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Optimum Profit Model for Determining Purchaser's Order Quantity and Producer's Order Quantity and Producer's Process Mean and Warranty Period

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Abstract— In this paper, the author presents a modified Chen and Liu's (2007) model for determining the optimum order quantity, process mean, and warranty period of product between the producer and the purchaser. Assume that the demand quantity of the end of customer and the quality characteristic of product are independent normally distributed. Taguchi's symmetric quadratic quality loss function is applied in measuring the product quality. The numerical result shows that the standard deviation of the quality characteristic, σ_y, the mean demand of the end of customer, μ_x, the parameter of demand, α_1, the parameter of selling price, *a*, and the sale price per unit for the end of customer, *R*, have a significant effect on the expected total profit of the society.

Keywords — Order quantity, process mean, quadratic quality loss function, warranty period.

1. INTRODUCTION

Product quality improvement is useful for increasing the customer's satisfaction. Hence, the maximum expected profit model between the producer and the purchaser is an important problem for the supply chain system. The producer needs to determinate the optimum product quality and the purchaser needs to consider the order quantity of product. Hence, the market needs to solve the problem o f "how to get a trade-off between them". Recently, many researchers have addressed this work.

Chen and Liu (2007) presented the optimum profit model between the producer and the purchaser for the supply chain system with pure procurement policy from the regular supplier and mixed procurement policy from the regular supplier and the spot market. In 2008, they further proposed an optimal consignment policy considering a fixed fee and a per-unit commission. Their model determines a higher manufacturer's profit than the traditional production system and coordinates the retailer to obtain a large supply chain profit. Li and Liu (2008) considered the problem about the retailer determining his optimal order quantity and the manufacturer determining his optimal reserve capacity. Their model can make both sides of the supply chain to increase their profit.

The optimum process mean setting has a major effect on the defective fraction of product, the inspection/ reprocessing cost, and the expected total profit/cost. There are considerable attentions paid to this work by applying Taguchi's (1986) quadratic quality loss function, e.g., Chen (2006), Chen and Lai (2007a, 2007b), and Chen and Khoo (2008, 2009).

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There are some works that assured a post-sale warranty cost where imperfection is prevalent in the production system. The works of Djamaludin et al. (1994), Yeh and Lo (1998), Yeh, et al. (2000), Wang and Sheu (2000, 2003), Wang (2004), and Yeh and Chen (2006) can be quoted along this line. Wang and Sheu (2000) suggested a speedy solution procedure for Djamaludin et al.'s (1994) model. Subsequently, Wang and Sheu (2003) investigated the lot size problem for repairable products sold under the free-repair warranty policy. They searched the optimal lot size for minimizing the expected total cost including the set-up cost, the inventory holding cost, and the warranty cost. Yeh and Chen (2006) examined the problem of free minimal repair warranty product in a deteriorating production system. They developed an approximate solution for obtaining the optimal lot size and the corresponding inspection policy. In the above mentioned models, an assumption is made where the warranty period of the product is a known quantity. In 2004, Ladany and Shore considered the problem of determining the optimal warranty period of product with lower specification limit. The products are assumed that sale-price per manufactured item increases linearly with the warranty period. Ladany and Shore (2007) further considered the joint problem of sale price and warranty period setting.

In Chen and Liu's (2007) model, they neglected the effect of product quality on the demand quantity of the end of customer and only considered the order quantity satisfying the uniform distribution. In 2009, Chen proposed a modified Chen and Liu's model based on the demand quantity of the end of customer correlated with producer's product quality. Taguchi's symmetric quadratic quality loss function is applied for measuring the product quality. The bivariate normal distribution is adopted for formulating the purchaser's expected profit model. The optimum purchaser's order quantity and supplier's process mean are jointly determined.

