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Recent Developments in Vacation Queueing Models : A Short Survey

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Abstract— Queueing systems with server vacations (server's performing non-queueing jobs) have been studied extensively since the late 70's. A considerable number of works in this area were completed in the early 80's and surveyed by Doshi in 1986. As an extension to the classical queueing system, allowing idle servers to work on non-queueing jobs makes the vacation models more applicable in a variety of systems including flexible manufacturing or service and computer communication systems. Motivated by these applications, more studies on vacation models have been done during the late 80's and 90's and surveyed in the book by Takagi in 1991 and the book by Tian and Zhang in 2006. This paper intends to provide a brief summary of the most recent research works on vacation queueing systems in the past 10 years.

Keywords — server's vacation, batch arrivals, modified vacation policy, working vacations, multi-server vacation models.

1. INTRODUCTION

The queueing model with server vacations (server absences) has been well studied in the past three decades and successfully applied in many areas such as manufacturing/service and computer/communication network systems. These vacation queueing models can be classified according to the arrival processes, service processes, and the vacation policies. Excellent surveys on the earlier works of vacation models have been reported by Doshi (1986), Takagi (1991), Tian and Zhang (2006). In this paper, we briefly survey the studies on vacation models for the past decade. The details of a queueing system with server vacations have been well described in the previous survey papers and books. To make this survey more concise, readers are referred to Doshi (2006) for the definitions, classification, and discussion of different vacation models. We only focus on reporting the types of models and the main methods (if non-traditional) used in the studies conducted over the past 10 years. For technical details of these

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models, again we refer the readers to the specific papers.

This paper is organized as follows: Section 2 is focused on the recent models with more complex arrival processes. Section 3 reviews the models dealing with different types of vacation policies and service processes. Section 4 presents the recent works on multi-server vacation models. Other new developments of vacation models in the past decade are reported in Section 5. Finally, Section 6 concludes with some possible future research directions.

2. VACATION MODELS WITH VARIANTS OF ARRIVAL PROCESSES

Extensive studies were conducted on the vacation models with batch arrivals. Katayama (2005) considered an $M^{[x]}/G/1$ queue with time-limited service and two types of vacations by using the level-crossing method. Arumuganathan and Ramaswami (2005) analyzed a $M^{[x]}/G(a, b)/1$ queue with two service rates and multiple vacations. Hur and Ahn (2005) studied an $M^{[x]}/G/1$ queue with server vacations and set-up times and obtained the steady-state queue length and waiting time distributions. Ke (2007a) investigated the operating characteristics of an $M^{[x]}/G/1$ queue with single or multiple vacation policy, server breakdown, and startup/closedown times. An $M^{[x]}/G/1$ queue with multiple vacations was investigated by Xu *et al.* (2007), who obtained the probability generating function of queue length, busy period, and the stationary waiting time under FCFS and LCFS service disciplines. At the same time, Wang *et al.* (2007) treated an $M^{[x]}/M/1$ queueing system with multiple vacations and server breakdowns. The maximum entropy principle was employed to develop the approximate probability distributions of various system performance measures. A comparative analysis between the exact and approximate results was performed. Later, Ke and Lin (2008) also used the maximum entropy approach to investigate an $M^{[x]}/G/1$ queue with N policy, server breakdowns, and single vacation policy. Moreover, Omey and Gulck (2008) showed that the maximum entropy estimate of Wang *et al.* (2007) could be significantly improved by including the exact probability of the empty system in the constraints. For the vacation model with both batch arrivals and services, Sikdar and Gupta (2008) analyzed an $M^{[X]}/G^{[Y]}/1/N$ queue with single/multiple vacation(s). They derived the stationary distributions of the number of customers and other performance measures. Some numerical results were presented to show the impact of changing some parameter on the system performance.

Another recent development in vacation model research is on the system with Markov Arrival Process (MAP). Gupta and Sikdar (2006) studied an MAP/G/1/N queue with single or multiple vacation policies and found the stationary distributions of number of customers at service completions, vacation terminations, pre-arrival, and arbitrary epochs. Furthermore, Banik *et al.* (2006) investigated a finite buffer MAP/G/1/N queue under single / multiple vacation policies and obtained the queue length distributions. Wu *et al.* (2009) studied a BMAP/G/1 type queue with both positive and negative customers, the second optional service, and multiple vacations. They developed the queue length distributions and the mean of the busy period by using the supplementary variable, the censoring technique, and the renewal theory.

