International Journal of
Operations Research

International Journal of Operations Research Vol. 8, No. 2, 8–17 (2011)

A Markov Decision Process Model for an Online Empty Container Repositioning Problem in a Two-port Fixed Route

Ning Shi[1] and Dongsheng Xu[2*]

[1,2]School of Business, Sun Yat-sen University, No. 135, West Xingang Road,

Guangzhou, China – 510275

Received December 2010; Revised January 2011; Accepted February 2011

Abstract—In this paper, we propose a Markov Decision Process model for an empty repositioning problem in a two-port system. We consider two cases. The first case is the offline case, where demand information is assumed as a random variable with known distribution. The second case is online case where demand information is partially known. In both cases, we figure out the optimal controlling policies. The second case enables the possibility to conduct an online optimization taking the advantages of real-time information.
Keywords—Empty container repositioning, markov decision process.

1. INTRODUCTION

Shipping industry is an old and traditional industry but it still carries around 90%'s international trade. Without shipping, the import and export of goods in the modern world would be impossible or much more expensive. In a flatter world, consumers enjoy the benefits of globalization, containerization and competing freight costs.

Liner shipping industry is one of the most capital intensive industries. Besides ship, container is an important asset category. Ocean carriers currently spend almost 100 billion dollars per year on operating container assets, and industry analysts estimate that approximately 16 billion of that is directly attributable to the total cost of repositioning empty containers. Empty repositioning generates no profits directly but has potential to meet future demands. Properly repositioning empty containers can improve the container assets' utilization and increase carriers' revenues, especially in imbalanced lines. Trade imbalance initiates repositioning empties: it is not unusual today for entire ships to be chartered to shift empties from surplus to demand locations. The trend of container flow imbalances in the main trades seems increasing, especially in the transpacific and Asia/Europe trades, as Table 1 shows. Such a trend highlights the empty container repositioning problem. It is still one of the most important managerial problems faced by ocean carriers.

Table 1. Container trade flow volumes of east/west axis (unit: 1000 TEU)

Year	Transpacific	N. Europe-Far East	Transatlantic	Mediterranean-Far East	Mediterranean North America
2000	3,436	1,399	585	650	454
2001	3,698	1,253	583	655	527
2002	4,867	1,402	637	715	574
2003	5,167	1,955	626	951	527
2004	5,723	2,279	674	1,119	554

Consider a shipping line which visits a fixed set of ports $\prod = \{1, 2, ..., T\}$. As a common practice, the vessel will visit those ports in a given and fixed sequence. Without loss of generality, we assume the sequence is 1, 2, .., n. In our

∗ Corresponding author's email: xudsh@mail.sysu.edu.cn

problem, we also consider the return trip which covers all ports and they are visited in a reversed sequence of the forward trip. The forward trip and return trip together form a round-trip. Figure 1 illustrates a fixed forward trip covering several ports from Asian to America.

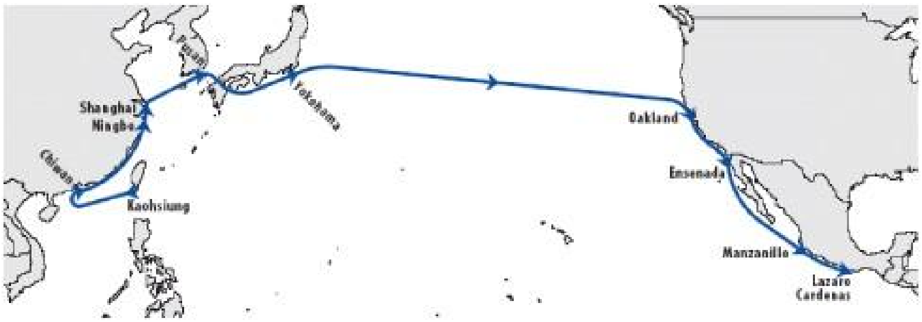

Figure 1. One shipping line covering multiple ports. (from www.apl.com)

