International Journal of
Operations Research

International Journal of Operations Research Vol. 7, No. 1. , 26-40 (2010)

The Optimal Cycle Time for EPQ Inventory Model of Deteriorating Items under Trade Credit Financing in the Fuzzy Sense

Gour Chandra Mahata[1,*] **and Anindya Goswami** [2]

[1]Department of Mathematics, Sitananda College, P.O. + P.S. – Nandigram, Dist. – Purba Medinipur, PIN – 721631, West Bengal, India.

[2]Department of Mathematics, Indian Institute of Technology, Kharagpur – 721302, India.

Received January 2010; *Revised May 2010*; *Accepted July 2010*

Abstract — Normally, the real-world inventory control problems are imprecisely defined and human interventions are often required to solve these decision-making problems. In this paper, a realistic inventory model with imprecise inventory costs have been formulated for deteriorating items under trade credit policy within the economic production quantity (EPQ) framework. We assume that the supplier would offer the retailer a delay period and the retailer also adopts the trade credit policy to stimulate his/her customer demand to develop the retailer's replenishment model for deteriorating items under the replenishment rate is finite. Under these conditions, we model the retailer's inventory system for deteriorating items as a cost minimization problem to determine the retailer's optimal inventory policy. We derive the expressions for the annual total inventory cost for the retailer both in the crisp and fuzzy sense. The total variable inventory cost in the fuzzy sense is defuzzified using Graded Mean Integration Representation method and it has been proved that there exists a unique optimal cycle time to minimize the annual total variable cost for the retailer. In addition, a theorem is developed to efficiently determine the optimal ordering policies for the retailer. For easy determination of optimal ordering policies, we have proposed three algorithms. Some previously published results of other authors will be special cases of this paper. Finally, numerical examples are used to illustrate all results obtained in this paper. Then, as well as, we obtain a lot of managerial insights from numerical examples.

Keywords — EPQ, Fuzzy inventory model, Graded mean integration, Fuzzy annual total variable cost, Fuzzy cost coefficients.

1. INTRODUCTION

The basic EOQ model assumes that the retailer's capitals are unrestricting and must be paid for the items as soon as the items are received. However, this may not be true. In today's business transactions, it is more and more common to see that suppliers usually offer some fixed time periods to the retailers in order to stimulate the demand for the products they produce. We term this period as trade credit period. Before the end of the trade credit period, the retailer can sell the goods and accumulate revenue and earn interest. A higher interest is charged if the payment is not settled at the end of the trade credit period. This brings some economic advantage to the retailers as they may earn some interest from the revenue realized during the period of permissible delay. In the real world, the supplier would allow a specified credit period (say, 30 days) to the retailer for payment without penalty to stimulate the demand of consumable products. This credit term in financial management is denoted as 'net 30'. The trade credit financing produces two benefits to the supplier: (1) it should attract new customers who consider it to be a type of price reduction; and (2) it should cause a reduction in sales outstanding, since some established customers will pay more promptly in order to take advantage of trade credit more frequently. In India, gas stations adopted a pricing policy that charged less money per gallon to the customer who paid by cash, instead of by a credit card. Likewise, a store owner in many China towns around the world usually charges a customer 5% more if the customer pays by a credit card, instead of by cash. As a result, the customer must decide which alternative to take when the supplier provides not only a cash discount but also a permissible delay.

One level trade credit financing refers that the supplier would offer the retailer trade credit but the retailer would not offer the trade credit to his/her customers. That is, the retailer could sell the goods and accumulate revenue and earn interest within the trade credit period but the customer would pay for the items as soon as the items are received from the retailer. Several interesting and relevant papers related to one level trade credit financing exist in the literature. Goyal (1985) first studied an EOQ model under the condition of permissible delay in payments. Chand and Ward (1987) analyzed Goyal's (1985) problem

* Corresponding author's email: gourmahata@yahoo.co.in

under assumptions of the classical EOQ model, obtaining different results. Chung (1989) presented the DCF (discounted cash flow) approach for the analysis of the optimal inventory policy in the presence of trade credit. Later, Shinn et al. (1996) extended Goyal's (1985) model and considered quantity discount for freight cost. Shah (1993), Aggarwal and Jaggi (1995) considered the inventory model with exponential deterioration rate under the condition of permissible delay in payments. Chu et al. (1998) and Chung et al. (2001) also extended Goyal's (1985) model for the case of deteriorating items. Liao et al. (2000) and Sarker et al. (2000) investigated this topic by introducing inflation rate. Jamal et al. (1997) and Chang and Dye (2001) extended this problem where they allowed shortages. Chung (2000) developed an alternative approach to modify Shah's (1993) solution procedure. Chang et al. (2001) extended this problem for the case of linear trend in demand. Shinn and Hwang (2003) determined the retailer's optimal price and order size simultaneously under the condition of order-size-dependent delay in payments. They assumed that the length of the credit period is a function of the retailer's order size, and also the demand rate is a function of the selling price. Arcelus et al. (2003) modelled the retailer's profit-maximizing retail promotion strategy, when confronted with a vendor's trade promotion offer of credit and/or price discount on the purchase of regular or perishable merchandises. Chung and Huang (2003) extended this problem within the EPQ framework and developed an efficient procedure to determine the retailers' optimal ordering policy. Chang et al. (2001) and Chung and Liao (2004) discussed the problem of determining the economic order quantity for exponentially deteriorating items under permissible delay in payments depending on the ordering quantity. Chang (2004) extended the model with inflation rate and finite time horizon. Huang (2004) investigated that the unit selling price and the unit purchasing price are not necessarily equal within the EPQ framework under supplier's trade credit policy.

All the aforementioned inventory models implicitly assumed one-level trade credit financing. But, in most business transactions, this assumption is unrealistic and usually the supplier offers a credit period to the retailer and the retailer, in turn, passes on this credit period to his/her customers. For example, in India, the TATA Company can delay the amount of purchasing cost until the end of the delay period offered by his supplier. The TATA Company also offers permissible delay payment period to his dealership. Recently, researchers developed inventory models under this two-level trade credit financing. Huang (2003) assumed that the retailer should also adopt the trade credit policy to stimulate his/her customer demand to develop the retailer's replenishment model. In most business transactions, this assumption is debatable. Furthermore, Huang (2003) assumed that the retailer's trade credit period offered by supplier M is not shorter than the customer's trade credit period offered by retailer N ($M \geq N$). The retailer cannot earn any interest in the situation, $M < N$.

Huang (2003) implicitly assume that the inventory level is depleted by customer's demand alone. This assumption is quite valid for nonperishable or non-deteriorating inventory items. However, there are numerous types of inventory whose utility does not remain constant over time. For example, volatile, liquids, medicines, materials, etc., in which the rate of deterioration is very large. In this case, inventory is depleted not only by customer's demand but also by the effect of deterioration. Therefore, the loss of items due to deterioration should not be neglected.

Another unrealistic assumption in Huang (2003) model is the infinite replenishment rate. Huang (2003) assumed that products obtained from an outside supplier and the entire lot size was delivered at the same time. In fact, when a product can be produced in-house, the replenishment rate is also the production rate, and is hence finite. So, we relax this assumption to finite replenishment rate. That is, the well-known economic production quantity (EPQ) framework. This viewpoint can be found in Chung and Huang (2003) model.

In the development of the inventory models discussed earlier, the researchers have assumed that the purchasing cost, the selling price, the holding cost and the set-up cost are constants. These kind of assumptions are not always true. It may not be possible to specify the values of these cost parameters precisely but they may contain some uncertain values such as "unit holding cost is about C_1", or "unit purchase cost is approximately C_p or more", etc. In other sense, these parameters may contain some uncertain values. For this reason, we consider in our inventory model the holding cost, purchase cost, selling price and ordering cost as fuzzy number.

The main purpose of this paper is to amend the paper of Huang (2003) and Chung and Huang (2003) with a view of making their model more relevant and so applicable to practice. Here, we propose a deteriorating inventory model with finite replenishment rate under the condition of permissible delay in payments in the fuzzy sense. We also assume that the supplier would offer the retailer a delay period and the retailer would also offer the trade credit period to his/her customer. Furthermore, the inventory costs namely holding cost, purchase cost, selling price and ordering cost may be flexible with some vagueness for their values. In real life situations, all these parameters in an inventory model are uncertain, imprecise and the determination of optimum cycle time becomes a non-stochastic vague decision making process. Again, for this type of models, statistical estimations proved to be inefficient because of the lack of statistical observations. In this situation, a suitable way to model these imprecise data is to use fuzzy sets. The ill-formed vagueness in the above parameters are introduced making them fuzzy in nature and then the model is formulated in a fuzzy environment. We use Graded Mean Integration Representation method for defuzzifying fuzzy total average cost. At first, this model shows that there exists a unique optimal cycle time to minimize the total variable cost per unit time. Then, a theorem is developed to determine the optimal ordering policies and consequently three algorithms have been developed. We deduce some previously published results of other authors as special cases. Finally, the theorem and the algorithms are illustrated with the help of numerical examples.