3. VACATION MODELS WITH VARIANTS OF VACATION POLICIES

Modified vacation policy:

Zhang and Tian (2001) investigated a Geo/G/1 queue with multiple adaptive vacations (MAV) where the server can take at most a certain number (*J*) of vacations continuously. Ke and Chu (2006) studied a batch arrival system with MAV. The MAV can be considered as a modified vacation policy. An MAV is reduced to the single or multiple vacation policy by setting the value of *J* to be one or infinity. Later, Ke (2007b) extended the model in Ke and Chu (2006) to the case with customer balking behavior. Ke *et al.* (2010a) generalized the model to the case with *N*-policy. Furthermore, Ke *et al.* (2010b) investigated the threshold model of Ke *et al.* (2010a) with a randomized control policy. Later, more works on the models with the modified vacation policies were done. For example, Ke and Chang (2009a) considered an M/G/1 retrial queue with modified vacation policy, customer balking, and feedbacks. Chang and Ke (2009) investigated an $M^{[x]}/G/1$ retrial queue with modified vacation policy by applying the supplementary variable technique. Ke and Chang (2009a) extended Chang and Ke's model to more general cases with impatience customers and feedback behaviors.

Bernoulli vacation policy:

Madan et al. (2003) analyzed an M/M/2 queue with a single Bernoulli schedule vacation policy and discussed two models under different conditions. Choudhury and Madan (2004) considered a batch arrival queueing system with two phase service and Bernoulli vacation. Choudhury and Madan (2005) further investigated the system with a modified Bernoulli vacation and N-policy. Tadj et al. (2006) studied a bulk service queueing system with random setup time under the Bernoulli vacation and N-policy. They developed an algorithm to determine the optimal policy. Later, Choudhury (2007) extended this model to a two-phase batch arrival retrial queueing system with Bernoulli vacations. Also Choudhury *et al.* (2007) examined an $M^{[x]}/G/1$ queue with two-phase service and Bernoulli vacation and multiple vacation policy. Choudhury (2008) analyzed an M/G/1 retrial queue with two-phase service and Bernoulli vacation schedule.

Recently, Kumar *et al.* (2009) studied an M/M/c retrial queueing system with Bernoulli vacations and obtained various system performance measures. Ke and Chang (2009b) investigated an $M^{[x]}/(G_1,G_2)/1$ retrial queue under Bernoulli vacation schedules with general repeated attempts and starting failures.

Working vacation policy (*WV*):

Servi and Finn (2002) first introduced the concept of working vacation in a single server system. In such a system denoted by M/M/1/WV, the server would work at a different rate rather than completely stop during the vacation period. The explicit formulae for the mean, variance, and distribution of the number and time in the system were presented. After that, the research interests on working vacation models grew fast. Liu *et al.* (2007) examined stochastic decomposition structures of the queue length and waiting time in an M/M/1/WV queue. Xu *et al.* (2009) extended the M/M/1/WV queue to a bulk input $M^{[x]}/M/1/WV$ queue and obtained the upper and lower bounds of the mean waiting time by using the properties of the conditional Erlang distribution.

For the general input queue, Baba (2005) analyzed a GI/M/1 queue with multiple working vacations and derived the distribution of sojourn time for an arbitrary customer. Later, the finite capacity GI/M/1 queue with multiple working vacations was studied by Banik *et al.* (2007). The GI/M/1 queue with working vacation and vacation interruption was discussed by Li *et al.* (2008). Afterward, the comparison analysis between the GI/M/1 and the GI/Geo/1 queues with single working vacation was provided by Chae *et al.* (2009).

For the general service time, Wu and Takagi (2006) investigated the M/G/1 queue with multiple working vacations and derived the stationary distributions for the queue size and the system time by utilizing the transient solution for the queue length of an M/G/1 queue. Li *et al.* (2009) used the matrix analytic method to analyze an M/G/1 queue with exponentially working vacations under a specific assumption. They obtained the conditional stochastic decomposition result and the joint distribution for queue length and service status.

Recently, Lin and Ke (2009) considered the multi-server system with single working vacation. The matrix-geometric approach was utilized to develop the computable explicit formula for the probability distributions of queue length and other performance measures. Yang, *et al.* (2010) treated the *F*-policy M/M/1/K queue with single working vacation and exponential startup times and derived the stationary distributions and related system characteristics, including an optimization numerical analysis. Jain and Jain (2010) investigated a single-server working-vacation model with server breakdowns of multiple types.

