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# A Review of Optimal Computing Budget Allocation Algorithms for Simulation Optimization Problem

**Loo Hay Lee[1,*], Chun-Hung Chen[2ψ], Ek Peng Chew[1], Juxin Li[1], Nugroho Artadi Pujowidianto[1], and Si Zhang[1]**

[1]Department of Industrial and Systems Engineering, National University of Singapore, Singapore - 119260

[2]Department of Systems Engineering and Operations Research, George Mason University, Fairfax, Virginia - 22030, USA

[2]Department of Electrical Engineering, National Taiwan University, Taipei, Taiwan - 106

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***Abstract*** — Simulation and optimization are two arguably most used operations research (OR) tools. Optimization intends to choose the best element from some set of available alternatives. Stochastic simulation is a powerful modeling and software tool for analyzing modern complex systems. This capability complements the inherent limitation of traditional optimization, so the combining use of simulation and optimization is growing in popularity. While the advance of new technology has dramatically increased computational power, efficiency is still a big concern because many simulation replications are required for each performance evaluation. Optimal Computing Budget Allocation (OCBA) algorithms have been developed to address such an efficiency issue with emphasis given on those aiming to maximize the probability of correct selection or other measures of selection quality given a limited computing budget. In this paper, we present a comprehensive survey on OCBA approaches for various simulation optimization problems together with the open challenges for future research.
***Keywords*** —Optimization; Discrete-event simulation; Simulation optimization; Ranking and selection; Computing budget allocation.

## 1. INTRODUCTION

There are two challenges in simulation optimization. One is to optimize or to find the best system or design where the number of alternatives may be huge. Due to uncertainties and dynamic relationships between the parts involved, many problems are too complex to be evaluated analytically. Therefore, the second challenge is to estimate the performance measures via simulation which is computationally intensive as multiple simulation replications are required for each alternative.

The simulation optimization problems have been widely studied. Many excellent reviews are available (Andradóttir, 1998; Fu, 2002; Fu et al., 2008, Hong and Nelson, 2009; Swisher et al., 2003; Tekin and Sabuncuoglu, 2004). Some of the approaches include metamodeling, sample average approximation, and gradient-based method. However, these methods may not be applicable when some of decision variables are discrete and the structure of the problems is unknown. Another alternative is the use of derivative-free, black-box simulation. When the number of alternatives to be selected is fixed, the problem comes down to a statistical selection problem called as Ranking and Selection.

The aim of ranking and selection procedures is to determine the number of simulation replications in selecting the best design. There are also a vast number of literatures on ranking and selection (Bechhofer et al., 1995; Goldsman and Nelson 1998; Kim and Nelson, 2003; Kim and Nelson, 2006; Kim and Nelson, 2007; Chick and Inoue, 2001ab; Branke et al., 2007). There are mainly two approaches. The first approach is to guarantee a desired probability of correct selection such as two-stage procedures by Dudewicz and Dalal (1975) and Rinott (1978), the two-stage procedure with screening by Nelson et al. (2001) or the fully-sequential procedure by Kim and Nelson (2001). In their procedures, a difference is considered significant if it is larger than a specified parameter or otherwise the decision maker is indifferent. Therefore, they are called as Indifference-zone (IZ) procedures.

Another popular approach is to maximize the probability of correct selection (PCS) given a computing budget called as Optimal Computing Budget Allocation (OCBA). Table 1 provides the key differences between IZ and OCBA approaches. For empirical comparison of the performance, see Branke et al. (2007) and Inoue et al. (1999). As simulation which is time consuming is used to estimate the performance measure, efficiency becomes a key issue. OCBA focuses on the efficiency issue by intelligently controlling the number of simulation replications based on the mean and variance information. Intuitively, to

* Corresponding author's email: iseleelh@nus.edu.sg
ψ Corresponding author's email: cchen9@gmu.edu, , cchen9@cc.ee.ntu.edu.tw

ensure a high probability of correctly selecting the desired optimal designs, a larger portion of the computing budget should be allocated to those designs that are critical in identifying the necessary ordinal relationships. The questions on how to identify the critical designs and how to allocate to critical and non-critical designs arise. It turns out that the answer actually depends on specific problem settings. We use a simple example to illustrate the ideas (Chen et al., 2008a). Figure 1(a) shows the 99% confidence intervals in a trivial case with 5 designs after initial simulation replications are performed. Given that the objective is to minimize cost, it is intuitive that design 1, 4, and 5 should not receive further simulation replications as they are clearly worse than design 2 and 3. In most cases such as the one shown in Figure 1(b), it is difficult to determine which designs to be further simulated and the amount of additional simulation replications to be made. OCBA therefore plays an important role in addressing these questions.