2. PRELIMINARIES

For achieving computational efficiency, we use the method of defuzzification of a generalized triangular fuzzy number by its graded mean integration representation. In 1998, Chen and Hsieh introduced Graded Mean Integration Representation method based on the integral value of graded mean h-level of generalized fuzzy number for defuzzifying generalized fuzzy number. Here, we first describe generalized fuzzy number as follows:

A generalized fuzzy number $\tilde{A}$ is described as any fuzzy subset of the real line R, whose membership function $m_{\tilde{A}}(x)$ satisfies the following conditions:

i) $m_{\tilde{A}}(x)$ is continuous mapping from R to the closed interval [0, 1],

ii) $m_{\tilde{A}}(x) = 0 \quad -\infty < x \le a,$

iii) $m_{\tilde{A}}(x) = L(x)$ is strictly increasing on [a, b],

iv) $m_{\tilde{A}}(x) = w \quad b \le x \le c,$ where $0 < w \le 1;$

v) $m_{\tilde{A}}(x) = L(x)$ is strictly increasing on [c, d],

vi) $m_{\tilde{A}}(x) = 0 \quad d \le x < \infty.$

Here a, b, c and d are real numbers. We denote this type of generalized fuzzy number as $\tilde{A} = (a,b,c,d;w)_{LR}$. When $w = 1$, it can be simplified as $\tilde{A} = (a,b,c,d)_{LR}$. Let L^{-1} and R^{-1} be the inverse functions of the functions L and R respectively, then the graded mean h-level value of generalized fuzzy number $\tilde{A} = (a,b,c,d;w)_{LR}$ is $h(L^{-1}(h) + R^{-1}(h))/2$ (see figure 1). The Graded Mean Integration Representation of $\tilde{A}$ is $P(\tilde{A})$ with grade w, where

$$P(\tilde{A}) = \int_0^w h\frac{L^{-1}(h) + R^{-1}(h)}{2}dh \Big/ \int_0^w hdh, \quad \text{with } 0 < h \le w \text{ and } 0 < w \le 1. \tag{1}$$

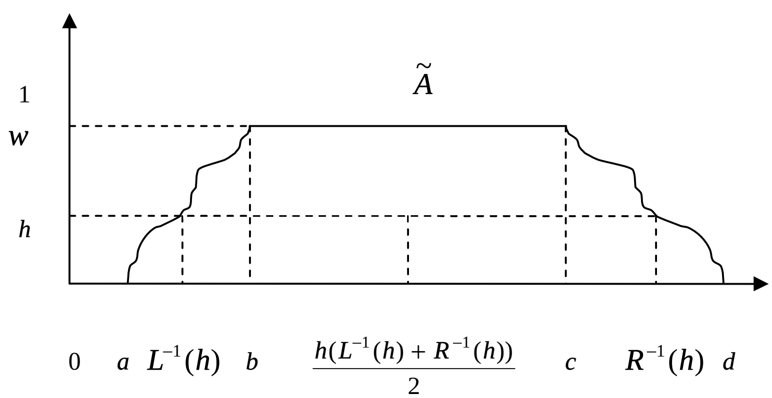

Figure 1: The graded mean h-level value of generalized fuzzy number $\tilde{A} = (a,b,c,d;w)_{LR}$

The generalized triangular fuzzy number (GTFN) $\tilde{B}$ is a special case of generalized fuzzy number, and be denoted as $\tilde{B} = (a,b,c;w)$. Its corresponding graded mean integration representation is

$$P(\tilde{B}) = \frac{\int_0^w h\{a + (b-a)h/w + c - (c-b)h/w\}/2dh}{\int_0^w hdh} = \frac{a + 4b + c}{6} \tag{2}$$

Remark: By formula (2), it is easy to observe that the graded mean integration representation of the GTFN $\tilde{B} = (a,b,c;w)$ is independent of w.

When w = 1, it can be simplified as $\tilde{B} = (a,b,c)$. The defuzzification of $\tilde{B} = (a,b,c)$ can be found by centroid or graded mean integration method. The centroid of the TFN $\tilde{B} = (a,b,c)$ is $C(\tilde{B}) = \frac{a+b+c}{3}$ and the graded mean integration of $\tilde{B} = (a,b,c)$ is $P(\tilde{B}) = \frac{a+4b+c}{6}$. The mid-point of the interval [a, c] is $M = \frac{a+b}{2}$. Thus

$$C(\tilde{B}) - P(\tilde{B}) = \frac{1}{3}(M-b), \quad P(\tilde{B}) - b = \frac{1}{3}(M-b), \quad \text{and} \quad M - C(\tilde{B}) = \frac{1}{3}(M-b).$$

(i) If $M > b$, then $a < b < P(\tilde{B}) < C(\tilde{B}) < M < c.$

(ii) If $M < b$, then $a < M < C(\tilde{B}) < P(\tilde{B}) < b < c.$

(iii) If $M = b$, then $a < M = C(\tilde{B}) = P(\tilde{B}) = b < c.$

From (i) and (ii), it is clear that $P(\tilde{B})$ is near *b*, and $C(\tilde{B})$ is near *M*. From figure 2 and 3, we observe that the membership grade of $\tilde{B}$ at *b* is 1 and the membership grade of $\tilde{B}$ at $P(\tilde{B})$ is greater than that at $C(\tilde{B})$. Then, we have $m_{\tilde{B}}(P(\tilde{B})) > m_{\tilde{B}}(C(\tilde{B}))$.

Property 1: From the membership grade viewpoint, it will be efficient to defuzzify the fuzzy number $\tilde{B}$ by $P(\tilde{B})$ instead of $C(\tilde{B})$.

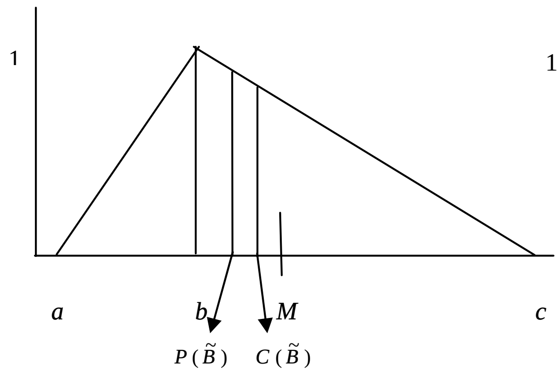

Figure 2: Case $M > b$

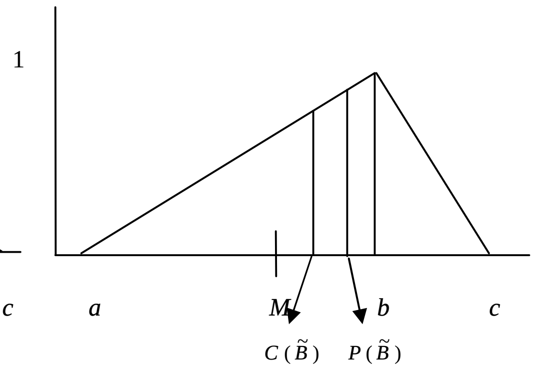

Figure 2: Case $M < b$

3. ASSUMPTIONS AND NOTATION

The mathematical model of the inventory system considered in this paper is basically an extension of the work of Huang (2003) and Chung and Huang (2003) model and is developed on the basis of the following assumptions and notation:

(i) Rate of replenishment is finite.

(ii) *D* is the annual demand rate, *K* is the annual production rate, which are assumed to be constant and $K>D$, $r=1-\frac{D}{K}$

(iii) $q_1(t)$ is the inventory level that changes with time *t* during production period and $q_2(t)$ is the inventory level that changes with time *t* during non-production period.

(iv) A constant fraction q, assumed to be small, of the on-hand inventory deteriorates per unit time.

(v) C_1: inventory holding cost per item per unit time; C_0: the replenishment (ordering) cost per order; C_p: the unit purchase cost; and C_s: the unit selling price per item of good quantity. In fuzzy sense, these quantities may be represented as $\tilde{C}_1=(C_1-\Delta_{C11}, C_1, C_1-\Delta_{C12})$, $\tilde{C}_0=(C_1-\Delta_{C01}, C_0, C_0-\Delta_{C02})$, $\tilde{C}_p=(C_p-\Delta_{p1}, C_p, C_p-\Delta_{p2})$, and $\tilde{C}_s=(C_s-\Delta_{s1}, C_s, C_s-\Delta_{s2})$.