In this paper, we consider a special case: A Two-Port system. In such a system, the shipping line travels between two ports back and forth. This system can be used to model the short-voyage shipping lines which cover only two ports. Such short-voyage shipping lines can be found in Pearl Delta River region where the containers are transported from Chinese factories via inland ports such as Fo Shan or Zhao Qin ports to the big port such as Hong Kong or Shenzhen ports. The system may be also used to perform a macro-economic analysis as well if we aggregate the ports in one region (like US) as a single port and aggregate the ports in another region (like China) as a single port. Since the demands are uncertain, the shipping line is concerned with how to control the amount of empty containers at those ports to deal with the uncertainty. The objective is to minimize the total costs including the leasing costs, holding costs and transportation costs. In this study, we formulate it as a stochastic dynamic program and we find that threshold-type optimal policy exists when the demands of two ports are not too imbalanced. The managerial insights from this model can supply some guidelines for the industry.

In this paper, we consider offline and online models. In the offline model, we interpret the demand of some port accumulated in the whole round trip can be interpreted as a random variable and the partially available information is ignored. In the online model, partially available information is explicitly considered. This enables the possibility of updating the information in a real-time fashion. In both models, we discover the structures of optimal policies.

The remainder of this paper is organized as follows: Section 2 introduces the literature review. Section 3 describes the mathematical model. Section 4 and Section 5 provides the analysis for offline and online models respectively. Section 6 concludes this paper.

2. LITERATURE REVIEW

In our work, we consider the operational planning for the ocean transportation of empty containers so as to balance the supply and demand of empty containers at ports. A lot of research has been done in developing optimal or near-to-optimal policies for container fleet management. The problem can be formulated as a dynamic fleet management if we regard an empty container as a piece of equipment. Crainic et al. (1993) propose both deterministic and stochastic network models for the in-land transportation of empty containers. Cheung and Chen (1998) proposes a two-stage stochastic network model and solves it by stochastic quasi-gradient method (SQG) numerically. This approach is based on a general network without assuming the topological structure of the maritime routes. However, that model assumes that the demand for empty container at each port is aggregated. In practice, the demand can be specified by an origin and destination pair. Variants of this model can be found in Jordan and Turnquist (1983); Cheung and Chen (1998); Crainic et al. (1993); Cheung and Powell (1996); Kochel et al. (2003). These models are usually solved by mathematical programming techniques such as mixed integer programming, stochastic programming, etc. However, mathematical programming models have on-line computation communication and data requirements. Moreover, the underlying logic of such models is hidden from the terminal managers who are responsible for fleet management (Du and Hall (1997)).

There is another modeling perspective: reusable inventory repositioning problem. Li et al. (2004) developed a two-level threshold policy to manage empty containers in a single port by assuming both import and export containers at the port are aggregated random variables. They showed that such threshold control policy is optimal for both finite and infinite horizon problems. Li et al. (2007) further showed that the threshold control policy is ε-optimal in multi-ports case. Again, the import and export containers at each port are aggregated.

Du and Hall (1997) used a threshold policy to redistribute empty equipment in a hub-and-spoke network. A decomposition approach was developed to find the optimal threshold values and fleet size subject to stock-out probability constraints. Song (2005) proved that optimal empty repositioning policy is of threshold control structure in two-depot service systems with the assumptions of exponentially distributed repositioning times and Poisson demand arrivals for continuous or periodic-review schemes. The work was further extended to a hub-and-spoke transportation system through a dynamic decomposition procedure Song and Carter (2008). They tested the threshold policy in a cyclic route numerically Song and Dong (2008), In this paper, we consider the empty container management from an ocean carrier's perspective. This perspective is same as Cheung and Chen (1998). However, different from Cheung and Chen (1998), Li et al. (2004, 2007), the demand at each port is not aggregate but explicitly associated with origin-destination pairs. Also, we relax the assumption that the repositioning times follow exponentially distributions and the demands are Poisson distributed as Song (2005).