The focus of the paper is to specifically review the main idea from Chen et al. (2008a) and Chen and Lee (2010) together with various extensions of OCBA and recent advances in OCBA research. The basics of OCBA are provided in the next section. Section 3 provides an overview of the different extensions of OCBA while Section 4 describes the application of OCBA in the real-world problem and its integration with search algorithm. The open challenges for future OCBA research are discussed in Section 5. Section 6 concludes this paper.

Table 1. Key differences between OCBA and IZ procedures

| Basis of Comparison | OCBA | IZ |
|---|---|---|
| Focus | Efficiency (maximizing PCS) | Feasibility (finding a feasible way to guarantee PCS) |
| Total number of simulation replications required | Equal to the computing budget which is set by the decision maker | Uncertain (it depends on when the stopping rule to guarantee PCS is met) |
| Value of PCS achieved | The actual PCS is unknown. However, it is also possible to guarantee PCS as long as the value of Approximate PCS (APCS) is greater than the desired PCS. (Note that APCS is easy to compute) | The desired PCS is guaranteed to be achieved. The actual PCS is unknown and it is usually much higher than the desired PCS as the procedure is developed based on the least favorable configuration |
| Assumptions | Both Bayesian and Frequentist view | Based on Frequentist view |
| Use of indifference-zone concept in making comparison | Does not incorporate the indifference-zone concept except in Teng et al. (2010) | Always incorporate the notion of indifference zone |
| Flexibility for alternative formulation | Easy to incorporate other objective than PCS such as minimizing opportunity cost | It is more difficult to develop stopping rule to guarantee other objective than PCS |

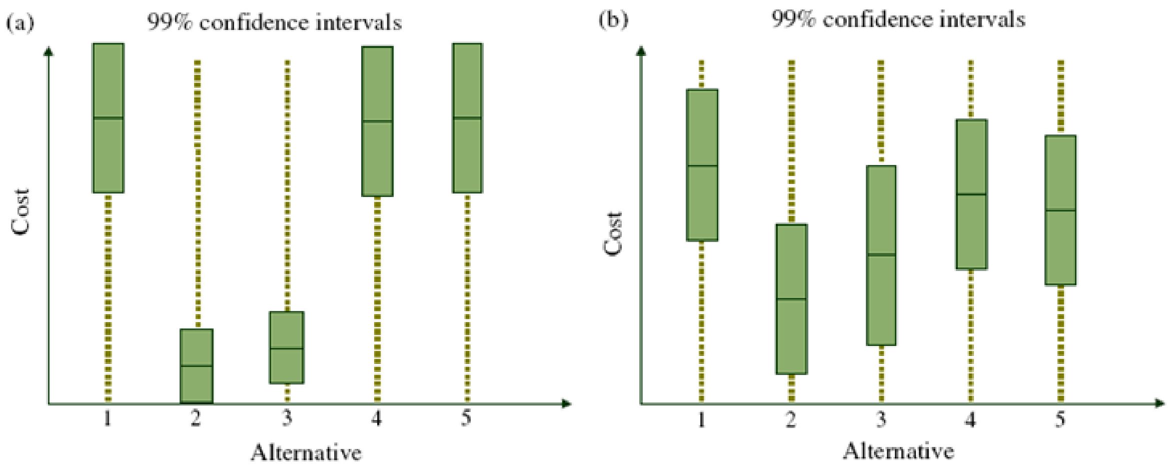


Figure 1. Illustration of 99% confidence intervals after some preliminary simulation in (a) a trivial case and (b) a more common case

## 2. BASICS OF OPTIMAL COMPUTING BUDGET ALLOCATION

The computing budget allocation problem falls into the traditional ranking and selection settings where ranking and selection is usually conducted based on the observed sample mean. Due to the underlying uncertainty with sampling, the selection result is not deterministic and probably varies. It is therefore necessary to establish the proper measurement of selection quality, which is highly depending on the decision makers' preference or on the specific interest with the problem domain. Then the computing budget allocation problem can be generally formulated as an optimization problem aiming to find the optimal allocation scheme, such that 1) the selection quality can be optimized under limited computing budget available, or 2) least computing budget would be needed to maintain the selection quality at a desired level. However, focusing only on the decision variables, or on the allocation scheme, the two types of formulation is indifferent (Chen et al. 2000a, Chen and Yücesan 2005) and only the first formulation will be discussed in the following text.

Common assumptions are usually made to make the evaluation of selection quality tractable. For the simulated systems, studies conducted usually assume implicitly that the steady state of the simulation system can be achieved with each run. However, the transient state of simulation systems may be of interest and sometimes is the only information available (Morrice et al., 2008, 2009). Another commonly made assumption is the independently and normally distributed observation applying to each individual design. This assumption is partially justified by the Law of Large Numbers, where the batch mean can be used as a single observation. Another commonly employed assumption is the equal unit cost with each simulation run under different designs, such that the computing budget can be simply interpreted as the total number of simulations totally available. For the case when computing time for one single run is different across the alternatives, see Chen et al. (1998) and Chen and Lee (2010). In addition, the concept from ordinal optimization is also generally used, based on which approximating relative order between alternatives is easier than estimating the performance measures accurately. For works with closer association with ordinal optimization, see Chen et al. (2000b), Chen et al. (2006), Dai (1996), Dai and Chen (1997), Dai et al. (2000), Ho et al. (2000), Lee et al. (1999), and Teng et al. (2007).