Although the economic order quantity, process mean setting, and warranty period of product setting are three different problems that occur in the inventory management, quality control, and warranty policy. If they are combined into an integrated model, then one can obtain the optimum decision parameters with maximum expected total profit of the society. In this paper, the author proposes a modified Chen and Liu's (2007) model for determining the optimum order quantity, process mean, and warranty period of product between the producer and the purchaser. Assume that the demand quantity of the end of customer and the quality characteristic of product are independent normally distributed. Ladany and Shore's (2007) model is integrated into the modified Chen and Liu's (2007) model for obtaining the optimum purchaser's order quantity, the expected lifetime of product, and the warranty period of product by maximizing the expected total profit of the society. The advantage of this modified Chen and Liu's (2007) model is to obtain the joint control of production quantity, process quality level, and warranty period. Taguchi's symmetric quadratic quality loss function is applied in measuring the product quality. Finally, the sensitivity analysis of parameters will be provided for illustration.

2. LITERATURE REVIEW

2.1 Chen and Liu's (2007) Model

Chen and Liu's (2007) pure procurement model is actually based on the standard news-vendor model without spot markets. They consider a single period supplier-buyer relationship in which a regular supplier produces short-life cycle products and a buyer orders products from the regular supplier and then sells to the end customer.

Assumptions:

1. A buyer purchases a finished product from a regular supplier and resells it at a price, R, to the end customer.
2. The regular supplier produces each unit at a cost, C.
3. The regular supplier and the buyer enter into a contract at a wholesale price, W.
4. The regular supplier sets the wholesale price to maximize his expected profit while offering the buyer a specific order quantity, Q.
5. When realized demand exceeds procurement quantity, unmet demand is lost; therefore, demand uncertainty exposes the buyer to risks associated with mismatches between the procurement quantity

and demand.

6. The procurement lead time is long relative to the selling season, so that the buyer cannot observe demand before placing the order.
7. The consumer demand, X, is an uniform distribution, i.e., $X \sim U[\mu_x - (\sigma_x/2), \mu_x + (\sigma_x/2)]$, where μ_x is the expected demand of customer and σ_x is the customer demand variability.

The buyer's profit is given by

$$\pi_{PS}^{R} = \begin{cases} RX - WQ + S(Q - X), & X < Q \\ RQ - WQ, & X \geq Q \end{cases} \tag{1}$$

where R is the sales price per unit; S is the salvage value per unit; Q is the quantity procured by the buyer from the regular supplier; X is the stochastic demand; W is the wholesale price per unit, paid by the buyer to the regular supplier.

The buyer's expected profit can be expressed as

$$E(\pi_{PS}^{R}) = \int_{\mu_x-(\sigma_x/2)}^{Q} [Rx - WQ + S(Q-x)] f(x)dx + \int_{Q}^{\mu_x+(\sigma_x/2)} [(R-W)Q] f(x)dx \tag{2}$$

where $f(x)$ is the probability distribution of X.

Let the partial derivative of the buyer's expected profit function with respect to Q be zero, i.e., $\frac{dE(\pi_{PS}^{R})}{dQ} = 0$. We have the optimal order quantity

$$Q^* = \left(\mu_x - \frac{\sigma_x}{2}\right) + \frac{(R-W)\sigma_X}{R-S} \tag{3}$$

The regular supplier maximizes his expected profit and determines the wholesale price per unit based on the buyer's order quantity Q^*. Hence, the regular supplier's expected profit is expressed as

$$E(\pi_{PS}^{S}) = (W-C)Q^* \tag{4}$$

Let the partial derivative of the regular supplier's expected profit function with respect to W be zero, i.e., $\frac{dE(\pi_{PS}^{S})}{dW} = 0$. The optimal W value of equation (4) yields

$$W^* = \frac{(R-S)\mu_x}{2\sigma_x} + \frac{(R+S+2C)}{4} \tag{5}$$

Substituting equation (5) into equation (3), the optimal Q value can be rewritten as

$$Q^* = \frac{1}{2}\left(\mu_x - \frac{\sigma_x}{2}\right) + \frac{(R-C)\sigma_X}{2(R-S)} \tag{6}$$