Our research is particularly interested to the demand model. We consider two demand models. In the first one, we interpret the uncertain demand as a random variable and ignore partially available information. In the second one, we interpret the uncertain demand as a random variable with partially available information. We call the first as offline model and the second online model. Literatures rarely consider online model for this problem.

3. MATHEMATICAL ANALYSIS：OFFLINE MODEL

A shipping line provides service for a cyclic route according to a pre-determined timetable. A customer demand is defined as a transportation requirement of moving containers from the origin port to the destination port. Empty containers are required to satisfy the customer demand, and once the demand is satisfied, it will generate laden containers. The containers are assumed of the same size (i.e.20-ft unit). The vessel, whose capacity is V, can pick up or drop off containers at ports.

In this problem, we need to make a sequence of decisions in a given planning horizon which consists of N round trips. Each round trip can be treated as a stage. Let us assume that the time of shipping from one port to the other is one period.

As shipping industry is very competitive, customers are not willing to wait for the shipping line if it cannot satisfy them at the desired trip. Therefore we assume the unsatisfied demands are lost. With this assumption, each port only needs to consider the accumulated demand between two consecutive visits. Note that the decision of updating the state of port 1 has to be made just at the time instant when the vessel departs from port 2 and before it arrives at port 1. As the accumulated demand from last visit to port 1 to the coming visit is only partially known, here for simplification, we assume it is a random variable D_t^i whose definition is given as follows,

D_t^i : The accumulated demand at port i from the t-1 visit to the t visit.

We can define the system state as,

x_t^i : The total number of available carrier-owned empty containers at port i when the vessel arrives at it.

When the vessel arrives at port i, the following events will happen sequentially (shown in Figure 2):

1. Unload the laden containers which are designated at port i.
2. The demand at port i, D_t^i is realized as d_t^i .
3. The laden containers are emptied and available for re-use.
4. If the amount of demand is less than x_t^i , they will be completely fulfilled by these empty containers. Then the carrier can determine the amount of empty containers, to be lifted on the vessel. If the demand is more than x_t^i , the carrier should satisfy all the demand.
5. Load the laden and empty containers (if there are) to the vessel.
6. The vessel leaves port i.

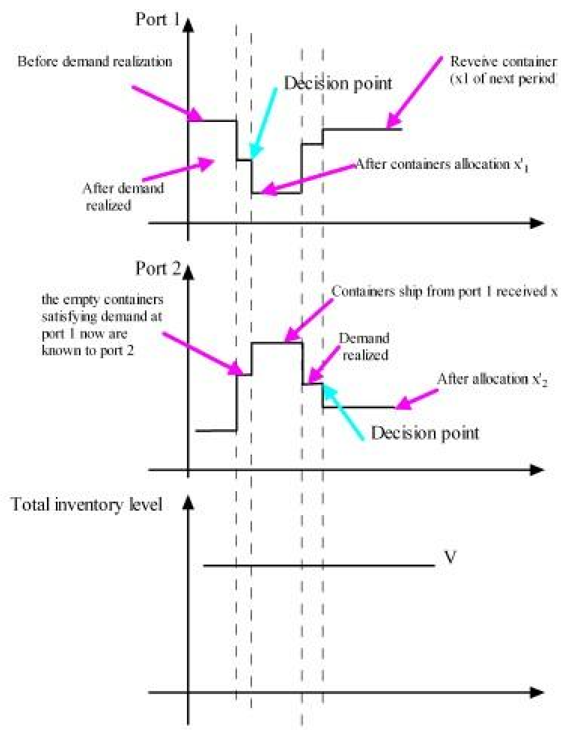

Figure 2. Sequence of events.