The most frequently used measurement of selection quality is the probability of correct selection, which treats correctly selecting the desired designs as a random event and measures the probability of such event. There are two different points of view on defining such a probability. From the frequentist's perspective, the true responses (mean and variance with given distribution) for each design are assumed known, and the problem is then formulated as to find the optimal computing budget allocation scheme based on the true information, such that the desired designs can be selected based on observation with the highest probability. On the other hand, the problem can also be considered from the traditional Bayesian point of view, where the true responses are assumed unknown and estimated by sample information, and the probability is defined as the probability that the selected designs are actually the desired ones. Let $k$ be the number of designs, $N_i$ be the number of simulation replications for design $i$, and $T$ be the total computing budget. Without loss of generality, the problem can be formulated as

$$\begin{aligned} &\max_{N_1,\dots,N_k} \quad PCS \\ &\text{s.t.} \quad N_1 + N_2 + \dots + N_k \le T \\ &\qquad N_i > 0 \end{aligned} \tag{1}$$

For evaluation of probability of correct selection, there is usually no mathematically closed form expression and a proper lower bound of it is used instead as the objective. The Karush-Kuhn-Tucker (KKT) conditions can then be applied to the formulation above and the optimality conditions can be derived. However, due to complexity in expressing PCS, direct solution from the KKT conditions is still intractable. Multiple versions of the lower bound have been developed attempting to achieve an easily derived expression of probability (Chen, 1996), and a variety of heuristics, the gradient search method, for instance, have been investigated to find the optimal solution (Chen, 1995; Chen et al., 1996; Chen et al., 1997). The allocation scheme derived in these papers is shown to be efficient. However, it is still not able to show optimality and intelligence or the underlying knowledge is not adequately interpreted. Chen et al. (1999b) and Chen et al. (2000a) firstly introduce the asymptotic OCBA framework by assuming that the total computing budget is infinite. Based on such a framework, the derivation based on KKT conditions can be simplified using limit theory and the optimal solutions can be derived under mild conditions. The optimal solution developed also provides evident insight in interpretation of the allocation rule. Let $N_i \equiv \alpha_i T$, design $b$ be the best design, $\sigma_i^2$ be the variance for design $i$, $\mu_i$ be the mean of design $i$, and $\delta_{b,i} \equiv \mu_i - \mu_b$. Based on the work by Chen et al. (2000a), PCS is asymptotically maximized when the relationship between two non-best design $i$ and $j$, $i \ne j \ne b$ is

$$\frac{N_i}{N_j} = \frac{\alpha_i}{\alpha_j} = \left( \frac{\sigma_i / \delta_{b,i}}{\sigma_j / \delta_{b,j}} \right)^2, \tag{2}$$

and the number of simulation replications for the best design is given as

$$N_b = \sigma_b \sqrt{\sum_{i=1, i \ne b}^{k} \frac{N_i^2}{\sigma_i^2}}. \tag{3}$$

It shows that the noisier the simulation output, the more replications are allocated. More replications are also given to the design of which mean is closer to that of the best design.

Moreover, given that each design will be allocated infinitely often under the asymptotic framework, it also validates the assumption of Gaussian distributions by Law of Large Numbers, without assuming using batch analysis. The asymptotic assumption also implies the convergence of the two allocation schemes derived from either frequentist' perspective or Bayesian perspective, as when the design is simulated for infinite times, the sample mean would actually converge to its true value almost surely, which would lead to the convergence of the Bayesian allocation scheme to the frequentist's one.

Alternative measures of selection quality are also widely developed. Trailovic and Pao (2004) attempt to minimize variance. Another important measure of selection quality is the expected opportunity cost, which penalizes particularly bad choices more than mildly bad choices (Chick and Wu, 2005; He et al. 2007). It can be seen that these alternative measures are also dependent on the correct (or incorrect, which is complement) selection result. In this paper, we focus mainly on the measurement using PCS.

