is based on the assumption that an item in stock remains to its 100 % efficiency for infinite time. Actually items like volatile liquids, blood, X – ray plates, medicines, electronic components, fashion goods, fruits and vegetable etc. looses their utility after some time. The most of the most of the articles were based on the fact that these deteriorated units are complete loss to the inventory system. See review articles by Raafat (1991), Shah and Shah (2000), Goyal and Giri (2001).

These days' advertisements and display of products plays a vital role in attracting mass customer. The advertisement through electronic media, news paper, internet, using innovative ways of display of product in the show room is the best tool for the promotion of a product. This attracted researcher to analyze the inventory problem when demand depends on stock displayed. Refer to Baker and Urban (1988), Mandal and Phaujdar (1989), Datta and Pal (1990), Padmanabhan and Vrat (1996), Giri et al. (1996), Sarkar et al. (1997) etc. The effect of advertisement on the demand of the product is studied by Bhunia and Maiti (1997), Goyal and Gunasekaran (1995), Pal et al. (1996, 2006) developed an inventory model for deteriorating items by taking into account the impact of marketing strategies viz pricing, advertisement and the displayed stock level of the demand rate of the system. These articles optimize total cost or net profit per time unit of an inventory system.

Misra (1979 a) studied the inventory decisions under inflationary conditions for EOQ model. Misra (1979 b) derived an inflation model for the EOQ, in which the time value of money and different inflation rates were considered. Gurunani (1983) gave the economic analysis of inventory systems and claimed that the discounting effects on EOQ were substantial. Related articles are by Queyranne (1985), Roundy (1986), Federgruen et al. (1992), Shah et al. (2003, 2004)

In this paper, a mathematical inventory model for time dependent deteriorating items is developed by considering demand to be function of advertisement and the displayed stock level. Shortages are not allowed. The storage capacity of the inventory system is finite. The objective is to minimize total cost.

2. ASSUMPTIONS AND NOTATIONS

The Mathematical model is derived using the following assumptions and notations

2.1 Notations

T : Cycle time (decision variable)

* Corresponding author's e-mail: nitahshah@gmail.com, shahnitah@gmail.com

t_0 : time point when the stock level is S_1
t_1 : time point when the stock level is $S_0 (S_1 > S_0)$
C : purchase cost per unit item
h : inventory holding cost per unit per unit time
G : cost of advertisement
O : ordering cost per order
λ : rate of change of frequency of advertisement
γ : salvage parameter for deteriorated units
A : frequency of advertisement in the cycle
a : fixed demand
b : rate of change of demand
$q(t)$: inventory level at any instant of time t during cycle time
$\theta(t)$: $\theta(t) = \alpha\beta t^{\beta-t}$ (Weibull distribution)
where $\alpha > 0$, $\beta > 0$. Here $\beta > 1$ is considered which means deterioration increases with time.
S : order quantity per cycle
K :total cost per time unit.

2.2 Assumptions

1. The inventory system deals with a single item.
2. Replenishment rate is infinite.
3. Shortages are not allowed and lead time is zero or negligible.
4. The planning horizon is finite.
5. The deterioration rate of units in inventory follows the Weibull distribution function given by $\theta(t) = \alpha\beta t^{\beta-t}$ where $\alpha > 0$, $\beta > 0$. Here $\beta > 1$ is considered which means deterioration increases with time.
6. Deteriorated units can neither be repaired nor replaced during the cycle time.
7. The demand rate $R(A,q)$ is a function of the frequency of advertisement A and the displayed inventory level in the super mall functional form as

$$
\begin{aligned}
R(A,q) &= A^{\lambda}(a + bS_1) \text{ for } q > S_1 \\
&= A^{\lambda}(a + bq(t)) \text{ for } S_0 < q \leq S_1 \\
&= A^{\lambda}(a + bS_0) \text{ for } q > S_0
\end{aligned}
$$

where $a, b > 0$, $a > b$.