We assume that the leased containers can be returned at any port and the leasing cost is proportional with the leasing time. In shipping industry, international leasing companies usually conduct business covering different ports around the world. It is a common practice that one ocean carrier leases an amount of empty containers from some port i and lease off those containers at the destinations. Considering that the international leasing companies would like to get their containers back for further leasing and the ocean carrier is willing to lease off unnecessary leased containers immediately, this assumption is quite reasonable. Furthermore, if the ocean carrier would like to hold those leased containers for further utilization, she can lease off them and then lease on some. In this way, we only need to take care of the management of carrier-owned empty containers.

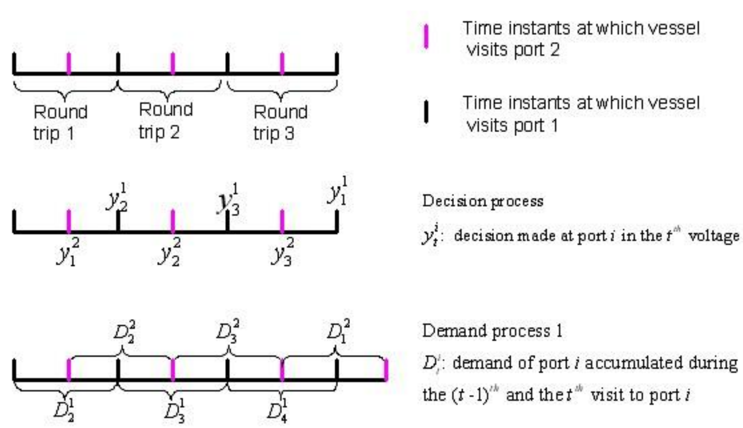

Figure 3. Offline Model: Ignore partially available demand information

We further define the system parameters:

c_i : The cost of shipping one empty container to port i.

h_i : The holding cost of one empty container at port i.

p_i : The penalty cost for shortage at port i.

F_i : The density function of the distribution of demand

b : The discounted factor

The holding and shortage costs for both ports at each period t can be denoted as

$$L_i^t(x) = p_i E(D_i^t - x)^+ + h_i E(x - D_i^t)^+$$

It is well known that $L_i^t(x)$ are convex with x, given i and x.

For stage t, let us define the optimal function for port 1 as $G_t(x)$ and the optimal function for port 2 as $J_t(x)$. The optimality equations can be stated as follows,

$$G_t(x_1) = \min_{x_1 \le y \le V} [c_1(y - x_1) + L_1(y) + b\,E(J_t(V - y + \min(y, D_1)))]$$

$$J_t(x_2) = \min_{x_1 \le y \le V} [c_2(y - x_2) + L_2(y) + b\,E(G_{t+1}(V - y + \min(y, D_2)))]$$

Explanation: The decision epoch for $G_t(x_1)$ is the time point right after port 2 has all laden containers to be unloaded at port 1 in round-trip t-1. According to the definition, x_1 consists of two parts. One is the remained empty container since round-trip t-1 at port 1, and the other is the laden containers to be unloaded at port 1. Therefore, $y - x_1$ is the number of empty containers to be shipped from port 2 to port 1. The term $c_1(y - x_1)$ represents the empty repositioning cost. The term $L_1(y)$ represents the expected total shortage and holding cost at port 1. Once the vessel arrives at port 1, the demand at port 1, denoted by D_t^1, is realized. It is free to use carrier-owned containers to satisfy the demand at port 2. If there are not enough empty containers to satisfy the demand, we have to lease from other companies, paying a much higher cost, the amount of leased containers is $\max(0, D_t^1 - y)$. Here we first assume that the vessel has infinite capacity. Under this assumption, the number of full-loaded self-owned containers to be shipped to port 2 is $\min(D_t^1, y)$. Therefore, the total number of available empty carrier-owned containers, namely the state, in port 2 will be $V - y + \min(D_t^1, y)$ (Note there are $V - y$ empty containers remained in port 2). So the decision of shipping empty containers to port 1 is equivalent to update the state of port 1. Such decision pays additional shipping costs for empty containers with potential savings on leasing empty containers. This logic is described in Figure 3.