The effectiveness of OCBA in saving computing budget is illustrated in Table 2 which is directly taken from Chapter 4 in Chen and Lee (2010). It provides two measures of OCBA performance relative to Equal Allocation (EA). EA is often used in simulation studies for comparing alternatives. It basically divides the computing budget equally across all designs. The first measure is called as speedup factor. It shows how fast OCBA relative to EA in reaching the same value of PCS, in this case 99% PCS is used. Let $T_{OCBA}$ and $T_{EA}$ be the computing budget for reaching 99% PCS. The speedup factor would then be equal to $T_{EA}/T_{OCBA}$ . For example when there are 10 designs, Table 2 shows that OCBA is 3.40 faster than EA. It is also shown that the savings of using OCBA instead of EA indicated by the speedup factor increases when the number of designs is increased. The second measure is called as Equivalent Number of Alternatives with a Fixed Computing Budget, ENAFCB($k$). This is equal to the number of designs divided by the speedup factor. The purpose of ENAFCB($k$) is to show the number of designs that can be simulated using EA given the computing budget needed by OCBA to simulate $k$ number of designs. For example, ENAFCB(100) is 4.99 meaning that OCBA is able to simulate 100 designs by only spending the same effort needed by EA to simulate less than 5 alternatives. The results show the potential of OCBA in enhancing the efficiency in simulation optimization. From the perspective of the first measure, OCBA requires less computing budget than EA to reach the same value of PCS. The second measure gives another angle to look at the advantage of OCBA that it is able to simulate more designs given the same computing budget.

Table 2. Illustration of the performance of OCBA compared to EA

| Number of designs, $k$ | 5 | 10 | 25 | 50 | 75 | 100 |
|---|---|---|---|---|---|---|
| Speedup factor using OCBA | 2.08 | 3.40 | 7.86 | 12.69 | 16.50 | 20.05 |
| ENAFCB ($k$) | 2.40 | 2.94 | 3.18 | 3.94 | 4.55 | 4.99 |

In the simulation practice, the simulation budget is usually allocated in a sequential approach as follows:

INPUT:
number of designs, total computing budget, initial number of replications, increment in each iteration

INITIALIZE:
Initial number of replications for each alternative is performed

LOOP: WHILE the total number of replications conducted so far is less than the total computing budget, DO:
  UPDATE:
  Calculate sample mean and variance; determine the best design based on the sample mean
  ALLOCATE:
  Add increment to the total replications conducted so far and determine the new number of replications for each alternative based on the OCBA rule and compute the additional replications that need to be conducted for each alternative.
  SIMULATE:
  Perform the additional number of replications for each alternative

END OF LOOP

For discussions on the choice of the initial number of replications and the increment, see Chen et al. (2008b), Chen et al. (2010), and Law and Kelton (2000).

## 3. ALLOCATION RULES FOR DIFFERENT SIMULATION OPTIMIZATION PROBLEMS

Problems may distinct from each other by various specifications. From the ranking and selection perspective, ranking of the simulated designs may vary depending on the following characteristics of a specific problem:

a) *Ranking criteria: objectives and constraints*

For the problem considered, it is usually aimed to find the design(s) that possess the optimality. However, under certain circumstances, feasibility of the designs may also be taken into consideration and such concerns would be formulated as specific constraints with the OCBA problem.

b) *Number of objectives: single or multiple*

Ranking of designs with only one objective is mostly intuitive by directly comparing their cardinal sample mean. However, for designs evaluated with multiple objectives, the direct cardinal comparison is not applicable as designs may be competing at different objectives. The transformation approach which reduces the problem into a single objective one by weighted average is indifferent from the basic single objectives problems, and it may not fully capture the problem structure due to the lack of preference over the objectives. The concept of Pareto optimality is employed instead, where the goodness of a design is measured in terms of domination. It should be noted that there are studies on duality between single objective problems with constraints and multi-objective problems without constraint. However, such duality is not completely equivalent and is not considered in the asymptotic OCBA framework. Moreover, ordinal comparison, rather than simulation precision, of the objective(s) with each design is of interest.

c) *Selection target*

For single objective problems, each design can be uniquely ranked, and the single best design is the most common target of selection for single objective problems with or without constraints. Alternative selection target involves the optimal subset containing top m designs, which is also distinct for single objective problems. For multi-objective problems, however, there is usually no single best design or exact top m designs since multiple designs may probably share the same rank and cannot be differentiated. Thus the Pareto set, $S_p$, containing all the non-dominating designs and the non-Pareto set, $\overline{S}_p$, containing all the dominated designs can be assessed. In general, either for single objective problems or multi-objective ones, the selection target can be generally those designs with their ranks up to a certain level, where, for instance, the top m designs are those with their ranks less than or equal to m (assuming that ranks start from 1), and when $m = 1$, the selection target reduces to the single best. For multi-objective problems, the selection target is also those designs with ranks up to a certain level, say $\gamma$, and when $\gamma = 0$ (assuming that ranks start from 0 here), the selection target would reduce to the Pareto set.

d) *Measurement of selection quality: Probability of Correct Selection*

Evaluation of the probability of correct selection depends on the underlying distributions of the sampled data, where only Gaussian distributions are discussed in this paper. The probability of correct selection depends on the sampling correlation among designs; and additionally, for multi-objective problems, the probability also depends on the sampling correlation among different objectives.