3. MATHEMATICAL MODEL

In this model, the cycle starts with on hand inventory level S at $t = 0$ after clearing shortages. Then inventory level depletes continuously and reaches to zero at $t = T$ due to demand and constant rate of deterioration of units. Meanwhile inventory reaches to S_1 at some time t_0 and it reaches to S_0 at some time t_1 as shown in the Figure 1.

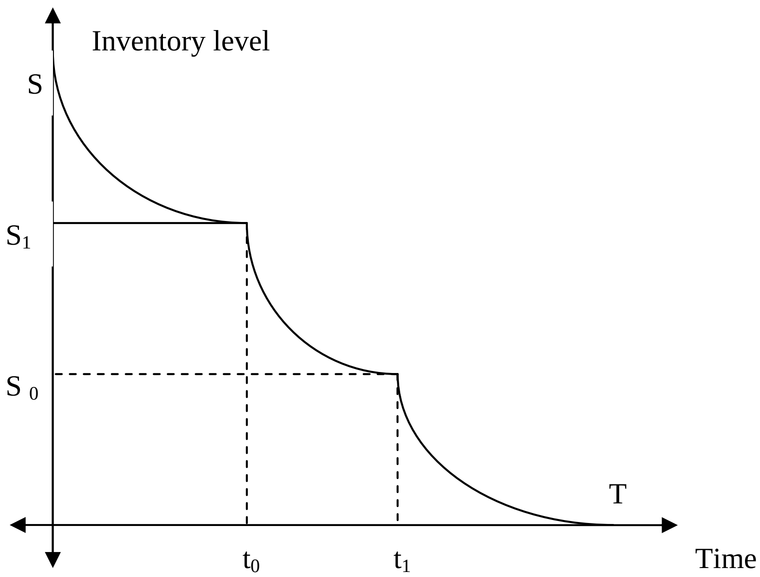

Figure 1. Time – inventory status.

The following differential equations represent the inventory level $q(t)$ at any instant of time t.

$$\frac{d\,q_1(t)}{dt}+\alpha\beta\,t^{\beta-1}\,q_1(t)=-A^{\lambda}\left(a+bS_1\right),\quad 0\le t\le t_0 \tag{1}$$

$$\frac{d\,q_2(t)}{dt}+\alpha\beta\,t^{\beta-1}\,q_2(t)=-A^{\lambda}\left(a+bq_2(t)\right),\quad t_0\le t\le t_1 \tag{2}$$

$$\frac{d\,q_3(t)}{dt}+\alpha\beta t^{\beta-1}\,q_3(t)=-A^{\lambda}\left(a+bS_0\right),\quad t_1\le t\le T \tag{3}$$

with boundary conditions $q_1(0)=S, q_2(t_0)=S_1, q_3(t_1)=S_0, q_3(T)=0$ (4)

The solution of differential Eq (1) – (3) using (4) is

$$q_1(t)=S\left(1-\alpha t^{\beta}\right)-A^{\lambda}\left(a+bS_1\right)\left(t-\frac{\alpha\beta t^{\beta+1}}{\beta+1}\right) \tag{5}$$

$$q_2(t)=S_1\left(1-\alpha\left(t^{\beta}-t_0^{\beta}\right)-bA^{\lambda}\left(t-t_0\right)\right)-aA^{\lambda}\begin{pmatrix} t-t_0-\dfrac{\alpha\beta}{\beta+1}\left(t^{\beta+1}-t_0^{\beta+1}\right)+\alpha t_0^{\beta}\left(t-1\right)+ \\ \alpha t_0\left(t^{\beta}-t_0^{\beta}\right)+bA^{\lambda}\left(tt_0-\dfrac{t^2}{2}-\dfrac{t_0^2}{2}\right)\end{pmatrix} \tag{6}$$

$$q_3(t)=S\left(1-\alpha\left(t^{\beta}-t_1^{\beta}\right)\right)-A^{\lambda}\left(a+bS_0\right)\left(t-t_1+\frac{\alpha}{\beta+1}\left(t^{\beta+1}-t_1^{\beta+1}\right)-\alpha t^{\beta}\left(t-t_1\right)\right) \tag{7}$$