(vi) I_e is the interest rate earned per year and I_c is the interest charged rate in stocks per year by the supplier, where $I_c \geq I_e$.

(vii) *M* is the retailer's trade credit period offered by supplier in years and *N* is the customer's trade credit period offered by retailer in years. It is assumed that $M \geq N$.

(viii) When $T \geq M$, the account is settled at $T = M$, the retailer pays off all units sold and keeps his/her profits and the retailer starts paying for the interest charges on the items in stock with rate I_c. When $T < M$, the account is settled at time $T = M$ and the retailer does not pay any interest charge.

(ix) The retailer can accumulate revenue and earn interest after his/her customer pays for the amount of purchasing cost to the retailer until the end of the trade credit period offered by the supplier. That is, the retailer can accumulate revenue and earn interest during the period *N* to *M* with rate I_e under the condition of trade credit.

4. MATHEMATICAL MODEL

A constant production rate starts at $t = 0$, and continues up to $t=t_1$ where the inventory level reaches the maximum level. Production then stops at $t=t_1$, and the inventory gradually depletes to zero at the end of the production cycle $t=T$ due to deterioration and consumption. Thereafter, during the time interval $(0, t_1)$, the system is subject to the effect of production, demand and deterioration. Then, the change in the inventory level can be described by the following differential equation:

$$\frac{dq_1(t)}{dt} + q\ q_1(t) = K - D; \qquad 0 \le t \le t_1, \tag{3}$$

with the initial condition $q_1(0) = 0.$

On the other hand, in the interval (t_1, T), the system is affected by the combined effect of demand and deterioration. Hence, the change in the inventory level is governed by the following differential equation:

$$\frac{dq_1(t)}{dt} + q\ q_1(t) = -D; \qquad t_1 \le t \le T, \tag{4}$$

with the ending condition $q_2(T) = 0.$

The solution of the differential equations (3) and (4) are respectively represented by

$$q_1(t) = \frac{K - D}{q}\left(1 - e^{-q\,t}\right), \qquad 0 \le t \le t_1 \tag{5}$$

and $$q_2(t) = \frac{D}{q}\left(e^{q(T-t_1)} - 1\right), \qquad t_1 \le t \le T. \tag{6}$$

In addition, using the boundary condition $q_1(t_1) = q_2(t_1)$, we obtain the following equations:

$$(K - D)\left(1 - e^{-q\,t}\right) = \ D\left(e^{q(T-t_1)} - 1\right) \tag{7}$$

and $$t_1 = \frac{1}{q}\ln\left\{1 + \frac{D}{K}\left(e^{q\,T} - 1\right)\right\}. \tag{8}$$

The annual total relevant cost consists of the following elements:

1. Annual ordering cost $= \dfrac{C_0}{T}$.

2. Annual stock holding cost (excluding interest charges) is

$$= \frac{C_1}{T}\left[\int_0^{t_1} q_1(t)dt + \int_{t_1}^{T} q_2(t)dt\right] = \frac{C_1}{q^2T}\left(q\,t_1 + e^{-q\,t_1} - 1\right)K + \frac{C_1}{q^2T}\left(e^{q(T-t_1)} - q\,T - e^{-q\,t_1}\right)D \tag{9}$$

Since $q_1(t_1) = q_2(t_1)$, which implies equation (9) can be rearranged as follows. Annual stock holding cost (excluding interest charges) = $\dfrac{C_1}{q\,T}(Kt_1 - DT).$

3. Annual cost due to deteriorated units $= \dfrac{C_p}{T}(Kt_1 - DT).$

4. According to assumption (viii), there are three cases to occur in interest charged for the items kept in stock per year.

   Case 1. $M \le T$

   Annual interest payable $= \dfrac{C_pI_c}{T}\displaystyle\int_M^T q_2(t)dt = \dfrac{C_pI_cD}{q^2T}\left(e^{q(T-M)} - q(T-M) - 1\right).$

   Case 2. $N \le T \le M$

   In this case, annual interest payable = 0.

   Case 3. $T \le N$

   Similar as case 2, annual interest payable = 0.

5. According to assumption (ix), three cases will occur in interest earned per year.

   Case 1. $M \le T$ (shown in figure 4)

Annual interest earned $= \frac{C_s I_e}{T} \int_N^M Dt\, dt = \frac{C_s I_e D}{2T}\left(M^2 - N^2\right).$

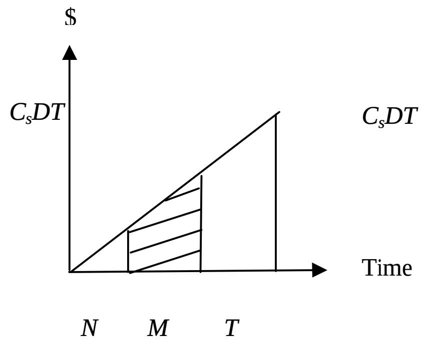

Figure 4: The total accumulation of interest earned when $M \le T$.

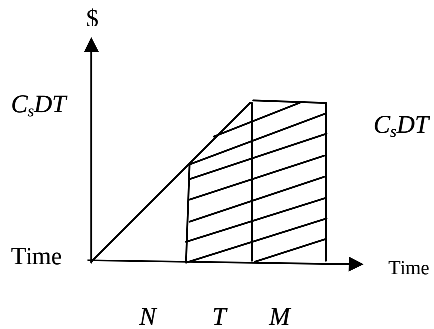

Figure 5: The accumulation of interest earned when $N \le T \le M$.

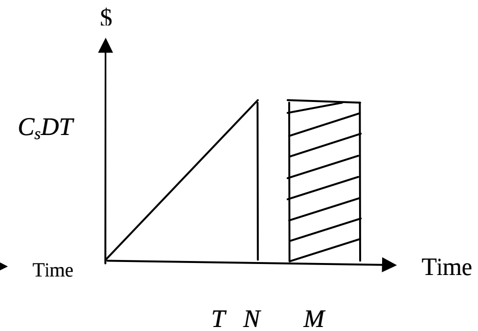

Figure 6: The accumulation of interest earned $T \le N$.

Case 2. $N \le T \le M$ (shown in figure 5)

Annual interest earned $= \frac{C_s I_e}{T}\left[\int_N^T Dt\, dt + DT(M-T)\right] = \frac{C_s I_e D}{2T}\left(2MT - N^2 - T^2\right).$

Case 3. $T \le N$ (shown in figure 6)

Annual interest payable $= \frac{C_s I_e}{T} DT(M-N).$

From the above arguments, the annual total relevant cost for the retailer can be expressed as

$TVC(T)$ = ordering cost + stock-holding cost + deterioration cost + interest payable – interest earned.

$$TVC(T) = \begin{cases} TVC_1(T), & \text{if } T \ge M, \\ TVC_2(T), & \text{if } N \le T \le M, \\ TVC_3(T), & \text{if } 0 < T \le N, \end{cases}$$

where $TVC_1(T) = \frac{C_0}{T} + \frac{C_1 + q\,C_p}{q\,T}(Kt_1 - DT) + \frac{C_p I_c D}{q^2 T}\left(e^{q(T-M)} - q(T-M) - 1\right) - \frac{C_s I_e D}{2T}\left(M^2 - N^2\right),$ (11)

$$TVC_2(T) = \frac{C_0}{T} + \frac{C_1 + q\,C_p}{q\,T}(Kt_1 - DT) - \frac{C_s I_e D}{2T}\left(2MT - N^2 - T^2\right), \tag{12}$$

$$TVC_3(T) = \frac{C_0}{T} + \frac{C_1 + q\,C_p}{q\,T}(Kt_1 - DT) - \frac{C_s I_e}{T} DT(M-N). \tag{13}$$

Inventory costs are normally assumed to be constant but always this is not true. In a perfect competitive market, ordering cost C_0, unit holding cost C_1, unit purchase cost C_p, unit selling price C_s etc. per day in a plan period T may fluctuate a little. For example, "unit holding cost is around C_1", "unit selling price is about C_s", etc. Suppose these cost parameters ordering cost, holding cost, purchase cost, selling price lie in the interval $[C_0 - \Delta_{C01}, C_0 + \Delta_{C02}]$, $[C_1 - \Delta_{C11}, C_1 + \Delta_{C12}]$, $[C_p - \Delta_{Cp1}, C_p + \Delta_{Cp2}]$, $[C_s - \Delta_{Cs1}, C_s + \Delta_{Cs2}]$. Similarly, corresponding to these intervals, we get the following fuzzy numbers: $\tilde{C}_0 = (C_0 - \Delta_{C01}, C_0, C_0 + \Delta_{C02})$, $\tilde{C}_1 = (C_1 - \Delta_{C11}, C_1, C_1 + \Delta_{C12})$, $\tilde{C}_p = (C_p - \Delta_{Cp1}, C_p, C_p + \Delta_{Cp2})$ and $\tilde{C}_s = (C_s - \Delta_{Cs1}, C_s, C_s + \Delta_{Cs2})$.