Theorem 1.
The function $G_t(x)$ and $J_t(x)$ are convex with x, given any t.

Proof.
See Appendix 1.

Theorem 2.
The optimal policy for offline model can be described as following: Given any state x, there exists state-independent optimal crucial point y^* such that when the state x is below than y^*, we should update the state to y^*, otherwise do nothing.

Proof.
This is a standard result from inventory theory. Refer to Ross (1990).

This optimal policy is a base-stock type policy which is independent with the state and therefore easy to implement.

4. MATHEMATICAL MODEL: ONLINE CASE

In Section 3, D_t^1 is the accumulated demand at port 1 during stage t-1 and t. However, the decision for problem $G_t(x_1)$ is made in the middle of stage t. More precisely, it is made at the time epoch when the vessel arrives at port 2 and all laden containers from port 2 to port 1 is loaded to the vessel (then we know the exact state x_t^1). However, actually, at that epoch, part of D_t^1 has already realized. In offline model, we basically ignore that partially realized information. The demand process is described in Figure 4. In this online model, we take this partially realized information into account. We define:

d_t^i : The accumulated demand at port *i* from the $(t-1)^{th}$ visit to the decision epoch.

D_t^i : The accumulated demand at port *i* from the decision epoch to the t^{th} visit.

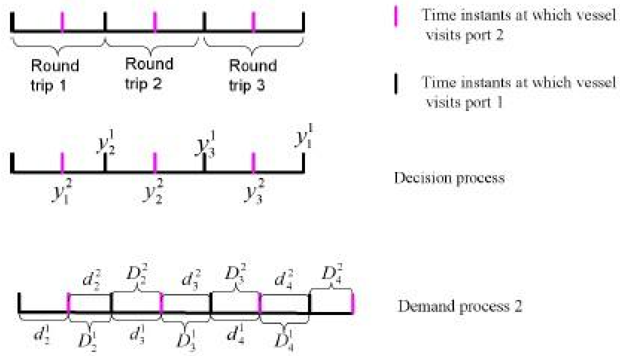

Figure 4. Online Model: Explicitly consider partially available demand information

Therefore, we can enhance the function $G_t(x)$ and $J_t(x)$ as,

$$G_t(x_1, d_t^1) = \min_{x_1 \le y \le V} [c_1(y - x_1) + L_1(y) + b\,E(J_t(V - y + \min(y, d_t^1 + D_t^1), D_t^2))]$$

$$J_t(x_2, d_t^2) = \min_{x_1 \le y \le V} [c_2(y - x_2) + L_2(y) + b\,E(G_{t+1}(V - y + \min(y, d_t^2 + D_t^2), D_{t+1}^1))]$$

Note here the holding cost and shortage cost function $L_1(x)$ and $L_2(x)$ are different from that in previous model.

$$L_i^t(x) = p_i E(d_i^t + D_i^t - x)^+ + h_i E(x - d_i^t - D_i^t)^+$$

With this assumption, we prove that the optimal equations are convex and sub-modular.

Theorem 3.

The optimal functions $G_t(x, d)$ and $J_t(x, d)$ are sub-modular and convex with x and d, given any t.

Proof.

See appendix 2.

Theorem 4.

The optimal policy for online model can be stated as follows: When the vessel departs from one port, decision maker should examine the number of total empty carrier-owned containers to be available at the other port, x, and the partially realized demand, d, when vessel arrives at the other port, if the realized demand is already greater than or equal to V, ship all empty containers; if the realized demand is less than V and the number of available empty containers is less than some crucial point $y^*(d)$, ship the difference $y^*(d) - x$; otherwise do nothing. The crucial point $y^*(d)$ is stage dependent, port dependent and monotonely increasing with d.

Different with the policy in the offline case, the policy in the online case is state-dependent. The computation of the critical value in the optimal policy is complicated and the practitioners need decision support systems to put it into practice.