From equations (2), (4), (5), and (6), the buyer's and supplier's expected profits can be expressed as

$$E(\pi_{PS}^{R}) = \frac{(R-S)[(Q^*)^2 - (\mu_X - \frac{\sigma_X}{2})^2]}{2\sigma_X} \tag{7}$$

and

$$E\left(\pi_{PS}^{S}\right)=[\frac{(R-S)\mu_X}{2\sigma_X}+\frac{R+S-2C}{4}]Q^* \tag{8}$$

2.2 Ladany and Shore's (2007) MODEL

Some assumptions of Ladany and Shore's (2007) model are as follows:

The lifetime of product, Y, is exponential with known upper specification limit (U) and unknown lower specification limit (L) which is also defined as the warranty period of product.

The selling price of product, r_s, is positive proportional to L. Let $r_S = a + b \cdot L$, where $a > 0$ and $b > 0$.

The selling price is r_S for the lifetime of product between L and U. The selling price is $r_S \cdot r_L$ for the lifetime of product below L, where $0 < r_L < 1$. The selling price is $r_S \cdot r_U$ for the lifetime of product beyond U, where $0 < r_U < 1$.

The expected profit per item is

$$\begin{aligned} EP &= r_s \cdot r_L \int_0^L f(y)dy + r_s \int_L^U f(y)dy + r_s \cdot r_U \int_U^\infty f(y)dy \\ &= (a+b\cdot L)[(r_L-1)P(Y \le L)-(r_U-1)P(Y \le U)+r_U] \end{aligned} \tag{9}$$

where $f(y)$ is the probability density function of Y and $P(Y \le .)$ is the cumulative distribution function of Y.

According to Cobb-Douglas type demand function, the demand quantity of product, *Q*, is negatively proportional to the selling price, r_s, and positively proportional to the warranty period of product lifetime, *L*. Hence, Eq. (10) shows the demand quantity of product

$$Q = \alpha_1 \times r_s^{\beta_1} \times L^r \tag{10}$$

where $\alpha_1 > 0$, $r > 0$, and $\beta_1 < 0$.

The expected total profit of product for Ladany and Shore's (2007) model is as follows:

$$\begin{aligned} &TEP \\ &= Q[R-C_1]-K_1 \\ &= \alpha_1 \cdot (a+b\cdot L)^{\beta_1} \cdot L^r \cdot \{[(r_L-1)P(Y \le L)-(r_U-1)P(Y \le U)+r_U]-C_1\}-K_1 \end{aligned} \tag{11}$$

where C_1 is the production cost per item and K_1 is the fixed cost associated with manufacturing and selling.

Ladany and Shore (2007) adopted the response modeling methodology for obtaining the optimum warranty period of product with the maximum expected total profit.

3. MODIFIED CHEN AND LIU's (2007) MODEL

Assume that demand quantity of the end of customer and the quality characteristic of product are independent normally distributed. The modified Chen and Liu's (2007) model is as follows:

The purchaser's profit is given by

$$\pi_{PS}^{R} = \begin{cases} RX - WQ + S(Q-X) - X\cdot Loss(Y), if\ X < Q,\ \ L \le Y \le U \\ RQ - WQ - Q\cdot Loss(Y), if\ X \ge Q,\ \ L \le Y \le U \end{cases} \tag{12}$$

where Y is the normal quality characteristic of the product, $Y \sim N(\mu_y, \sigma_y^2)$; μ_y is the unknown mean of Y; σ_y is the known standard deviation of Y; $Loss(Y)$ is the quality loss per unit, $Loss(Y) = k(Y - y_0)^2$; k is the quality loss coefficient; y_0 is the target value of product.; $X \sim N(\mu_x, \sigma_x^2)$; μ_x is the known mean of X; σ_x is the known standard deviation of X.