Through (11) – (13), for any $T > 0$, we get fuzzy total costs

$$T\tilde{V}C_1(T) = X_{11}\tilde{C}_0 + X_{12}\tilde{C}_1 + X_{13}\tilde{C}_p - X_{14}\tilde{C}_s, \qquad T \ge M, \tag{14}$$

$$T\tilde{V}C_2(T) = X_{21}\tilde{C}_0 + X_{22}\tilde{C}_1 + X_{23}\tilde{C}_p - X_{24}\tilde{C}_s, \qquad N \le T \le M, \tag{15}$$

$$T\tilde{V}C_3(T) = X_{31}\tilde{C}_0 + X_{32}\tilde{C}_1 + X_{33}\tilde{C}_p - X_{34}\tilde{C}_s, \qquad 0 < T \le N, \tag{16}$$

where $X_{11} = X_{21} = X_{31} = \dfrac{1}{T}$, $X_{12} = X_{22} = X_{32} = X_{23} = X_{33} = \dfrac{Kt_1 - DT}{qT}$, $X_{13} = \dfrac{Kt_1 - DT}{qT} + \dfrac{I_c D}{q^2 T}\left\{e^{q(T-M)} - q(T-M) - 1\right\}$,

$X_{14} = \dfrac{I_e D}{2T}\left(M^2 - N^2\right)$, $X_{24} = \dfrac{I_e D}{2T}\left(2MT - N^2 - T^2\right)$ and $X_{34} = I_e D(M - N)$.

Here we assume that $\tilde{C}_1 = (C_{11}, C_1, C_{12})$, $\tilde{C}_0 = (C_{01}, C_0, C_{02})$, $\tilde{C}_p = (C_{p1}, C_p, C_{p2})$, and $\tilde{C}_s = (C_{s1}, C_s, C_{s2})$ are nonnegative triangular fuzzy numbers, where $C_{11} = C_1 - \Delta_{C11}$, $C_{12} = C_1 + \Delta_{C12}$, $C_{01} = C_0 - \Delta_{C01}$, $C_{02} = C_0 + \Delta_{C02}$, $C_{p1} = C_p - \Delta_{Cp1}$, $C_{p2} = C_p + \Delta_{Cp2}$, $C_{s1} = C_s - \Delta_{Cs1}$, $C_{s2} = C_s + \Delta_{Cs2}$. Then we get $T\tilde{V}C_1(T)$ as

$$T\tilde{V}C_1(T) = \frac{1}{6}\left[X_{11}C_{01} + X_{12}C_{11} + X_{13}C_{p1} - X_{14}C_{s2},\ X_{11}C_0 + X_{12}C_1 + X_{13}C_p - X_{14}C_s,\ X_{11}C_{02} + X_{12}C_{12} + X_{13}C_{p2} - X_{14}C_{s1}\right].$$

We defuzzify $T\tilde{V}C_1(T)$ by formula (2) and obtain the graded mean integration representation of $T\tilde{V}C_1(T)$ as

$$\begin{aligned} P(T\tilde{V}C_1(T)) &= \frac{1}{6}\left[(X_{11}C_{01} + X_{12}C_{11} + X_{13}C_{p1} - X_{14}C_{s2}) + 4(X_{11}C_0 + X_{12}C_1 + X_{13}C_p - X_{14}C_s) + (X_{11}C_{02} + X_{12}C_{12} + X_{13}C_{p2} - X_{14}C_{s1})\right] \\ &= \frac{F_1}{T} + \frac{F_2 + qF_3}{qT}(Kt_1 - DT) + \frac{F_3 I_c D}{q^2 T}\left\{e^{q(T-M)} - q(T-M) - 1\right\} - \frac{F_4 I_e D}{2T}\left(M^2 - N^2\right); \quad T \ge M, \end{aligned} \tag{17}$$

where $F_1 = \dfrac{C_{01} + 4C_0 + C_{02}}{6}$, $F_2 = \dfrac{C_{11} + 4C_1 + C_{12}}{6}$, $F_3 = \dfrac{C_{p1} + 4C_p + C_{p2}}{6}$ and $F_4 = \dfrac{C_{s1} + 4C_s + C_{s2}}{6}$.

Similarly, the graded mean integration representation of $P(T\tilde{V}C_2(T))$ and $P(T\tilde{V}C_3(T))$ are respectively,

$$P(T\tilde{V}C_2(T)) = \frac{F_1}{T} + \frac{F_2 + qF_3}{qT}(Kt_1 - DT) - \frac{F_4 I_e D}{2T}\left(2MT - N^2 - T^2\right); \quad N \le T \le M, \tag{18}$$

$$P(T\tilde{V}C_3(T)) = \frac{F_1}{T} + \frac{F_2 + qF_3}{qT}(Kt_1 - DT) - F_4 I_e D(M - N); \quad 0 < T \le N. \tag{19}$$

Thus the total cost in the fuzzy sense based on graded mean integration representation is

$$P(T\tilde{V}C(T)) = \begin{cases} P(T\tilde{V}C_1(T)), & if\ T \ge M, \\ P(T\tilde{V}C_2(T)), & if\ N \le T \le M, \\ P(T\tilde{V}C_3(T)), & if\ 0 < T \le N, \end{cases} \tag{20}$$

Where $P(T\tilde{V}C_1(T))$, $P(T\tilde{V}C_2(T))$, $P(T\tilde{V}C_3(T))$ are given by (18), (19), (20). Since $P(T\tilde{V}C_1(M)) = P(T\tilde{V}C_2(M))$ and $P(T\tilde{V}C_2(N)) = P(T\tilde{V}C_3(N))$, $P(T\tilde{V}C(T))$ is continuous and well-defined, all $P(T\tilde{V}C_1(T))$, $P(T\tilde{V}C_2(T))$, $P(T\tilde{V}C_3(T))$ and $P(T\tilde{V}C(T))$ are defined on $T > 0$. The objective here is to find an optimal cycle time to minimize the total variable cost per unit time. For this, the optimal cycle time T_1^*, obtained by setting the derivative of equation (17) with respect to T equal to 0, is the root of the following equation:

$$-F_1 + \frac{K(F_2 + qF_3)}{q}\left\{T\frac{dt_1}{dT} - t_1\right\} + \frac{F_3 I_c D}{q^2}\left\{q\,Te^{q(T-M)} - e^{q(T-M)} - q\,M + 1\right\} + \frac{F_4 I_e D}{2}\left(M^2 - N^2\right) = 0. \tag{21}$$

Equation (21) gives the optimal value of T_1 of equation (17). Let

$$f_1(T) = -F_1 + \frac{K(F_2 + qF_3)}{q}\left\{T\frac{dt_1}{dT} - t_1\right\} + \frac{F_3 I_c D}{q^2}\left\{q\,Te^{q(T-M)} - e^{q(T-M)} - q\,M + 1\right\} + \frac{F_4 I_e D}{2}\left(M^2 - N^2\right). \tag{22}$$

Then both $f_1(T)$ and $P(T\tilde{V}C_1(T))'$ have the same sign and domain. We also have

$$f_1'(T) = \frac{K(F_2 + qF_3)}{q}T\frac{d^2 t_1}{dT^2} + F_4 I_e D T e^{q(T-M)} > 0; \text{ if } T > 0, \tag{23}$$

Where $\frac{d^2t_1}{dT^2} = \frac{DKq\, e^{qT} r}{(D e^{qT} + Kr)^2} > 0$. Hence $f_1(T)$ is increasing on $(0, \infty)$. Also

$f_1(0) = -F_1 - \frac{F_3 I_c D}{q^2}\left\{e^{-qM} - 1 + q\, M\right\} + \frac{F_4 I_e D}{2}\left(M^2 - N^2\right)$ and $\lim_{T\to\infty} f_1(T) = \infty > 0$. Hence we see that

$$\frac{dP(T\tilde{V}C_1(T))}{dT}\begin{cases} < 0; & if \quad T \in (0, T_1^*) & (a) \\ = 0; & if \quad T = T_1^* & (b) \\ > 0; & if \quad T \in (T_1^*, \infty) & (c)\end{cases} \tag{24}$$

Provided that $f_1(0) < 0$. Therefore T_1^* is the unique non-negative solution of (21). If $f_1(0) > 0$, then (21) has no solution for *T*.