Under the asymptotic OCBA framework, the following problems are studied with the common assumption of normally distributed but non-correlated samples, which are summarized as follow:

OCBA1: single objective problem without constraints, aiming to select the single best design (Chen et al. (2000a)
OCBA-m: single objective problem without constraints, aiming to select top m designs (Chen et al. 2007; Chen et al. 2008b)
OCBA-CO: single objective problem with one stochastic constraint, aiming to select the best feasible design (Pujowidianto et al. 2009)
MOCBA: multi-objective problem without constraints, aiming to select the Pareto set (Lee et al., 2004; Chen and Lee, 2009; Lee et al., 2010)

The optimal budget allocated to each design in the asymptotic OCBA framework is as follows:

$$\alpha_i = \frac{\beta_i}{\sum_{i=1}^{k} \beta_i}, \tag{4}$$

where the corresponding values of $\beta_i$ are shown in Table 3. Note that $w_i^{OCBA1}$, $w_i^{MOCBA}$, $w_i^{OCBA-CO}$, $w_i^{OCBA-m}$ are the weights of the replications of design i's to the replications allocated to the selected design(s) in different OCBA procedures. For example, in OCBA1, where s is the best design,

$$w_i^{OCBA1} = \frac{\sigma_s}{\sigma_i}. \tag{5}$$

$\eta_i^{OCBA1}$, $\eta_i^{MOCBA}$, $\eta_i^{OCBA-CO}$, $\eta_i^{OCBA-m}$ are the noise-to-signal ratio of design i in different OCBA procedures. The noise-to-signal ratio indicates how likely an incorrect decision is made. The higher the ratio of a non-desired design, the more chance it is to be incorrectly selected as the desired designs. The noise refers to the uncertainty in the performance measure(s) while the signal of a non-desired design measures its distance to the desired designs. Therefore, the insight from this ratio is that we should allocate more replications to the non-desired designs with greater noise or smaller signal to minimize the probability of incorrect selection. For illustration purpose, the following is the noise-to-signal ratio in OCBA1,

$$\eta_i^{OCBA1} = \frac{\sigma_i}{\mu_i - \mu_s}. \tag{6}$$

$\eta_i^{MOCBA}, \eta_i^{OCBA-CO}, \eta_i^{OCBA-m}$ also have the similar expression, namely the ratio of standard deviation to the sample means' difference, but they are a little different due to different problems. For detailed expressions of $w_i^{MOCBA}, w_i^{OCBA-CO}, w_i^{OCBA-m}$, $\eta_i^{MOCBA}, \eta_i^{OCBA-CO}, \eta_i^{OCBA-m}$, please refer to Lee et al. (2004, 2010), Pujowidianto et al. (2009) and Chen et al. (2008b).

Table 3. Allocation schemes for various problems

| Different OCBA procedures | Design(s) with multiple comparisons | Other Designs |
|---|---|---|
| OCBA1 | The best design: $\beta_s = \sqrt{\sum_{i \neq s}^{k} \left(w_i^{OCBA1} \beta_i\right)^2}$ | $\beta_i = \left(\eta_i^{OCBA1}\right)^2$ |
| MOCBA | Designs playing the role of dominating $\beta_s = \sqrt{\sum_{\substack{i \in \{\bar{S}_p \cap \{j \mid \text{Design } s \\ \text{is the design dominates} \\ \text{desgin } j \text{ most}\}\}}} \left(w_i^{MOCBA} \beta_i\right)^2}$ | Designs playing the role of being dominated $\beta_i = \left(\frac{\eta_i^{MOCBA}}{\eta_m^{MOCBA}}\right)^2$ in which design $m$ is any one in $\bar{S}_p$. |
| OCBA-CO | The best feasible design: $\beta_s = \max\left(\sqrt{\sum_{\substack{i \in \{\text{Designs in the} \\ \text{optimality dominance set}\}}} \left(w_i^{OCBA-CO} \beta_i\right)^2}, \left(\frac{\eta_s^{OCBA-CO}}{\eta_i^{OCBA-CO}}\right)^2 \beta_i\right)$ | $\beta_i = \left(\eta_i^{OCBA-CO}\right)^2$ |
| OCBA-m | $\beta_s = \left(\eta_s^{OCBA-m}\right)^2$ | $\beta_i = \left(\eta_i^{OCBA-m}\right)^2$ |

It can be seen from Table 3 that the allocation schemes for various problems share some common properties. First, it is necessary to distinguish the type of comparisons that is incurred to each design, which is critical in identifying whether the design is desired (optimal) or not. There are two types of comparison in general, i.e. individual comparison with each design, or multiple comparisons with a certain design. Individual comparison means that a certain design is only compared with a certain value or some other design once, and then the allocation related to this design can be determined accordingly. Multiple comparisons are related to a certain design where the goodness of the design can only be determined by comparing with several other designs. For instance, for the OCBA1 problem, the best design should be compared with all other designs to determine its superiority, and thus multiple comparisons are incurred with it, whereas for all designs other than the best, it is only necessary for them to compare with the best design to show their inferiority, and thus individual comparison is incurred with each design.