Hence, the purchaser's expected profit is

$$\begin{aligned} &E(\pi_{PS}^{R}) \\ &= \int_{-\infty}^{Q}\int_{L}^{U}\left[Rx - WQ + S(Q-x)\right] f(x,y)\,dydx + \int_{Q}^{\infty}\int_{L}^{U}\left[R-W\right]Qf(x,y)\,dydx \\ &+ \int_{-\infty}^{Q}\int_{L}^{U} x\cdot Loss(y)\, f(x,y)\,dydx + \int_{Q}^{\infty}\int_{L}^{U} QLoss(y)\, f(x,y)\,dydx \end{aligned} \tag{13}$$

where $f(x, y)$ is the joint probability density function of X and Y.

The purchaser's expected profit can be rewritten as

$$\begin{aligned} &E\left(\pi_{PS}^{R}\right) \\ &= \{(R-S)[\mu_x\Phi(\frac{Q-\mu_x}{\sigma_x}) - \sigma_x\varphi(\frac{Q-\mu_x}{\sigma_x})] + Q(S-W)\Phi(\frac{Q-\mu_x}{\sigma_x})\}\cdot [\Phi(\frac{U-\mu_y}{\sigma_y}) - \\ &\Phi(\frac{L-\mu_y}{\sigma_y})]\ - [\mu_x\Phi(\frac{Q-\mu_x}{\sigma_x}) - \sigma_x\varphi(\frac{Q-\mu_x}{\sigma_x})]\int_{L}^{U} Loss(y)\, f(y)\,dy + (R-W)Q[1-\Phi(\frac{Q-\mu_x}{\sigma_x})] \\ &[\Phi(\frac{U-\mu_y}{\sigma_y}) - \Phi(\frac{L-\mu_y}{\sigma_y})] - Q\Phi(\frac{Q-\mu_x}{\sigma_x})\cdot\int_{L}^{U} Loss(y)\, f(y)\,dy \end{aligned} \tag{14}$$

where

$$\begin{aligned} &\int_{L}^{U} Loss(y) f(y) dy \\ &= k\{[(\mu_y - y_0)^2 + \sigma_y^2]\cdot\left[\Phi\left(\frac{U-\mu_y}{\sigma_y}\right) - \Phi\left(\frac{L-\mu_y}{\sigma_y}\right)\right] + \sigma_y[(\mu_y 2y_0 + L)\varphi(\frac{L-\mu_y}{\sigma_y}) - \\ &(\mu_y - 2y_0 + U)\varphi(\frac{U-\mu_y}{\sigma_y})]\} \end{aligned} \tag{15}$$

$\Phi(\cdot)$ is the cumulative distribution function of the standard normal random variable; $\phi(\cdot)$ is the probability density function of the standard normal random variable.

Consider the conforming product sold to the primary market and the non-conforming product scrapped and sold to the secondary market. The supplier's profit is given by

$$\pi_{PS}^{S} = \begin{cases} W - z - cY - i, & L \le Y \le U \\ S_p - z - cY - i, & Y < L \text{ or } Y > U \end{cases} \tag{16}$$

where W is the selling price per unit for the conforming product; S_p is the discounted price per unit for the non-conforming product scrapped; z is the constant product cost per unit; c is the variable product cost per unit; i is the inspection cost per unit.

Hence, the supplier's expected profit per unit is

$$E_1\left(\pi_{PS}^{S}\right)$$
$$= P(L<Y<U)[W-(z+c\mu_y+i)]\frac{1}{P(L<Y<U)} \tag{17}$$
$$+S_p[1-P(L<Y<U)]\frac{1}{P(L<Y<U)}$$

The supplier needs to produce $\frac{Q}{P(L<Y<U)}$ items in order to satisfy the buyer's order quantity. Hence, the supplier's expected profit is

$$E\left(\pi_{PS}^{S}\right)$$
$$=\frac{Q}{P(L<Y<U)}\cdot E_1(\pi_{PS}^{S})$$
$$=WQ-\left(z+c\mu_y+i\right)\frac{Q}{P\left(L<Y<U\right)}+S_p\left[\frac{Q}{P\left(L<Y<U\right)}-Q\right]$$
$$=Q\left\{W-\frac{z+c\mu_y+i-S_p\left[1-\Phi\left(\frac{U-\mu_y}{\sigma_y}\right)+\Phi\left(\frac{L-\mu_y}{\sigma_y}\right)\right]}{\Phi\left(\frac{U-\mu_y}{\sigma_y}\right)-\Phi\left(\frac{L-\mu_y}{\sigma_y}\right)}\right\} \tag{18}$$