Similarly, the optimal cycle time T_2^*, obtained by setting the derivative of equation (18) with respect to T equal to 0, is the root of the following equation:

$$-F_1 + \frac{K(F_2 + q\, F_3)}{q}\left\{T\frac{dt_1}{dT} - t_1\right\} + \frac{F_4 I_e D}{2}\left(T^2 - N^2\right) = 0. \tag{25}$$

Equation (18) is the optimality condition of equation (25). Let

$$f_2(T) = -F_1 + \frac{K(F_2 + q\, F_3)}{q}\left\{T\frac{dt_1}{dT} - t_1\right\} + \frac{F_4 I_e D}{2}\left(T^2 - N^2\right) \tag{26}$$

and $f_2'(T) = \frac{K(F_2 + q\, F_3)}{q} T \frac{d^2t_1}{dT^2} + F_4 I_e DT > 0$; if $T > 0$. Therefore $f_2(T)$ is increasing on $(0, \infty)$. Evidently, from

$f_2(0) = -F_1 - \frac{F_4 I_e DN^2}{2} < 0$ and $\lim_{T\to\infty} f_2(T) = \infty > 0$, we have

$$\frac{dP(T\tilde{V}C_2(T))}{dT}\begin{cases} < 0; & if \quad T \in (0, T_2^*) & (a) \\ = 0; & if \quad T = T_2^* & (b) \\ > 0; & if \quad T \in (T_2^*, \infty) & (c)\end{cases} \tag{27}$$

Likewise, the optimal cycle time T_3^*, obtained by setting the derivative of equation (19) with respect to T equal to 0, is the root of the following equation:

$$-F_1 + \frac{K(F_2 + q\, F_3)}{q}\left\{T\frac{dt_1}{dT} - t_1\right\} = 0. \tag{28}$$

Let $f_3(T) = -F_1 + \frac{K(F_2 + q\, F_3)}{q}\left\{T\frac{dt_1}{dT} - t_1\right\}$, we have $f_3'(T) = \frac{K(F_2 + q\, F_3)}{q} T \frac{d^2t_1}{dT^2} > 0$; if $T > 0$. Therefore $f_3(T)$ is increasing

on $(0, \infty)$. Also, $f_3(0) = -F_1 < 0$ and $\lim_{T\to\infty} f_3(T) = \infty > 0$ implies

$$\frac{dP(T\tilde{V}C_3(T))}{dT}\begin{cases} < 0; & if \quad T \in (0, T_3^*) & (a) \\ = 0; & if \quad T = T_3^* & (b) \\ > 0; & if \quad T \in (T_3^*, \infty) & (c)\end{cases} \tag{29}$$

Also,

$$P(T\tilde{V}C_1(M))' = P(T\tilde{V}C_2(M))' = \frac{f_1(M)}{M^2} = \frac{f_2(M)}{M^2} = \frac{\Delta_1}{M^2} \text{ and } P(T\tilde{V}C_2(N))' = P(T\tilde{V}C_3(N))' = \frac{f_2(N)}{N^2} = \frac{f_3(N)}{N^2} = \frac{\Delta_2}{N^2}. \tag{30}$$

Where $\Delta_1 = -F_1 + \frac{K(F_2 + q\, F_3)}{q}\left\{T\frac{dt_1}{dT} - t_1\right\}_{T=M} + \frac{F_4 I_e D}{2}\left(M^2 - N^2\right)$ (31)

And $\Delta_2 = -F_1 + \frac{K(F_2 + q\, F_3)}{q}\left\{T\frac{dt_1}{dT} - t_1\right\}_{T=N}$. (32)

We have $\Delta_1-\Delta_2=\dfrac{K(F_2+q\,F_3)}{q}\left\{\left(T\dfrac{dt_1}{dT}-t_1\right)_{T=M}-\left(T\dfrac{dt_1}{dT}-t_1\right)_{T=N}\right\}+\dfrac{F_4 I_e D}{2}\left(M^2-N^2\right).$ (33)

Lemma 1.

$(i)\ \ T\dfrac{dt_1}{dT}-t_1>0$ and $(ii)\ \ \left(T\dfrac{dt_1}{dT}-t_1\right)_{T=M}>\left(T\dfrac{dt_1}{dT}-t_1\right)_{T=N}$.

Proof.

Let $h(T)=T\dfrac{dt_1}{dT}-t_1$, if $T>0$, then $h'(T)=T\dfrac{d^2t_1}{dT^2}>0.$ Hence $h(T)$ is increasing for all $T>0$. Consequently, $h(T)>h(0)=0$ if $T>0$ and also $h(M)>h(N)$ as $M>N.$ Thus, we have $T\dfrac{dt_1}{dT}-t_1>0$ and $\left(T\dfrac{dt_1}{dT}-t_1\right)_{T=M}>\left(T\dfrac{dt_1}{dT}-t_1\right)_{T=N}$. This completes the proof.

Therefore, we have from (33), $\Delta_1>\Delta_2$.

5. DECISION RULES OF THE OPTIMAL CYCLE TIME T^*

In this section, we develop efficient decision rules to find the optimal cycle time for the retailer.

Theorem 1.

(A) If $\Delta_1>0$ and $\Delta_2\ge 0$, then $P(T\tilde{V}C(T^*))=P(T\tilde{V}C_3(T_3^*))$ and $T^*=T_3^*$.

(B) If $\Delta_1>0$ and $\Delta_2<0$, then $P(T\tilde{V}C(T^*))=P(T\tilde{V}C_2(T_2^*))$ and $T^*=T_2^*$.

(C) If $\Delta_1\le 0$ and $\Delta_2<0$, then $P(T\tilde{V}C(T^*))=P(T\tilde{V}C_1(T_1^*))$ and $T^*=T_1^*$.

Proof.

(A) If $\Delta_1>0$ and $\Delta_2\ge 0$, then we have $f_1(M)=f_2(M)>0$ and $f_2(N)=f_3(N)\ge 0,$ i.e. $P(T\tilde{V}C_1(M))'=P(T\tilde{V}C_2(M))'>0$ and $P(T\tilde{V}C_2(N))'=P(T\tilde{V}C_3(N))'\ge 0$. So $T_1^*<M$, $T_2^*<M$, $T_3^*<N$ and $T_2^*<N$. Equation (24), (27) and (29) imply that (i) $P(T\tilde{V}C_1(T))$ is increasing on $[M,\infty)$, (ii) $P(T\tilde{V}C_2(T))$ is increasing on $[N,M]$, (iii) $P(T\tilde{V}C_3(T))$ is decreasing on $(0,T_3^*)$ and increasing on $[T_3^*,N]$. Combining (i)-(iii) and equation (20), we see that $P(T\tilde{V}C(T))$ is decreasing on $(0,T_3^*)$ and increasing on $[T_3^*,\infty)$. Consequently, $T^*=T_3^*$.

(B) If $\Delta_1>0$ and $\Delta_2<0$, then we have $f_1(M)=f_2(M)>0$ and $f_2(N)=f_3(N)<0,$ i.e. $P(T\tilde{V}C_1(M))'=P(T\tilde{V}C_2(M))'>0$ and $P(T\tilde{V}C_2(N))'=P(T\tilde{V}C_3(N))'<0$. So $T_1^*<M$, $T_2^*<M$, $T_2^*>N$ and $T_3^*>N$. Equation (24), (27) and (29) imply that (i) $P(T\tilde{V}C_1(T))$ is increasing on $[M,\infty)$, (ii) $P(T\tilde{V}C_2(T))$ is decreasing on $[N,T_2^*]$ and increasing on $[T_2^*,M]$, (iii) $P(T\tilde{V}C_3(T))$ is decreasing on $[0,N]$. Combining (i)-(iii) and equation (20), it can be easily verified that $P(T\tilde{V}C(T))$ is decreasing on $(0,T_2^*)$ and increasing on $[T_2^*,\infty)$. Consequently, $T^*=T_2^*$.