It should be noted that identification of comparison types may not be straightforward for complicated problems. For instance, for multi-objectives problems, because of the underlying complexity in cross comparisons of designs, all designs can be grouped by the roles they are playing, either dominating or being dominated. For designs playing the role of dominating, each of them is dominating multiple designs, and thus multiple comparisons are incurred; for designs playing the role of being dominated, each design is only dominated by one design, and hence the individual comparison is incurred.

In general, for designs with individual comparison, the related allocation would follow the noise-to-signal ratio scheme, which in the statistical sense, is critical in identifying the significant difference from one value to the other; whereas for designs with multiple comparisons, the related allocation would follow the sum of weighted variance scheme, where the variance would rule out the comparisons for significant differences. It should be noted that the noise-to-signal ratio scheme follows the assumption of Gaussian distributions, where the ratio represents the exponent of the probability density function of Gaussian distributions, and plays the role of convergence rate with regard to the probability of comparison result.

Both the OCBA procedures for single objective and multi-objective problems have been extended in several ways. Table 4 lists the works extending OCBA for single objective while that for multi-objective are provided in Table 5. For instance, the extensions consider the correlation between the designs (Fu et al., 2004, 2007) or use non-normal distributions (Glynn and Juneja, 2004) which will make the formulation of PCS to be different and consequently affecting the optimal allocation rules. Other extensions that have been made are to combine OCBA with regression (Brantley et al., 2008) or splitting (Shortle and Chen, 2008). Combining OCBA and regression will also result in different PCS. Splitting is combined with OCBA when the problem is to evaluate rare-event probabilities. In this case, the decision variable becomes the number of simulation replication for each design in each splitting level while the objective is changed to minimize the variance of the rare-event probability estimator.

Table 4. Literatures extending OCBA for single objective

| References | Extensions made to OCBA |
|---|---|
| Brantley et al. (2008) | Incorporate regression analysis which can utilize the information from the underlying function |
| Chen et al. (2003b) | Introduce minor random perturbation to the original OCBA |
| Chick et al. (2010) | Develop a sequential one-step myopic allocation procedure |
| Frazier and Powell (2008) | Consider the problem with a correlated multivariate normal prior belief |
| Fu et al. (2004, 2007) | Account for correlation between designs |
| Glynn and Juneja (2004) | Address performance measure that is not normally distributed |
| Morrice et al. (2008, 2009) | Deal with transient mean that is a function of other variable such as time |
| Shortle and Chen (2008) | Deal with rare event simulation by minimizing variance of its estimator |

Table 5. Literatures extending OCBA for multi-objectives

| References | Extensions made to MOCBA |
|---|---|
| Lee et al. (2007) | Minimize expected opportunity cost |
| Branke and Gamer (2007) | Transform multiple objectives into single objective with ability to interactively update the weight distribution |
| Teng et al. (2010) | Incorporate the indifference-zone concept |

## 4. OCBA APPLICATIONS AND INTEGRATION WITH SEARCH ALGORITHMS

Because of their well performance to get a high confidence level under certain computing budget constraint, OCBA procedures show great potential in improving simulation efficiency for tackling simulation optimization problems, finding or selecting the best solutions for a system in which the performance of solutions is evaluated based on the output of the simulation model of this system. Therefore, the application of OCBA procedures is studied by many researchers. The application can be classified by different ways in which OCBA is applied. For the simulation optimization problems given a fixed set of alternatives, OCBA can be directly applied to select the optimal one among all these solutions. For the simulation optimization problems with enormous size or continuous solution space, the application of OCBA is indirect by integrating it with search algorithms.

### 4.1 Direct application of OCBA to simulation optimization problems

In real industry, facing with the competitive and fast-changing business environment, companies focus on enhancing their core competence and strengthening the corporation with other related companies. The improvement on companies' own manufacturing and service operations and the constantly expanding information, economic and trade exchange and cooperation among companies bring on the increasing complexity of companies' networks. Many problems in such large, complex, and stochastic networks are large scaled, without an analytical structure of the problem, and with high uncertainties. For example, the product design problems, operation scheduling problems and vehicle routing problems all belong to the combinatorial optimization problems and their structures all change dynamically.

With these difficulties, traditional simulation approaches usually cannot handle these problems because of the high computational cost. On the other side, OCBA can effectively reduce computing cost by intelligently determining the number of simulation replications to different designs and shows superiority over ordinal optimization which has exponential convergence rate under certain conditions.