Using boundary condition (4), we have

$$S=S_1(1+\alpha t_0^{\beta})+(a+bS_1)\left(t_0+\frac{\alpha t_0^{\beta+1}}{\beta+1}\right) \tag{8}$$

Now total inventory from (0, T) is

$$\text{IT}=\int_0^{t_0} q_1dt+\int_{t_0}^{t_1} q_2dt+\int_{t_1}^{T} q_3dt \tag{9}$$

Number of deteriorated during positive inventory time interval is

$$DU=S=A\lambda a-A\lambda b(S_1t_0+S_0(T-t_1))-A^{\lambda}b\int_{t_0}^{t_1} q_2(t)dt \tag{10}$$

Hence, total cost per time unit T is

$K(T)$ = (Inventory holding cost + Advertisement cost + ordering cost + cost due to Deterioration – salvage value of the deteriorated units) / T .

$$= \frac{1}{T}(b \times IT + A \times G + OC + C \times DU - C \times \gamma \times DU)$$

The necessary condition for $K(T)$ to be minimum is $\frac{\partial K(T)}{\partial T}$ =0 and solving it for T by a suitable mathematical software For obtained T , $K(T)$ is minimum only if $\frac{\partial^2 K}{\partial T^2}$ > 0 for all T > 0.

4. COMPUTATIONAL ALGORITHM

The following steps are to be completed for the optimal solution

Step 1: start with $A = 1$.

Step 2: compute $t_0, t_1, T, K(T)$.

Step 3: Increment A by 1.

Step 4: Perform step 2 until $K(A-1, t_0, t_1, T) \geq K(A, t_0, t_1, T) \leq K(A+1, t_0, t_1, T)$.

Step 5: stop.

5. NUMERICAL EXAMPLE

Consider an inventory system with following parameters in proper units :
$[O, h, C, G, \lambda, a, b, \gamma, \alpha, \beta, S_0, S_1] = [100, 1, 10, 100, 1.3, 200, 0.3, 0.20, 0.10, 3.5, 40, 150]$.

The optimum value of frequency A is 3, t_0 = 1.8980 years, t_1 = 2.5002 years, T = 3.6685 years, $K(T)$ = \$ 6072.22, S = 4479.04 units (see Figure 2). The sensitivity analysis is carried out to study the effect of deterioration of units; α and β , stock dependent parameter; b , rate of change of frequency; λ , and salvage parameter; γ of deteriorated units on the objective function are exhibited in following tables:

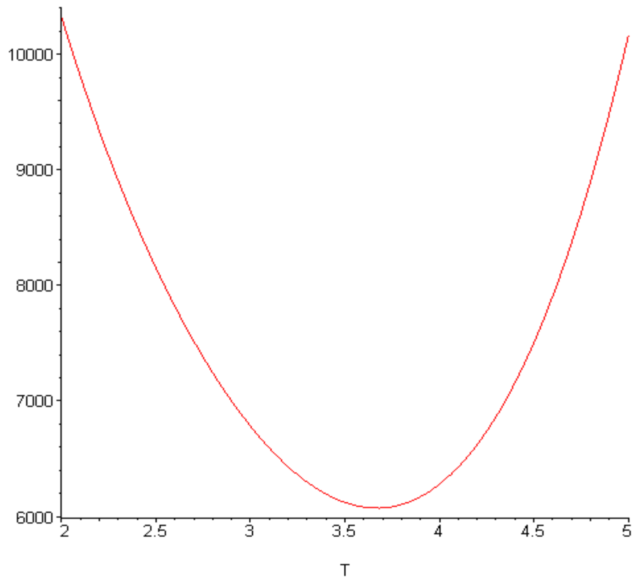

Figure 2. Convexity of $K(T)$.