The lifetime of product is one of the quality characteristics. One usually assumes that the quality characteristic of product is normally distributed. Hence, the normal lifetime distribution is considered in the Ladany and Shore's (2007) model. Let $r_L = r_U$, $r_S = W$, and $r_S \cdot r_L = S_p$, where $r_S = a + b\cdot L$ and $0 < r_L < 1$. By substituting some parameters of Ladany and Shore's (2007) model into the modified Chen and Liu's (2007) model, the supplier's expected profit can be rewritten as

$$E(\pi_{PS}^{S}) = Q\left\{W-\frac{z+c\mu_y+i-S_p\left[1-\Phi\left(\frac{U-\mu_y}{\sigma_y}\right)+\Phi\left(\frac{L-\mu_y}{\sigma_y}\right)\right]}{\Phi\left(\frac{U-\mu_y}{\sigma_y}\right)-\Phi\left(\frac{L-\mu_y}{\sigma_y}\right)}\right\} \tag{19}$$

Where

$$Q=\alpha_1\cdot r_S^{\beta_1}\cdot L^r \tag{20}$$
$$W=r_S=a+b\cdot L \tag{21}$$
$$S_p=r_S\cdot r_L \tag{22}$$

The expected total profit of the society including the buyer and the supplier is

$$ETP=E(\pi_{PS}^{R})+E(\pi_{PS}^{S}) \tag{23}$$

It is difficult to show that the Hessian's matrix is a negative definite matrix for Eq. (23) with optimum solution. One cannot obtain a closed-form solution. For the given parameters, we can adopt the direct search method for obtaining the optimal warranty period of product, L^*, the optimal process mean, μ_y^*, and the optimal order quantity, Q^*. The procedure of approximate optimum solution for the above integrated model (23) is as follows:

Step 1. Set the maximum warranty period *L*, $L_{\max} = U$.

Step 2. Set the minimum searched warranty period *L*, $L_{\min} = 0.01$, and let the expected lifetime searched value $\mu_y = L_{\min} + 0.01$. Compute *ETP* using Eq. (23).

Step 3. Let $\mu_y = \mu_y + 0.01$. Compute *ETP* using Eq. (24). Repeat this step until $\mu_y = U - 0.01$.

Step 4. Let $L = L_{\min} + 0.01$ and $\mu_y = L + 0.01$. Compute *ETP* using Eq. (23). Repeat steps 3 and 4 until $L = U - 0.02$.

Step 5. Select the maximum expected total profit of the society from above Steps 1-4 as the best policy. The corresponding parameters of L^*, μ_y^*, and Q^* values having the maximum *ETP* is the optimum solution.

The optimal solution of integrated model depends on the several cost and profit parameters. The influences of them need to be illustrated by considering the sensitivity analysis of parameters.

4. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

Assume that some parameters are as follows: *a* = 10, *b* =2, $W = r_S = a + bL,$ $R = 3W$, $S = 0.2W$, $r_L = 0.4, \mu_x = 100$, $S_p = 0.4W$, $\sigma_x = 20$, $y_0 = \frac{L+U}{2}$, $\sigma_y = 0.8$, $i = 0.05$, $k = 10$, $c = 5$, $\alpha_1 = 100,\ \beta_1 = \text{-}0.1,\ r = 0.1,$ and *U* = 3. By solving Eq. (23), one obtain the optimal order quantity $Q^* = 75$, the optimal expected lifetime of product $\mu_y^* = 1.42$, the optimal warranty period of product $L^* = 0.61$ with the expected profit of buyer $E(\pi_{PS}^R) = 1305.74$, the expected profit of supplier $E(\pi_{PS}^S) = 215.01$ and the expected total profit of the society $ETP = 1520.76$.