Therefore, OCBA provides us an effective way to solve these difficult operation problems, such as the combinatorial optimization problems which include machine clustering problems (Chen et al., 1999a), electronic circuit design problems (Chen et al., 2003a), semiconductor wafer fab scheduling problems (Hsieh et al., 2001; Hsieh et al. 2007). In Chen and He (2005), the authors apply OCBA to a design problem in US air traffic management due to the high complexity of this system. For multi-objective problems, Lee et al. (2005) employ MOCBA to optimally select the non-dominated set of inventory policies for the differentiated service inventory problem and an aircraft spare parts inventory problem. In these papers, although certain changes to OCBA are made according to different problems, the main idea is still retained. It builds the real industry problem as a design(s) selection problem and would like to allocate the limited computing budget to each design such that the probability of correct selection can be maximized. A greedy approach or an asymptotically optimal allocation rule to solve this model is then provided by OCBA. The numerical result in these papers all show that OCBA can save much computing cost compared with the traditional ordinal optimization methods and so on.

### 4.2 Indirect application of OCBA to simulation optimization problems by integrating with search algorithms

When we directly use OCBA to get the optimal solution for a simulation optimization problem, every solution should be given beforehand. In addition, OCBA allocates every solution certain replications at the initial stage to get a general idea about these solutions. If the solution space is of enormous size, continuous, or even unbounded, the total computing replications needed will be prohibitively high. Thus, direct application of OCBA is only suitable for simulation optimization problems whose solution spaces are discrete, bounded and within certain size.

To tackle the simulation optimization problems with continuous or enormous sized solution spaces, many search algorithms have been proposed to search good solutions in the solution space by efficient ways instead of sampling all solutions to get the optimal one. At the same time, in the search process, search algorithms need to repeatedly evaluate and compare candidate solutions to decide the next search direction. Because the objective functions in simulation optimization problems are in the stochastic form, certain computational effort should also be spent on getting the estimates of the objective function at these candidate solutions, besides the need to search the space for new candidate solutions. In this evaluation and comparison step, we already know the solutions required to be compared and the total number of candidate solutions at each iteration is relatively small, so OCBA can be applied to enhance the simulation efficiency of this step. Therefore, the integration of OCBA and search algorithms is better than OCBA or search algorithm individually in dealing with difficult simulation optimization problems.

Some frameworks about how to integrate OCBA with search algorithms have been developed. Lee et al. (2006b) propose a framework for the integration of MOCBA with search algorithms which is also applicable for general OCBA procedures. Most papers about the integration of OCBA with search algorithms actually follow the basic idea of this framework. OCBA is applied to determine the right replications allocated to each candidate solution, which are generated by search algorithms at each iteration, to accurately estimate the fitness of these solutions and compare them.

We can classify the related papers based on the different search algorithms integrated with OCBA. For the integration with Nested Partition (NP), Shi et al. (1999) show its application in discrete resource allocation. Shi and Chen (2000) then give a more detailed hybrid NP algorithm and prove its global optimal convergence. Brantley and Chen (2005) use OCBA with mesh moving algorithm for searching the most promising region. Chew et al. (2009) integrate MOCBA with NP to handle multi-objective inventory policies problems. For the integration with evolutionary algorithms, Lee et al. (2008) discuss the integration of MOCBA with Multi-objective Evolutionary Algorithm (MOEA). In Lee et al. (2009), Genetic Algorithm (GA) is integrated with MOCBA to deal with the computing budget allocations for Data Envelopment Analysis. The integration of OCBA with Coordinate Pattern Search for simulation optimization problems with continuous solution space is considered in Romero et al. (2006). Chen et al. (2008) show numerical examples about the performance of the algorithm combining OCBA-m with Cross-Entropy (CE). The theoretical part about the integration of OCBA with CE is then further analyzed in He et al. (2010).

In the papers mentioned above, the hybrid algorithms developed by integrating OCBA with search algorithms show a great improvement by numerical experiments, but they mostly just consider the computing cost optimization in estimating the performance values of candidate solutions and forget the computing cost optimization in sampling and searching the sample space. To avoid this limitation, Lin and Lee (2006) develop a more general framework that can dynamically determine the computing resources allocated to the process of estimating the performance values of candidate solutions (named as "depth process" in the paper) and the process of searching the solution space (named as "breath process" in the paper). The insights here are that in simulation optimization, the estimates will be less accurate if we spend most of the efforts in selecting as many designs in the search space as possible due to the uncertainties in the performance measure. However, if we spend most of the efforts in getting accurate estimate of the performance measure, the number of designs selected is small and the best one may not be near the global optimal. Therefore, the decision maker needs to balance the efforts for these two processes. Lee et al. (2006a) goes a step further by also considering the selection of an appropriate degree of information used in the sampling method to sample a design. In all these papers, the numerical result demonstrates the significant improvements gained by integrating OCBA with search algorithms.

## 5. OPEN CHALLENGES

Although OCBA approaches have great potential in dealing with simulation optimization problems, there still exist some challenges in this field. Andradóttir et al. (2005) and Kim and Nelson (2007) give excellent discussion about the challenges of simulation optimization approaches and ranking and selection procedures respectively. In this section, we will only focus on challenges of OCBA procedures.