Table 1. Effect of λ

λ	1.30	1.32	1.34	1.36
T	3.6685	3.6571	3.6460	3.6350
t_0	1.8980	1.8916	1.8854	1.8792
t_1	2.5002	2.4860	2.4720	2.4579
K	6072.22	6107.90	6144.48	6181.97
S	4479.04	4508.36	4538.68	4570.05

Table 2. Effect of α

α	0.10	0.12	0.18	0.20
T	3.6684	3.5444	3.2935	3.2335
t_0	1.8898	1.8423	1.7310	1.7048
t_1	2.500	2.4286	2.280	2.2434
K	6072.22	6276.40	6289.49	6997.95
S	4479.09	4511.09	4630.82	4672.89

Table 3. Effect of β

β	3.3	3.5	3.7	4.0
T	3.8777	3.6685	3.4887	3.2623
t_0	1.9590	1.8980	1.8448	1.7767
t_1	2.5808	2.5002	2.4299	2.3396
K	5873.83	6072.22	6270.42	6571.11
S	4531.17	4479.04	4445.010	4452.64

Table 4. Effect of γ

γ	0.10	0.20	0.30	0.40
T	3.7067	3.6684	3.6280	3.5855
t_0	1.8930	1.8980	1.9044	1.9128
t_1	2.4937	2.5002	2.5083	2.5185
K	6562.48	6072.21	5576.38	5074.64
S	4452.36	4479.04	4512.10	4554.08

Table 5. Effect of b

b	0.1	0.15	0.2
T	4.0012	3.8264	3.6684
t_0	2.2152	2.0419	1.8980
t_1	3.0462	2.8499	2.5002
K	8411.55	7211.87	6182.03
S	6617	5678	4236

Table 6. Effect of α & γ on decision variables

α \ γ		1.30	1.32	1.34
0.10	t_0	2.0940	2.0875	2.0810
	t_1	2.8579	2.8487	2.8394
	T	3.9900	3.9801	3.9035
	$K(T)$	5367.27	5367.27	5398.99
	S	3977.77	3988.51	3999.24
0.12	t_0	2.0371	2.0306	2.0242
	t_1	2.7665	2.7580	2.7493
	T	3.8263	3.8527	3.8430
	$K(T)$	5566.32	5583.09	5599.77
	S	4019.99	4031.17	4042.20
0.14	t_0	1.9921	1.9851	1.9787
	t_1	2.6913	2.6833	2.6752
	T	3.7594	3.7500	3.7405
	$K(T)$	5747.17	5764.93	5782.58
	S	4060.30	4071.89	4083.43

Table 7. Effect of β & λ on decision variables.

β \ λ		1.30	1.32	1.34
3.3	t_0	2.1640	2.1572	2.1505
	t_1	2.9605	2.9505	2.9405
	T	4.2170	4.2066	4.1963
	$K(T)$	5120.77	5137.61	5154.07
	S	3636.27	3979.37	4025.38
3.5	t_0	2.0940	2.0874	2.0810
	t_1	2.8579	2.8487	2.8394
	T	3.9900	3.9801	3.9703
	$K(T)$	5367.27	5383.14	5398.98
	S	3977.78	3988.51	3999.24
3.7	t_0	2.0333	2.0269	2.0207
	t_1	2.7674	2.7589	2.7503
	T	3.7950	3.7856	3.7763
	$K(T)$	5620.41	5635.24	5650.00
	S	4003.05	4012.70	4022.33

Table 8. Effect of λ & γ on decision variables

λ \ γ		1.10	1.20	1.30
1.30	t_0	2.0856	2.0940	2.1055
	t_1	2.8638	2.8579	2.8510
	T	4.0352	3.9900	3.9425
	$K(T)$	5851.71	5367.27	4876.15
	S	3999.82	3977.78	3952.49
1.32	t_0	2.0874	2.0792	2.0986
	t_1	2.8486	2.8541	2.8424
	T	3.9801	4.0251	3.9329
	$K(T)$	5866.88	5383.14	4892.69
	S	3988.51	5866.88	3965.31
1.34	t_0	2.0729	2.0810	2.1005
	t_1	2.8443	2.8394	2.8190
	T	4.0151	3.9703	3.8649
	$K(T)$	5882.06	5398.99	4909.18
	S	4017.60	3999.24	3923.97