5. CONCLUSIONS AND DISCUSSIONS

In this paper, we consider a simplified empty container repositioning problem with a fixed route that covers two ports. We formulate as a stochastic dynamic programming model and analyze the structures of optimal policies of empty repositioning decisions. In the offline model where the partially available demand information is ignored, we find that the optimal repositioning policy is indeed a base-stock policy. This policy is easy to implement and state-independent. In the online model where the partially available demand information is explicitly considered, we find that the optimal repositioning policy is a two-index threshold-type policy. This policy, however, is relatively difficult to implement as these crucial points are stage dependent, port dependent and state dependent. However, as the online model takes the advantage of real-time information, it could provide more economic value if it is well implemented in a decision support system. It is very interesting to investigate theoretical bounds between the optimal objective values between the offline model and online model. It is also interesting to study multiple port case by treating two-port case as a sub-problem. They deserve future research.

ACKNOWLEDGEMENT

This research was supported in part by the National Natural Science Foundation of China under grant 71001109, and by the Ministry of Education of the PRC under grant 09YJC630235. The authors are very grateful to the anonymous referees for their valuable comments and suggestions to improve the paper in the present form.

Appendix 1: Proof of Theorem 1.

Four supporting functions are needed to facilitate the proof.

$$\begin{aligned} G_t(x_1) &= \min_{x_1 \le y \le V}[c_1(y-x_1)+L_1(y)+b\,E(J_t(V-y+\min(y,D_1)))] \\ &= \min_{x_1 \le y \le V}[c_1 y+L_1(y)+b\,E(J_t(V-y+\min(y,D_1)))]-c_1x_1 \\ &= Q_t(x_1)-c_1x_1 \end{aligned}$$

And,

$$\begin{aligned} J_t(x_2) &= \min_{x_1 \le y \le V}[c_2(y-x_2)+L_2(y)+b\,E(G_{t+1}(V-y+\min(y,D_2)))] \\ &= H_t(x_2)-c_2x_2 \end{aligned}$$

Further, $Q_t(x_1)=\min_{x_1 \le y \le V}[S_t(y)]$ and $H_t(x_2)=\min_{x_2 \le y \le V}[Z_t(y)]$.

Proposition 1.
$H_t(x)$ and $Q_t(x)$ are monotonely increasing and convex function if $Z_t(y)$ and $S_t(y)$ are convex. Specifically, suppose $Z_t(y)$ attains its minimum at a positive number x_2^* and $S_n(x)$ attains its minimum at a positive number x_2^* :

$$H_n(x)=\begin{cases} Z(x_2^*) & V \ge x^* \ge x \\ Z(x) & x^* < x \\ Z(V) & x^* > V \end{cases} \quad \text{and,} \quad Q_n(x)=\begin{cases} S(x_2^*) & V \ge x^* \ge x \\ S(x) & x^* < x \\ S(V) & x^* > V \end{cases}$$

Proof.
According to the theorem in [2]: if $Z(y)$ is convex on $(-\infty,+\infty)$ and attain its minimum at y^* , then

$$\min_{a \le y \le b}[Z(y)] = Z^L + Z^U,$$

where,

$$Z^L = Z(\max(a, y^*)),$$
$$Z^U = Z(b) - Z(\max(b, y^*)).$$

Let $a = x_2, b = V$, we can derive that,

$$\min_{x_2 \le y \le V}[Z(y)]=\begin{cases} Z(y^*) & V \ge y^* \ge x_2 \\ Z(x) & y^* < x \\ Z(V) & y^* > V \end{cases}$$

The relationship between $Z(y)$ and $H(y)$ can be described as Figure 5.