All OCBA procedures are still the conservative, not optimal, allocation rules because they are developed based on the lower bounds of the probability of correct selection. These lower bounds are derived to simplify the probability of correct selection. Improving these lower bounds is still an open research problem. Moreover, it is also a deserved research problem using large deviation theory formulates the computing budget allocation problem to avoid the hardness in building the expression of probability of correct selection, so that we can find the optimal allocation rules. For references in comparing alternatives using large deviation theory, see Blanchet et al. (2008); Glynn and Juneja (2004); Szechtman and Yücesan (2008).

It is also observed that OCBA procedures evaluate the goodness of a design by its mean. Sometimes, using mean as a performance metric is far from enough. Quantile, another metric of designs' performance, owns the flexibility to adjust the performance metric among the downside risk, the central tendency, and upside risk (Batur and Choobineh, 2010). It can provide the decision maker a most appropriate criterion of the problem under consideration. Hence, developing the allocation rule when the selection is based on quantile instead of mean is also an open challenge problem for OCBA procedures.

Another limitation of some OCBA works is that the allocation is based on an asymptotically optimal solution to the approximate problem. Although OCBA performs well in the numerical experiments conducted, there is no theoretical proof to show how good the finite-time performance of OCBA is with respect to the real problem. In addition, OCBA does not reduce the computing budget required for a single simulation replication. If each simulation replication takes a very long time due to the complexity, it is often not possible to run the simulation more than one single replication for each design. Therefore, OCBA is not applicable in this case.

For the application of OCBA, integrating OCBA with search algorithms can effectively enhance the simulation efficiency, but it cannot improve the search part about the algorithm. He et al. (2010) point out that it is still an open challenge to develop more approaches in integrating OCBA with search algorithms which can optimally allocate computing budget according to the objective of the search algorithm leading to improvement in the search and overall efficiency.

The previous sections show OCBA's application to simulation optimization problems. In fact, OCBA is more than an efficient simulation optimization approach. From a generalized view, OCBA is applicable to the broader domains than simulation optimization. The generalized OCBA model is shown in Figure 2 which is directly taken from Chapter 8 in Chen and Lee (2010). In the generalized model, the total budget T is the amount of resources in all, which should be allocated to different processors. Each processor gets the budget Ni and generates the output Xi. All these outputs are then input into a synthesizer to obtain a final outcome. In this framework, the generalized OCBA model plays the role that optimally determines how to allocate all resources to each processor such that the synthesizer can obtain the optimal result.

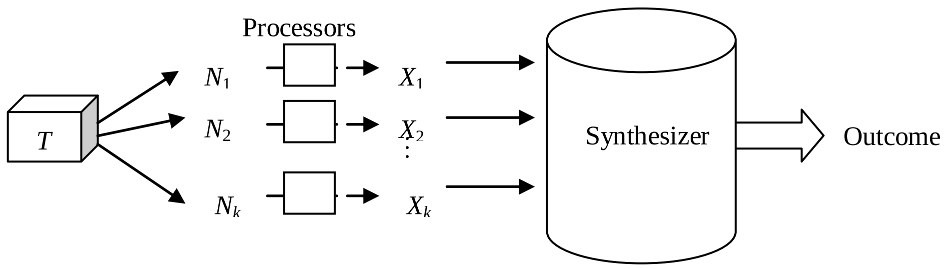


Figure 2. A generic view of the OCBA framework.

All different OCBA approaches follow the generalized framework, an optimization model determining the best way to allocate the budget to maximize a specific objective related to the outcome. Beyond optimization, Shortle and Chen (2008) extend the idea to problem of estimating a rare-event probability using simulation. The generalized OCBA model can also be extended to problem without simulations or optimizations. Lee et al. (2009) shows an example on how OCBA can help determine the budget allocation for data collection which maximizes the accuracy of an efficiency prediction in data envelopment analysis (DEA). Certainly the OCBA notion can be applied or extended to many other problems with or without simulations or optimizations.

## 6. CONCLUSION

Optimal Computing Budget Allocation (OCBA) is a concept of intelligently allocating simulation budget for maximizing the desired selection quality in finding the best or the top $m$ alternatives. The concept is especially useful given a limited computing budget which is common as simulation is computationally intensive. OCBA procedures utilize both mean and variance in determining the number of replications for each alternative and they are implemented sequentially. The numerical experiments in various papers show the efficient performance of OCBA compared to other procedures. They also indicate that OCBA performs well in finite time although the allocation rule in the procedures is developed in asymptotic condition, numerical experiments in the literatures show that it performs well in finite time.

The contribution of this paper is to present various extensions of the original OCBA to handle many different simulation optimization problems which can be used as a guide in future research. In addition, it also highlights that there are plenty future works that can be done in customizing OCBA to facilitate its integration with search algorithms leading to an improved efficiency in handling simulation optimization problems with large number of alternatives.

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