Table 9. Effect of b & λ on decision variables

λ \ b		0.2	0.3	0.4
1.30	t_0	2.2179	2.0904	2.0395
	t_1	3.0407	2.8579	2.7270
	T	4.1316	3.9900	3.9116
	$K(T)$	6113.98	5367.27	4909.85
	S	4483.36	3977.78	3697.30
1.32	t_0	2.2086	2.0875	2.0340
	t_1	3.0328	2.8487	2.7174
	T	4.1206	3.9801	3.9024
	$K(T)$	6149.49	5383.14	4922.50
	S	4500.18	3988.50	3706.72
1.34	t_0	2.1995	2.0810	2.0284
	t_1	3.0248	2.8394	2.7078
	T	4.1099	3.9703	3.8931
	$K(T)$	6164.79	5398.99	4935.24
	S	4516.88	3999.24	3716.27

Table 10. Effect of β & b on decision variables

β \ b		0.2	0.3	0.4
3.3	t_0	2.2951	2.1640	2.1061
	t_1	3.1631	2.9605	2.8184
	T	4.3707	4.2170	4.1328
	$K(T)$	5835.49	5120.78	4689.10
	S	4474.55	3967.44	3694.35
3.5	t_0	2.2179	2.0940	2.0395
	t_1	3.0407	2.8579	2.7270
	T	4.1316	3.9900	3.9116
	$K(T)$	6113.98	5367.27	4909.85
	S	4483.36	3977.78	3679.30
3.7	t_0	2.1508	2.0333	1.9818
	t_1	2.9326	2.7674	2.6467
	T	3.9262	3.7950	3.7218
	$K(T)$	6398.40	5620.41	5136.53
	S	4506.91	4003.05	3715.25

Table 11. Effect of α & β on decision variables

α \ β		3.3	3.4	3.5
0.10	t_0	2.1640	2.0940	2.0331
	t_1	2.9605	2.8579	2.7674
	T	4.2170	3.9900	3.7950
	$K(T)$	5120.77	5367.27	5620.41
	S	3967.44	3977.78	4003.05
0.12	t_0	2.1004	2.0375	1.9821
	t_1	2.8509	2.7665	2.6829
	T	4.0732	3.8624	3.6808
	$K(T)$	5305.13	5566.32	5834.22
	S	3999.36	4019.99	4054.26
0.14	t_0	2.0495	1.9916	1.9410
	t_1	2.7792	2.6913	2.6131
	T	3.9574	3.7594	3.5884
	$K(T)$	5473.37	5747.17	6027.63
	S	4031.54	4060.30	4101.65

Table 12. Effect of α & γ on decision variables

α \ γ		0.10	0.20	0.30
0.10	t_0	2.0856	2.0940	2.1055
	t_1	2.8638	2.8579	2.8511
	T	4.0352	3.9900	3.9425
	$K(T)$	5851.82	3977.78	3952.49
	S	3999.82	3977.78	3952.49
0.12	t_0	2.0293	2.0371	2.0477
	t_1	2.7752	2.7665	2.7563
	T	3.9081	3.8624	3.8142
	$K(T)$	6088.03	5566.32	5037.93
	S	4054.83	4019.99	3980.22
0.14	t_0	1.9843	1.9916	2.0015
	t_1	2.7023	2.6913	2.6786
	T	3.8057	3.7549	3.7107
	$K(T)$	6303.66	5747.17	5184.24
	S	4106.71	4060.30	4007.61