(C) If If $\Delta_1\le 0$ and $\Delta_2<0$, then we have $f_1(M)=f_2(M)\le 0$ and $f_2(N)=f_3(N)<0,$ i.e. $P(T\tilde{V}C_1(M))'=P(T\tilde{V}C_2(M))'\le 0$ and $P(T\tilde{V}C_2(N))'=P(T\tilde{V}C_3(N))'<0$. So $T_1^*>M$, $T_2^*>M$, $T_2^*>N$ and $T_3^*>N$. Equation (24), (27) and (29) imply that (i) $P(T\tilde{V}C_1(T))$ is decreasing on $[M,T_1^*]$ and increasing on $[T_1^*,\infty)$, (ii) $P(T\tilde{V}C_2(T))$ is increasing on $[N,M]$, (iii) $P(T\tilde{V}C_3(T))$ is decreasing on $[0,N]$. Combining (i)-(iii) and equation (20), we observe that $P(T\tilde{V}C(T))$ is decreasing on $(0,T_1^*)$ and increasing on $[T_1^*,\infty)$. Consequently, $T^*=T_1^*$.

This completes the proof of Theorem 1.

Using Intermediate value theorem namely "let $f(x)$ be a continuous function on $[a,b]$ and $f(a).f(b)<0$, then there exists a number $c\in(a,b)$ such that $f(c)=0$", we can determine the values of T^*,T_1^*,T_2^* and T_3^* as follows:

a) Suppose that $\Delta_1>0$ and $\Delta_2\ge 0$, then T_3^* exists, $T^*=T_3^*$ and $0<T_3^*<N$. Recall that T_3^* denotes the unique root of equation (27). Since $f_3(T)=-F_1+\dfrac{K(F_2+q\,F_3)}{q}\left\{T\dfrac{dt_1}{dT}-t_1\right\}$, we have $f_3{}'(T)=\dfrac{K(F_2+q\,F_3)}{q}T\dfrac{d^2t_1}{dT^2}>0$; if $T>0$.

Therefore $f_3(T)$ is increasing on $(0,\infty)$. We see that $f_3(0)=-F_1<0=f_3(T_3^*)<f_3(N)$. So $f_3(0).f_3(N)<0$. Consequently, we are in a position to outline the algorithm to find T_3^*.

Step 1: Let $e>0$.

Step 2: Let $T_L=0$ and $T_U=N$.

Step 3: Let $T_{opt}=\dfrac{T_L+T_U}{2}$.

Step 4: If $|f_3(T_{opt})|<e$, go to Step 6. Otherwise, go to Step 5.

Step 5: If $f_3(T_{opt})>0$, set $T_U=T_{opt}$. If $f_3(T_{opt})<0$, set $T_L=T_{opt}$. Then go to step 3.

Step 6: Set $T_3^*=T_{opt}$.

b) Suppose that $\Delta_1>0$ and $\Delta_2<0$, then T_2^* exists, $T^*=T_2^*$ and $N<T_2^*<M$. T_2^* denote the root of the equation (25).

Since
$$f_2(T)=-F_1+\frac{K(F_2+q\,F_3)}{q}\left\{T\frac{dt_1}{dT}-t_1\right\}+\frac{F_4I_eD}{2}\left(T^2-N^2\right), \tag{34}$$

then we have $f_2'(T)=\dfrac{K(F_2+q\,F_3)}{q}T\dfrac{d^2t_1}{dT^2}+F_4I_eDT>0$; if $T>0$. Therefore $f_2(T)$ is increasing on $(0,\infty)$. We have

$f_2(N)<0=f_2(T_2^*)<f_2(M)$. Consequently, we are in a position to outline the algorithm to find T_2^*.

Step 1: Let $e>0$.

Step 2: Let $T_L=N$ and $T_U=M$.

Step 3: Let $T_{opt}=\dfrac{T_L+T_U}{2}$.

Step 4: If $|f_2(T_{opt})|<e$, go to Step 6. Otherwise, go to Step 5.

Step 5: If $f_2(T_{opt})>0$, set $T_U=T_{opt}$. If $f_2(T_{opt})<0$, set $T_L=T_{opt}$. Then go to step 3.

Step 6: Set $T_2^*=T_{opt}$.

c) Suppose that $\Delta_1\le 0$ and $\Delta_2<0$, then T_1^* exists and $T^*=T_1^*$. T_1^* denote the root of the equation (21). Let

$$T_1^U=\frac{2q^2F_1+2F_3I_cDe^{-qM}+2q^3M+q^4N^2}{2qF_3I_cDe^{-qM}} \tag{35}$$

Then we have the following lemma:

Lemma 2. Suppose that $\Delta_1\le 0$ and $\Delta_2<0$, then $T_1^U>T_1^*>M$, where T_1^U is given by (35).

Proof. Since $0<e^{-qT}<1$,

$$\begin{aligned}
f_1(T)&=-F_1+\frac{K(F_2+q\,F_3)}{q}\left\{T\frac{dt_1}{dT}-t_1\right\}+\frac{F_3I_cD}{q^2}\left\{q\,Te^{q(T-M)}-e^{q(T-M)}-q\,M+1\right\}+\frac{F_4I_eD}{2}\left(M^2-N^2\right)\\
&>-F_1+\frac{F_3I_cD}{q^2}\left\{qTe^{q(T-M)}-e^{q(T-M)}-qM-\frac{q^2N^2}{2}\right\}\\
&=e^{qT}\left\{-F_1e^{-qT}+\frac{F_3I_cD}{q^2}\left(qTe^{-qM}-e^{-qM}-qMe^{-qT}-\frac{q^2N^2}{2}e^{-qT}\right)\right\}.
\end{aligned}$$

Hence $f_1(T_1^U)>0=f_1(T_1^*)$. Since $f_1(T)$ is increasing on $[M,\infty)$ and $f_1(T_1^U)>f_1(T_1^*)>f_1(M)$, we obtain $T_1^U>T_1^*>M$. Consequently, we have complete the proof.

By lemma 2, we are in position to outline the algorithm to find T_1^*.

Step 1: Let $e > 0$.

Step 2: Let $T_L = M$ and $T_U = T_1^U$.

Step 3: Let $T_{opt} = \frac{T_L + T_U}{2}$.

Step 4: If $|f_1(T_{opt})| < e$, go to Step 6. Otherwise, go to Step 5.

Step 5: If $f_1(T_{opt}) > 0$, set $T_U = T_{opt}$. If $f_1(T_{opt}) < 0$, set $T_L = T_{opt}$. Then go to step 3.

Step 6: Set $T_1^* = T_{opt}$.

6. SPECIAL CASES

In this section, we obtain some previously published results of other authors as special cases.

6.1) Huang's model (2003)

When the deterioration rate is ignored, there is no deterioration cost: thus by using L'Hospital's rule, we have

$$\lim_{q\to 0}\left[\frac{C_0}{T} + \frac{C_1 + q\,C_p}{q\,T}(Kt_1 - DT) + \frac{C_p I_c D}{q^2 T}\left(e^{q(T-M)} - q(T-M) - 1\right) - \frac{C_s I_e D}{2T}\left(M^2 - N^2\right)\right]$$

$$= \frac{C_0}{T} + \frac{DTC_1 r}{2} + \frac{C_p I_c D(T-M)^2}{2T} - \frac{C_p I_e D(M^2 - N^2)}{2T}, \tag{37}$$

$$\lim_{q\to 0}\left[\frac{C_0}{T} + \frac{C_1 + q\,C_p}{q\,T}(Kt_1 - DT) - \frac{C_p I_e D}{2T}\left(2MT - N^2 - T^2\right)\right] = \frac{C_0}{T} + \frac{DTC_1 r}{2} - \frac{C_p I_e D}{2T}\left(2MT - N^2 - T^2\right) \tag{38}$$

$$\lim_{q\to 0}\left[\frac{C_0}{T} + \frac{C_1 + q\,C_p}{q\,T}(Kt_1 - DT) - C_p I_e D\,(M - N)\right] = \frac{C_0}{T} + \frac{DTC_1 r}{2} - -C_p I_e D\,(M - N) \tag{39}$$

Moreover, in the crisp sense, when $K \to \infty$ and $C_p = C_s$, the total variable cost per unit $TVC(T)$ can be expressed as

$$TVC(T) = \begin{cases} TVC_4(T), & if\ T \ge M, \\ TVC_5(T), & if\ N \le T \le M, \\ TVC_6(T), & if\ 0 < T \le N, \end{cases} \tag{40}$$

Where
$$TVC_4 = \frac{C_0}{T} + \frac{DTC_1 r}{2} + \frac{C_p I_c D(T-M)^2}{2T} - \frac{C_p I_e D(M^2 - N^2)}{2T} \tag{41}$$

$$TVC_5 = \frac{C_0}{T} + \frac{DTC_1 r}{2} - \frac{C_p I_e D}{2T}\left(2MT - N^2 - T^2\right) \tag{42}$$

$$TVC_6 = \frac{C_0}{T} + \frac{DTC_1 r}{2} - -C_p I_e D\,(M - N)\cdot \tag{43}$$

Let $T_4^* = \sqrt{\frac{2C_0 + C_p D\left[M^2(I_c - I_e) + N^2 I_e\right]}{D\left(C_1 + C_p I_c\right)}}$, $T_5^* = \sqrt{\frac{2C_0 + C_p D N^2 I_e}{D\left(C_1 + C_p I_e\right)}}$ and $T_6^* = \sqrt{\frac{2C_0}{DC_1}}$. Then $TVC_i'(T_i^*) = 0$ for $i = 4, 5, 6$. Equations (40) are consistent with equations (1a-c) in Huang's model (2003), respectively. Hence, Huang's model (2003) is a special case of this model.