Table 1 lists the $\pm 20\%$ change for parameter values and presents the effect on the order quantity, the expected lifetime, the warranty period, the expected profit of buyer, the expected profit of supplier, and the expected total profit of the society. If the change percentage of the expected profit is larger than 10%, then the parameter has a major effect on the expected profit. From Table1, one has the following conclusions:

(1) The standard deviation of the quality characteristic, σ_y, the mean demand of the end of customer, μ_x, the upper specification limit of product lifetime, *U*, the variable product cost per unit , *c*, the parameter of demand, α_1, the parameter of selling price, β_1 , and the sale price per unit for the end of customer , *R*, have a significant effect on the purchaser's order quantity.

(2) The standard deviation of the quality characteristic, σ_y, the mean demand of the end of customer, μ_x, the upper specification limit of product lifetime, *U*, the variable product cost per unit , *c*, the parameter of demand, α_1, the parameter of warranty period, *r*, the parameter of selling price, *a*, the parameter of selling price, *b*, and the sale price per unit for the end of customer , *R*, have a significant effect on the expected lifetime and warranty period of product.

(3) The standard deviation of the quality characteristic, σ_y, the mean demand of the end of customer, μ_x, the upper specification limit of product lifetime, *U*, the variable product cost per unit , *c*, the parameter of demand, α_1, the parameter of selling price, *a*, and the sale price per unit for the end of customer , *R*, have a significant effect on the expected profit of buyer.

(4) The standard deviation of the quality characteristic, σ_y, the mean demand of the end of customer, μ_x, the upper specification limit of product lifetime, *U*, the variable product cost per unit , *c*, the parameter of demand, α_1, the parameter of selling price, *a*, and the sale price per unit for the end of customer , *R*, have a significant effect on the expected profit of supplier.

(5) The standard deviation of the quality characteristic, σ_y, the mean demand of the end of customer, μ_x, the parameter of demand, α_1, the parameter of selling price, *a*, and the sale price per unit for the end of customer , *R*, have a significant effect on the expected profit of expected total profit of the society.

5. DISCUSSION AND CONCLUSIONS

The parameters having a significant effect on the combination (*L*, μ_y) of parameters are the standard deviation of the quality characteristic, σ_y, the mean demand of the end of customer, μ_x, the upper specification limit of product lifetime, *U*, the variable product cost per unit , *c*, the parameter of demand, α_1, the parameter of selling price, *a*, and the sale price per unit for the end of customer , *R*. The parameters having a significant effect on the order quantity are the standard deviation of the quality characteristic, σ_y, the mean demand of the end of customer, μ_x, the upper specification limit of product lifetime, *U*, the variable product cost per unit , *c*, the parameter of demand, α_1, the parameter of selling price, β_1 , and the sale price per unit for the end of customer , *R*. The parameters having a significant effect on the expected total profit of the society are the standard deviation of the quality characteristic, σ_y, the mean demand of the end of customer, μ_x, the parameter of demand, α_1, the parameter of selling price, *a*, and the sale price per unit for the end of customer , *R*. Hence, one needs to have an exact estimation of σ_y, μ_x, α_1, *a*, and *R* in order to obtain the optimal control of the product quality and the maximum expected total profit of the society.

In this paper, the author has presented an integrated Ladany and Shore's (2007) model into the modified Chen and Liu's (2007) model with quality loss and warranty period of product. The warranty period of product, the expected lifetime of product, and the order quantity are simultaneously determined in the modified Chen and Liu's (2007) model. The extension to an integrated model with rectifying sampling inspection plan may be left for further study.