Table 13. Effect of α & b on decision variables

α \ b		0.2	0.3	0.4
0.10	t_0	2.2179	2.0940	2.0395
	t_1	3.0407	2.8579	2.7270
	T	4.1316	3.990	3.9116
	$K(T)$	6113.98	5369.27	4909.85
	S	4483.36	3977.78	3697.30
0.12	t_0	2.1605	2.0371	1.9823
	t_1	2.9338	2.7664	2.5541
	T	3.9992	3.862	3.7860
	$K(T)$	6303.55	5566.32	5104.19
	S	4506.28	4019.99	3742.84
0.14	t_0	2.1144	1.9916	1.9367
	t_1	2.8460	2.6913	2.5772
	T	3.8920	3.7594	3.6848
	$K(T)$	6472.74	5747.17	5283.22
	S	4528.60	4060.30	3783.57

Table 14. Effect of β & γ on decision variables

β \ γ		0.10	0.20	0.30
3.3	t_0	2.1551	2.1640	2.1761
	t_1	2.9646	2.96050	2.9558
	T	4.2652	4.2170	2.9558
	$K(T)$	5571.69	5120.76	4663.26
	S	3981.24	3967.44	3951.56
3.5	t_0	2.0856	2.0940	2.1055
	t_1	2.8638	2.8579	2.8510
	T	4.0351	3.9900	3.9425
	$K(T)$	5851.71	5367.27	4876.15
	S	3999.82	3977.78	3952.49
3.7	t_0	2.0254	2.0330	2.0441
	t_1	2.7750	2.7674	2.758
	T	3.8375	3.7950	3.7501
	$K(T)$	6140.53	5620.41	5093.71
	S	3389.32	4003.05	3967.87

Table 15. Effect of γ & b on decision variables

γ \ b		0.2	0.3	0.4
0.10	t_0	2.1997	2.0856	2.0338
	t_1	3.0561	2.8638	2.7279
	T	4.1771	4.0352	3.9554
	$K(T)$	6716.63	5851.71	5332.03
	S	4548.55	3999.82	3700.35
0.20	t_0	2.2179	2.0940	2.0395
	t_1	3.0407	2.8579	2.7270
	T	4.1316	3.9900	3.9116
	$K(T)$	6113.98	5367.27	4909.85
	S	4483.36	3977.78	5498.80
0.30	t_0	2.2438	2.1055	2.0471
	t_1	3.0233	2.8510	2.7260
	T	4.0860	3.9425	3.8653
	$K(T)$	5504.87	4876.15	4481.65
	S	4411.22	3952.49	3693.83

Observations

As deterioration parameter α and β increases total cycle time decreases whereas total cost increases. Total cost and cycle time both decreases as salvage parameter γ increases. This is because there is some savings by selling deteriorated items instead of just throwing it away.

As stock displayed parameter b increases total cost and cycle time both decreases. The retailer will have to put orders frequently resulting increase in the total cost. Cycle time decreases whereas total cost increases as rate at which advertisement is displayed increases. All these are critical factors in deciding optimal ordering policy to minimize the objective function total cost.

This sensitivity analysis done here shows that stock dependent demand increases total cost decreases significantly. Frequent advertisement is going to increase in total cost but if salvage parameter increases significantly increment in total cost can be checked.

6. CONCLUSIONS

In market, the units deteriorate due to vaporization; damages while loading and unloading, perishablility and many other factors. So the deterioration effect should not be ignored while computing inventory cost of the retailer. For products like food grains, blood components, vegetables and fruits, pharmaceuticals etc. utility decreases with passage of time and hence retailer needs to find trade off among inventory carrying cost and deterioration cost. On the other hand, retailer uses media for the sale of the product through advertisement and displayed stock to attract the customers. This concept encourage author to develop propose model. This model can be extended to the incorporate selling price and advertisement dependent demand.