6.2) Chung and Huang's model (2003)

When $q \to 0$, using L'Hospital's rule, we have $\lim_{q\to 0} t_1 = \lim_{q\to 0}\frac{1}{q}\ln\left\{1 + \frac{D}{K}\left(e^{qT} - 1\right)\right\} = \frac{DT}{K}$ and $\lim_{q\to 0} t_M = \lim_{q\to 0}\frac{1}{q}\ln\left\{1 + \frac{K}{D}\left(e^{qM} - 1\right)\right\} = \frac{DM}{K}$. In this paper, suppose that N = 0, it means that the supplier would offer the retailer a delay period but the retailer would not offer the delay period to his/her customer i.e. one level trade credit and $q \to 0$, i.e. deterioration rate is ignored, then in the crisp sense, using L'Hospital's rule and considering $N = 0$ and $C_p = C_s$, equations (11) – (13) yield that

$$\lim_{q\to 0} TVC_1(T) = \frac{C_0}{T} + \frac{DTC_1 r}{2} + \frac{C_p I_c r}{T}\left(\frac{DT^2}{2} - \frac{KM^2}{2}\right) - \frac{C_p I_e}{T}\left(\frac{DM^2}{2}\right) = TVC_7(T);$$

$$\lim_{q\to 0} TVC_2(T) = \frac{C_0}{T} + \frac{DTC_1 r}{2} + \frac{C_p I_c}{T}\left(\frac{D(T-M)^2}{2}\right) - \frac{C_p I_e}{T}\left(\frac{DM^2}{2}\right) = TVC_8(T);$$ and

$$\lim_{q\to 0} TVC_3(T) = \frac{C_0}{T} + \frac{DTC_1 r}{2} - \frac{C_p I_e}{T}\left(\frac{DT^2}{2} + DT(M - T)\right) = TVC_9(T);$$

Equation (10) will be reduced as follows:

$$TVC(T)=\begin{cases} TVC_7(T), & if\ T\geq\frac{KM}{D}, \\ TVC_8(T), & if\ M\leq T\leq\frac{KM}{D}, \\ TVC_9(T), & if\ 0<T\leq M, \end{cases} \tag{44}$$

Let $T_7^*=\sqrt{\frac{2C_0+DM^2C_p(I_c-I_e)-KM^2C_pI_e}{Dr(C_1+C_pI_c)}}$, $T_8^*=\sqrt{\frac{2C_0+DM^2C_p(I_c-I_e)}{D(C_1r+C_pI_e)}}$ and $T_9^*=\sqrt{\frac{2C_0}{D(C_1r+C_pI_e)}}$. Then $TVC_i'(T_i^*)=0$ for $i=7,8,9$. Equations (44) are consistent with equations (6a-c) in Chuang and Huang model (2003), respectively. Hence, Chuang and Huang (2003) model will be a special case of this paper.

6.3) Goyal's model (1985)

When $q\to 0,\quad K\to\infty$, $N=0$ and $C_p=C_s$, let

$TVC_{10}(T)=\frac{C_0}{T}+\frac{DTC_1}{2}+\frac{C_pI_cD(T^2-M^2)}{2T}-\frac{C_pI_e}{T}\left(\frac{DM^2}{2}\right)$, $TVC_{11}(T)=\frac{C_0}{T}+\frac{DTC_1}{2}-C_pI_eD\left(M-\frac{T}{2}\right)$, $T_{10}^*=\sqrt{\frac{2C_0+DM^2C_p(I_c-I_e)}{D(C_1+C_pI_c)}}$

and $T_{11}^*=\sqrt{\frac{2C_0}{D(C_1+C_pI_e)}}$. Then $TVC_i'(T_i^*)=0$ for $i=10,11$. Equation (10) will be reduced as follows:

$$TVC(T)=\begin{cases} TVC_{10}(T), & if\ T\geq M, \\ TVC_{11}(T), & if\ 0<T\leq M, \end{cases} \tag{45}$$

Equations (45) exactly matches with equations (1) and (4) in Goyal's model (1985), respectively. Hence, Goyal's model (1985) is a special case of this model.

6.4) Shah's model (1993)

In the crisp case, when $K\to\infty$, $N=0$ and $C_p=C_s$, let

$TVC_{12}(T)=\frac{C_0}{T}+\frac{(C_1+qC_p)D}{q^2T}\left(e^{qT}-1-qT\right)+\frac{C_pI_cD}{q^2T}\left\{e^{q(T-M)}-q(T-M)-1\right\}-\frac{C_pI_eDM^2}{2T}$,

$TVC_{13}(T)=\frac{C_0}{T}+\frac{(C_1+qC_p)D}{q^2T}\left(e^{qT}-1-qT\right)-C_pI_eD\left(M-\frac{T}{2}\right)$.. Equation (10) will be reduced as follows:

$$TVC(T)=\begin{cases} TVC_{12}(T), & if\ T\geq M, \\ TVC_{13}(T), & if\ 0<T\leq M, \end{cases} \tag{46}$$

Equations (46) is consistent with Shah's (1993) model. Hence, Shah's model (1993) is a special case of this model.

7. NUMERICAL EXAMPLES

To illustrate the results of the proposed model, we solve the following numerical examples. Let $\tilde{C}_p=Rs.(45,50,55)/unit$, $\tilde{C}_s=Rs.(70,75,80)/unit$, $\tilde{C}_1=Rs.(12,15,18)/unit/year$ $I_c=Rs.0.15/year$ $I_e=Rs.0.12/year$ $M=0.1\,year$ and $q=0.02(0.02)0.10$.

Example I: *When* $\tilde{C}_0=Rs.(140,150,160)/order$, $D=2500\,units/year$, $K=3000\,units/year$ *and* $N=0.06\ year$, *then* $\Delta_1<0$ *and* $\Delta_2<0$. The results are given in Table 1.

Example II: *When* $\tilde{C}_0=Rs.(140,150,160)/order$, $D=3000\,units/year$, $K=5000\,units/year$ *and* $N=0.06\ year$, *then* $\Delta_1>0$ *and* $\Delta_2<0$. The results are given in Table 2.

Example III: *When* $\tilde{C}_0=Rs.(45,50,55)/order$, $D=2500\,units/year$, $K=5000\,units/year$ *and* $N=0.08\ year$, *then* $\Delta_1>0$ *and* $\Delta_2>0$. The results are given in Table 3.

Table 1: The optimal cycle time and the minimum annual total cost with various values of θ when $\Delta_1<0$ and $\Delta_2<0$.

Cases	θ	Δ_1	Δ_2	T_1^*	$P(\tilde{TVC}_1(T_1^*))$
(i)	0.02	< 0	< 0	0.1202	1181.034
(ii)	0.04	< 0	< 0	0.1193	1205.753
(iii)	0.06	< 0	< 0	0.1184	1230.252
(iv)	0.08	< 0	< 0	0.1174	1254.537
(v)	0.10	< 0	< 0	0.1166	1278.613

Table 2: The optimal cycle time and the minimum annual total cost with various values of θ when $\Delta_1>0$ and $\Delta_2<0$.

Cases	θ	Δ_1	Δ_2	T_2^*	$P(\tilde{TVC}_2(T_2^*))$
(i)	0.02	> 0	< 0	0.0927	1583.655
(ii)	0.04	> 0	< 0	0.0915	1638.813
(iii)	0.06	> 0	< 0	0.0904	1693.268
(iv)	0.08	> 0	< 0	0.0893	1747.047
(v)	0.10	> 0	< 0	0.0883	1800.174

Table 3: The optimal cycle time and the minimum annual total cost with various values of θ when $\Delta_1>0$ and $\Delta_2>0$.