4. MULTI-SERVER VACATION MODELS

Multi-server vacation models were studied by a number of researchers over the past decade. The servers in these models can either take the same vacation together (called synchronous vacation) or take individual vacations (asynchronous vacations) independently. Zhang and Tian (2004) first studied the multi-server model with asynchronous vacations which represents a service system with multi-task employees. More multi-server vacation models are based on synchronous vacations. Under such a policy, a group of servers take vacations together when the vacation condition is met. Zhang and Tian (2003a, 2003b) first analyzed the Markovian multi-server queueing system with single/multiple synchronous vacations. Moreover, Tian and Zhang (2003) investigated a more general GI/M/c queueing system with phase-type vacations where all servers take multiple vacations together until waiting customers exist at a vacation completion instant. Tian and Zhang (2006) considered a multi-server queueing system with a threshold type (*d*,*N*) vacation policy under which *d* idle servers keep taking multiple synchronous vacations until the number of customers reaches or exceeds a threshold *N*. A computational study is presented for determining the optimal values of *d* and *N*. Another multi-server vacation model with single vacation and threshold policy was treated by Xu and Zhang (2006). Zhang (2005) presented an analysis on the multi-server vacation model with three threshold policy. Yue *et al.* (2006) studied a finite buffer multi-server queue with balking, reneging, and single synchronous vacation policy. They obtained the stationary distributions of the queue length and some other performance measures in matrix forms. A multi-server model with MAP and synchronous phase-type vacations was investigated by Chakravarthy (2007). Several special cases with MAPs and numerical examples were presented, including the table of optimal values of system parameters and the corresponding system performances measures. Recently, Ke *et al.* (2009) studied the optimal (*d*, *c*) vacation policy for finite capacity M/M/c/N queue with unreliable servers and repairs. Chakravarthy (2009) analyzed the MAP/M/c system with phase type vacation and presented some interesting numerical illustrations. Recently, Gharbi and Ioualalen (2010) studied the finite-source multi-server queueing systems with single/multiple vacation policies and developed some the algorithms for computing the system performance measures.

5. OTHER VACATION MODELS

Rahmoune and Aissani (2008) used the strong stability method to study the characteristics of the M/G/1/N queue with multiple vacations where the rate of the vacations is sufficiently small. This method is also useful in analyzing complex queueing systems such as retrial queueing systems with unreliable servers, with batch arrivals, or with priorities. Thangaraj and Vanitha (2009) studied a two-phase M/G/1 queue with Bernoulli feedback and multiple-vacation policy where the two stage service and rejoining behavior (feedback) of customers are considered. Both single server and multi-server vacation models with impatient customers were discussed by Altman and Yechiali (2006). Customers waiting in the queue may leave the system before getting the service if their waiting times are too long. Multi-server vacation models with this feature may be more appropriate for analyzing call centers. As an extension to the M/G/1 type vacation model, Zhang, *et al.* (2001) studied the optimal service policies that minimize the long-term average cost of a vacation system with multiple vacation types. Furthermore, Zhang (2006) also presented the proof of some convexity of the average cost function in the threshold in a single server vacation model. Choudhury (2002) analyzed the M/G/1 queue with multiple vacations of two types and obtained the stationary queue length waiting time distributions.

6. CONCLUDING REMARKS

Although extensive works have been done in the vacation model area over the past three decades as surveyed in Doshi (1986), Takagi (1991), Tian and Zhang (2006), and some survey papers including this paper, there are still many open problems for further studies. Here we just mention a few directions for future research.

Most past works were focused on one-stage queueing systems with server vacations and queueing networks with some stations having server vacations have not been investigated. Investigating queueing network with vacations can be a research topic. Also, there are many possible extensions on the multi-server vacation models which have not been explored. For example, the multi-server vacation model with heterogeneous servers has not been considered in the literature. One reason for these open problems may be due to the "curse of dimensionality" caused by the complexity of the systems. Developing good approximations to these complex vacation models will be a fruitful future research direction. Another direction is to analyze the vacation models with more complex vacation policies motivated by real systems. For example, it is worth extending the two threshold policy to the multiple threshold policy in a multi-server vacation models where the number of servers on vacations depends on the queue length. Finally, generalizing the single server model with multi-types of vacations to the multi-server settings is an interesting future research topic.

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