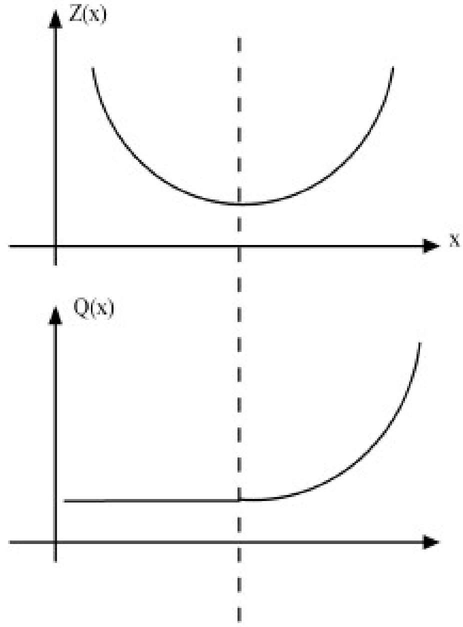

Figure 5. Relationship between H and Q.

Similarly, we can reach the relationship between $Q_t(x)$ and $S_t(y)$. Therefore, we can see that $H_t(x)$ and $Q_t(x)$ are monotonely increasing and convex function if $Z_t(x)$ and $S_t(x)$ are convex

Proposition 2.

The function $Z_t(x)$ and $S_t(x)$ are convex, given any x.

Proof.

This can be done by mathematical induction. Given $Z_{N+1}(x)=0$ and both $c_2(y-x)$ and $L_2(y)$ are convex with y. According to Proposition 1, the function $Z_N(x)=c_2(y-x_2)+L_2(y)$ is convex with x. Therefore, $H_N(x)$ and $G_N(x)$ are convex according to Proposition 1. Then, we can prove that $S_N(x)$ and $J_N(x)$ are convex. Then assuming that $Z_{t+1}(x)$ and $S_{t+1}(x)$ are covex, we can prove that $G_t(x)$ and $J_t(x)$ are convex. By mathematical induction, we can prove that all Z, S, G, J are convex with x.

We can reach Theorem 1 according to Proposition 2.

Appendix 2: Proof of Theorem 3.

First, we can four supporting functions,

$G_t(x_1,d_t^1)=Q_t(x_1,d_t^1)-c_1x_1$,

$J_t(x_2,d_t^2)=H_t(x_2,d_t^2)-c_2x_2$,

$Q_t(x_1,d_t^1)=\min\limits_{x_1\le y\le V}(S_n(y,d_t^1))$,

$H_t(x_2,d_t^2)=\min\limits_{x_2\le y\le V}(Z_n(y,d_t^2))$.

Proposition 1 and Proposition 2 are all hold for above four functions as well. Convexity can be shown as Theorem 1. Consider two cases. In the first case, $y\le d_t^1$, function $S_t(y,d_t^1)=c_1y+L_1(y)+b\,E(J_{t+1}(V,D_2)$ is clearly convex and sub-modular.

In the second case, $y > d_t^1$, we need to derive $\frac{d^2(S(y,d_t^1)}{dyd(d_t^1)}$. We derive $\frac{d(S_n(y,r))}{dy}$ first.

$$\frac{d(S_t(y,d_t^1))}{dy} = c_1 + (p_1+h_1)F_1(y-d_t^1) - p_1 + b\,\frac{d\int_0^{y-d_t^1}\int_0^{\infty}(J_t(V-y+d_t^1+z,r))dF_2(r)dF_1(z)}{dy}$$
$$+b\,\frac{d\int_{y-d_n^1}^{\infty}\int_0^{\infty}(J_t(V,r))dF_2(r)dF_1(z)}{dy}$$
$$= c_1 + (p_1+h_1)F_1(y-d_t^1) - p_1 + b\,\frac{d\int_0^{y-d_t^1}\int_0^{\infty}H_t(V-y+d_t^1+z,r) - c_2(V-y+z)dF_2(r)dF_1(z)}{dy}$$
$$+b\,\frac{d\int_{y-d_t^1}^{\infty}\int_0^{\infty}(J_n(V,r))dF_2(r)dF_1(z)}{dy}$$
$$= c_1 + (p_1+h_1+b\,c_2)F_1(y-d_t^1) - p_1 + b\int_0^{y-d_t^1}\int_0^{\infty}\frac{\partial(H_t(V-y+d_t^1+z,r))}{\partial y}dF_2(r)dF_1(z)$$
$$= c_1 + (p_1+h_1+b\,c_2)F_1(y-d_t^1) - p_1 + b\int_0^{y-d_t^1}\frac{\partial(E(H_t(V-y+d_t^1+z,D_t^2))}{\partial y}dF_1(z)$$