Cases	θ	Δ_1	Δ_2	T_3^*	$P(\tilde{TVC}_3(T_3^*))$
(i)	0.02	> 0	> 0	0.0757	970.2486
(ii)	0.04	> 0	> 0	0.0747	1017.262
(iii)	0.06	> 0	> 0	0.0736	1063.608
(iv)	0.08	> 0	> 0	0.0726	1109.314
(v)	0.10	> 0	> 0	0.0717	1154.407

To study the effect of the inventory deteriorating rate on the optimal cost and the optimal cycle time derived by the proposed method, we solve the above three examples with various values of q. The following inference can be made based on Tables 1, 2 and 3.

- When the deterioration rate increases, from Tables 1, 2 and 3, it is observed that, the optimal cost increases whereas the optimal cycle time decreases.
- As the deterioration rate increases, the situation of $\Delta_1 < 0$ and $\Delta_2 < 0$ in Table 1, $\Delta_1 > 0$ and $\Delta_2 < 0$ in Table 2 and $\Delta_1 > 0$ and $\Delta_2 > 0$ in Table 3 will not changes. Cases (i) – (v) of Tables 1, 2 and 3 show the computed results.

8. CONCLUSIONS

In reality the value or utility of goods decreases over time for deteriorating items, which in turn suggests smaller cycle length. In this article, we have developed an EPQ model for deteriorating items in the fuzzy sense where delay in payments for both retailer and customer are permissible to reflect realistic situations. We assume that the supplier would offer a trade credit period to the customer. The inventory costs namely holding cost, ordering cost, purchasing cost and selling price are assumed as fuzzy numbers instead of crisp or probabilistic in nature. First, we investigate the retailer's inventory system as a cost minimization problem to determine the retailer's optimal inventory policy. Using Graded Mean Integration Representation method, we have defuzzified the fuzzy total variable cost per unit time. From the view point of the costs, decision rules to find the optimal cycle time T^* contain three cases: (i) $T \le N$ (ii) $N \le T \le M$ and (iii) $T \ge M$. In order to obtain the optimal ordering policy, we propose a theorem and three algorithms. With the help of Theorem 1 and corresponding lemmas, simple algorithms are provided for obtaining the optimum cycle time and the annual total relevant cost for the retailer.

In the crisp case, when $q \to 0$, $K \to \infty$ and $C_p = C_s$, Huang (2003) can be treated as a special case of this paper. In the crisp case, when $q \to 0$, $N = 0$ and $C_p = C_s$, Chung and Huang (2003) can be treated as a special case of this paper. In the crisp case, when $K \to \infty$, $N = 0$ and $C_p = C_s$, the inventory model discussed in this paper is reduced to Shah (1993) model. In the crisp case, when $q \to 0$, $K \to \infty$, $N = 0$ and $C_p = C_s$, the inventory model discussed in this paper is reduced to Goyal (1985) model.

Some numerical examples are studied to illustrate the theoretical results. To study the effect of q on the optimal cycle time T^* and on the optimal annual total cost $P(T\widetilde{V}C(T^*))$, there are some managerial phenomena from Table 1 - Table 3: (i) a higher value of deterioration rate causes lower value of cycle time and (ii) a higher value of deterioration rate causes higher value of annual total relevant cost. The proposed model can assist the retailer in accurately determining the optimal cycle time and the optimal annual total relevant cost. Moreover, the proposed model can be used in inventory control of certain deteriorating items such as food items, photographic film, electronic components, radio active materials, and fashionable commodities, and others.

The proposed model can be extended in several ways. For instance, we may extend the constant deterioration rate to a two-parameter Weibull distribution. In addition, we could consider the demand as a function of time, selling price, product quality, and others. Finally, we could generalize the model to allow for shortages, quantity discounts, and others.

9. ACKNOWLEDGEMENTS

The authors greatly appreciate the editors and the anonymous referees for their valuable comments and suggestions to improve the paper in the present form. This research work is fully supported by the University Grant Commission (UGC), INDIA, for providing a Minor Research Project (MRP, UGC) under the research grant no. PSW – 150/09-10, SNO: 95958.

REFERENCES

1. Aggarwal, S.P. and Jaggi, C.K. (1995). Ordering policies of deteriorating items under permissible delay in payments, *Journal of Operation Research Society*, 46, 658-662.
2. Arcelus, F.J., Shah, N.H. and Srinivasan, G. (2003). Retailers pricing, credit and inventory policies for deteriorating items in response to temporary price/credit incentives, *International Journal of Production Economics*, 81-82, 153-162.
3. Chand, S. and Ward, J. (1987). A note on economic order quantity under conditions of permissible delay in payments, *Journal of the Operational Research Society*, 38, 83-84.
4. Chang, C.T. (2004). An EOQ model with deteriorating items under inflation when supplier credits linked to order quantity, *International Journal of Production Economics*, 88, 307-316.
5. Chang, C.T., Ouyang, L.Y. and Teng, J.T. (2003). An EOQ model for deteriorating items under supplier credits linked to ordering quantity, *Applied Mathematical Modelling*, 27, 983-996.
6. Chang, H.J. and Dye, C.Y. (2001). An inventory model for deteriorating items with partial backlogging and permissible delay in payments, *International Journal of Systems Science*, 32, 345-352.
7. Chang, H.J., Hung, C.H. and Dye, C.Y. (2001). An inventory model for deteriorating items with linear trend demand under the condition of permissible delay in payments, *Production Planning and Control*, 12, 274 - 282.
8. Chen, S. H. and Hsieh, C.H. (1999). Graded mean integration representation of generalized fuzzy number, *Journal of Chinese Fuzzy Systems*, 5(2), 1-7.
9. Chen, S.H. (1985). Operations on fuzzy numbers with function principle, *Tamkang Journal of Management Sciences*, 6(1), 13-26.
10. Chu, P., Chung, K.J. and Lan, S.P. (1998). Economic order quantity of deteriorating items under permissible delay in payments, *Computers and Operations Research*, 25, 817-824.
11. Chung, K.J. (1998). A theorem on the determination of economic order quantity under conditions of permissible delay in payments, *Journal of Information and Optimization Sciences*, 25, 49-52.
12. Chung, K.J. and Huang, Y.F. (2003). The optimal cycle time for EPQ inventory model under permissible delay in payments, *International Journal of Production Economics*, 84, 307-318.
13. Chung, K.J., Chang, S.L. and Yang, W.D. (2001). The optimal cycle time for exponentially deteriorating products under trade credit financing, *The Engineering Economist*, 46, 232 - 242.
14. Chung, K.J. and Liao, J.J. (2004). Lot-sizing decisions under trade credit depending on the ordering quantity, *Computers and Operations Research*, 31, 909-928.
15. Goyal, S.K. (1985). Economic order quantity under conditions of permissible delay in payments, *Journal of Operation Research Society*, 36, 335-338.
16. Hwang, H. and Shinn, S.W. (1997). Retailer's pricing and lot sizing policy for exponentially deteriorating product under the condition of permissible delay in payments, *Computer and Operations Research*, 24, 539-547.
17. Huang, Y.F. (2004). Optimal retailers replenishment policy for the EPQ model under suppliers trade credit policy, *Production Planning and Control*, 15, 27-33.
18. Huang, Y.F. (2003). Optimal retailer's ordering policies in the EOQ model under trade credit financing, *Journal of the Operational Research Society*, 54, 1011-1015.
19. Jamal, A.M.M., Sarker, B.R. and Wang, S. (1997). An ordering policy for deteriorating items with allowable shortages and permissible delay in payment, *Journal of Operation Research Society*, 48, 826-833.

20. Liao, H.C., Tsai, C.H. and Su, C.T. (2000). An inventory model with deteriorating items under inflation when a delay in payment is permissible, *International Journal of Production Economics*, 63, 207-214.
21. Sarker, B.R., Jamal, A.M.M. and Wang, S. (2000). Supply chain model for perishable products under inflation and permissible delay in payment, *Computers and Operations Research*, 27, 59-75.
22. Shah, N.H. (1993). A lot-size model for exponentially decaying inventory when delay in payments is permissible, *Cahiers du CERO*, 35, 115-123.
23. Shinn, S.W., Hwang, H.P. and Sung, S. (1996). Joint price and lot size determination under conditions of permissible delay in payments and quantity discounts for freight cost, *European Journal of Operational Research*, 91, 528-542.
24. Shinn, S.W. and Hwang, H. (2003) Optimal pricing and ordering policies for retailers under order size dependent delay in payments, *Computers and Operations Research*, 30, 35-50.