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Table 1. The effect of parameters on the optimal solution

k	Q	μ_y	L	$E(\pi_{PS}^R)$	$E(\pi_{PS}^S)$	ETP
8	75	1.42	0.61	1314.64	215.01	1529.65
12	75	1.42	0.61	1296.85	215.01	1511.86
σ_y	Q	μ_y	L	$E(\pi_{PS}^R)$	$E(\pi_{PS}^S)$	ETP
0.64	77	1.66	0.83	1504.77	173.79	1678.56
0.96	71	0.33	0.32	712.39	738.51	1450.90
σ_X	Q	μ_y	L	$E(\pi_{PS}^R)$	$E(\pi_{PS}^S)$	ETP
16	75	1.43	0.62	1343033	212.13	1555.46
24	75	1.42	0.61	1267.30	215.01	1482.31
μ_X	Q	μ_y	L	$E(\pi_{PS}^R)$	$E(\pi_{PS}^S)$	ETP
80	68	0.23	0.22	531.15	759.30	1290.45
120	76	1.46	0.65	1380.61	203.33	1583.94
U	Q	μ_y	L	$E(\pi_{PS}^R)$	$E(\pi_{PS}^S)$	ETP
2.4	71	0.34	0.33	725.94	736.41	1462.35
3.6	78	1.89	0.91	1488.41	81.72	1570.13
z	Q	μ_y	L	$E(\pi_{PS}^R)$	$E(\pi_{PS}^S)$	ETP
0.4	75	1.42	0.61	1305.74	224.12	1529.87
0.6	75	1.42	0.60	1306.60	205.05	1511.64
a	Q	μ_y	L	$E(\pi_{PS}^R)$	$E(\pi_{PS}^S)$	ETP
8	71	0.25	0.24	557.787	577.08	1134.86
12	74	1.49	0.65	1552.61	349.46	1902.07
b	Q	μ_y	L	$E(\pi_{PS}^R)$	$E(\pi_{PS}^S)$	ETP
1.6	74	1.33	0.50	1256.32	221.94	1478.26
2.4	76	1.51	0.72	1359.25	212.80	1572.05

Table 1. (Continued)

α_1	Q	μ_y	L	$E(\pi_{PS}^{R})$	$E(\pi_{PS}^{S})$	ETP
80	61	1.46	0.65	1098.33	162.66	1260.99
120	89	1.33	0.50	1372.29	284.88	1657.17
β_1	Q	μ_y	L	$E(\pi_{PS}^{R})$	$E(\pi_{PS}^{S})$	ETP
-0.08	79	1.41	0.60	1340.45	228.65	1569.10
-0.12	72	1.43	0.62	1265.30	202.11	1467.41
r	Q	μ_y	L	$E(\pi_{PS}^{R})$	$E(\pi_{PS}^{S})$	ETP
0.08	76	1.38	0.56	1304.24	227.88	1532.12
0.12	75	1.46	0.66	1308.62	202.42	1511.04
c	Q	μ_y	L	$E(\pi_{PS}^{R})$	$E(\pi_{PS}^{S})$	ETP
4	77	1.58	0.76	1346.20	313.72	1659.92
6	69	0.25	0.24	688.55	721.94	1410.49
i	Q	μ_y	L	$E(\pi_{PS}^{R})$	$E(\pi_{PS}^{S})$	ETP
0.04	75	1.42	0.61	1305.74	215.92	1521.67
0.06	75	1.42	0.61	1305.74	214.10	1519.84
R	Q	μ_y	L	$E(\pi_{PS}^{R})$	$E(\pi_{PS}^{S})$	ETP
2.4*W*	69	0.26	0.25	474.72	753.95	1228.67
3.6*W*	76	1.55	0.67	1764.49	170.75	1935.23
S	Q	μ_y	L	$E(\pi_{PS}^{R})$	$E(\pi_{PS}^{S})$	ETP
0.16*W*	75	1.42	0.61	1305.38	215.01	1520.39
0.24*W*	75	1.42	0.61	1306.11	215.01	1521.12
S_P	Q	μ_y	L	$E(\pi_{PS}^{R})$	$E(\pi_{PS}^{S})$	ETP
0.32*W*	75	1.43	0.59	1310.93	195.63	1506.56
0.48*W*	75	1.41	0.63	1299.76	236.17	1535.94