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REFERENCES

1. Baker, R. C. and Urban, L. A. (1988). Deterministic inventory system with an inventory level – dependent demand rate, *Journal of the Operational Research Society*, 29: 823 – 831.
2. Bhunia, A. K., Maiti, M. 1997. An inventory model for decaying items with selling price, frequency of advertisement and linearly time dependent demand with shortages. IAPQR Transactions, 22: 41 – 49.
3. Datta, T. K. and Pal, A. K. (1990). A note on an inventory model with inventory - level - dependent demand rate, *Journal of the Operational Research Society*, 41: 971 – 975.
4. Federgruen, A., Queryranne, M. and Zheng, Y. S. (1992). Simple power of two policies are close to optimal in general class of production / distribution networks with general joint setup costs, *Mathematics of Operations Research*, 17(4): 951 – 963.
5. Giri, B. C., Pal, S., Goswami, A., Chaudhuri, K. S. 1996. An inventory model for deteriorating items with stock – dependent demand rate. *European Journal of Operations Research,* 95: 604 – 610.
6. Goyal, S. K., Gunasekaran, A. 1995. An integrated production- inventory-marketing model for deteriorating items. *Computers and Industrial Engineering,* 28: 755 – 762.
7. Goyal, S. K. and Giri, B. C. (2001). Recent trends in modeling deteriorated inventory. *European Journal of Operational Research*, 134: 1 – 16.
8. Gurnani, C. (1983). Economic analysis of inventory systems, *International Journal of Production Research*, 21 (2): 261 – 277.
9. Mandal, B. N. and Phaujdar, S. (1989). An inventory model of deteriorating items and stock dependent consumption rate, *Journal of Operational Research Society*, 40: 483 – 488.
10. Misra, R. B. (1979 - a). A study of inflation effects of inventory system, Logistics *Spectrum*, 9: 260 – 268.
11. Misra, R. B. (1979 - b). A note on optimal inventory management under inflation, *Naval Research Logistics*, 26: 161 – 165.
12. Padmanabhan, G. and Vrat, P. (1995). EOQ models for perishable items under stock dependent selling rate, *European Journal of Operational Research*, 86: 281 – 292.
13. Pal, S., Goswami, A., Chaudhuri, K. S., Pal, A. K. (1996). An inventory model with two component demand rate and shortages. *Journal of the Operational Research Society,* 47: 1029 – 1036.
14. Pal, A. K., Bhunia, A. K. and Mukharjee , R. N. (2006). Optimal lot size model for deteriorating items with demand rate dependent on displayed stock level (DSL) and partial backordering. *European Journal of Operational Research,* 175: 977 – 991.
15. Queryanne, M. (1985). A polynomial - time sub-modular extension to Roundy's 98 % effective heuristic for production / inventory systems, working paper no.1136, Faculty of Commerce and Business Administration, University of British Columbia, Vancouver, B. C. Canada.

16. Raafat, F. (1991). Survey of literature on continuously deteriorating inventory models, *Journal of the Operational Research Society*, 40: 27 – 37.
17. Roundy, R. (1986). A 98 % - effective lot sizing rule a multi – product, multi – stage production – inventory system, *Mathematics of Operations Research*, 11: 699 – 727.
18. Sarker, B. R., Mukharjee, S. and Balan, C. V. (1997). An order – level of lot size inventory model with inventory level dependent demand and deterioration, *International Journal of Production Economics*, 48: 227 – 236.
19. Shah, Nita H., and Shah, Y. K. (2000). *Literature survey on inventory model for deteriorating items, Economic Annals*, XLIV: 221 – 237.
20. Shah, Nita H., Shah B. J. and Shah, Y. K. (2003). Present value formulation of economic lot size model for inventory system with variable deteriorating items, *Modeling Measurement and Control (France)*, 24(2): 17 – 28.
21. Shah, Nita H., Shah B. J. and Shah, Y. K. (2004). An order level lot size model with stock dependent demand for time dependent rate of deterioration of items under permissible delay in payments, *For East Journal of Theoretical Statistics,* 12(2): 201 – 220.