Then, we can derive $\frac{d^2(S(y,r)}{dydr}$,

$$\frac{d(S_n(y,r))}{dydr} = -(p_1+h_1+b\,c_2)\,f_1(y-r) + b\,\frac{d\int_0^{y-r}\frac{\partial(E(H_t(V-y+r+z,D_t^2))}{\partial y}dF_1(z)}{dr}$$
$$= -(p_1+h_1+b\,c_2)\,f_1(y-w) + b\int_0^{y-w}\frac{\partial(E(H_t(V-y+w+z,D_t^2))}{\partial y\partial w}dF_1(z) + \frac{\partial(E(H_t(y',D_t^2))}{\partial y'}\,f_1(y-w)\,|_{y'=V}$$

By induction, we can prove the sub-modularity. Assume that H_t is convex. The first term is clearly negative. The third term is negative as function H_t is monotone decreasing in terms of y. The second term is also negative as $H_t(V-y+w+z,D_t^2)$ is convex with $V-y+w+z$. Similarly, we can prove that, $Z_t(y,D_t^2)$ is also sub-modular. Then, by observing the relationships between functions Z, S, H, Q, J, G, we can see sub-modularity preserves.

REFERENCES

1. Ross (1990). Introduction to Stochastic Dynamic Programming, Academic Press.
2. Cheung, R. K and Chen, C.-Y. (1998). A two-stage stochastic network Model and Solution methods for the dynamic empty container allocation problem. *Transportation Science*, 32(2: 142-162.
3. Cheung, R. K and Powell, W. B (1996). An algorithm for multistage dynamic Networks with Random Arc Capacities, with an Application to Dynamic Fleet Management. *Operations Research*, 44(6): 951-963.
4. Crainic, T.G., Gendreau, M. and Dejax, P. (1993). Dynamic and stochastic models for the allocation of empty containers. *Operations Research*, 411: 102-127.
5. Du, Y. and Hall, R. (1997). Fleet sizing and empty equipment redistribution for center-terminal transportation networks. *Management Science*, 43(2): 145-157.
6. Jordan, W. C. and Turnquist, M. A. (1983). A stochastic dynamic network model for railroad car distribution. *Transportation Science*, 17(2): 123-145.
7. Kochel, P., Kunze, S., and Nielander, U. (2003). Optimal control of a distributed service system with moving resources: an application to the fleet sizing and allocation problem. *International Journal of Production Economics*, 81-82: 443-459.
8. Li, J.A., Leung, S.C., Wu, Y., and Liu, K. (2007). Allocation of empty containers between multi-ports. *European Journal of Operational Research*, 182(1): 400-412.
9. Li, J.A, Liu, K., Leung, S.C. and Lai, K.K. (2004). Empty container management in a port with long-run average criterion. *Mathematical and Computer Modeling*, 40(1): 85-100.
10. Powell, W.B. (2007). Approximate dynamic programming: solving the curses of dimensionality (Wiley Series in Probability and Statistics), Wiley-Interscience, 1st edition.
11. Song, D.P. and Carter, J. (2008). Optimal empty vehicle redistribution for hub-and-spoke transportation systems. *Naval Research Logistics*, 55(2): 156-171.

12. Song, D.P. (2005). Optimal threshold control of empty vehicle redistribution in two depot service systems. *IEEE Transactions on Automatic Control*, 50(1): 87-90.
13. Song, D.P. and Dong, J.X. (2008). Empty container management in cyclic shipping routes. *Maritime Economics and Logistics*, 10(4):335-361.