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$\mathcal{K}$-terminal reliability Evaluation of a Telecommunications Network represented by a Discret and a Dynamic model

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Abstract—In this work a discrete and a dynamic model for evaluating the reliability of telecommunication networks, are defined, in the case where the set of terminals $\mathcal{K}$ is arbitrary. Three methods are presented. The first one is exact type called generalized method of Ahmad, while the two others are approximate and simulation-based type. The three methods are compared by an application to the Bejaia district telecommunications network. The results show that the network is susceptible to failures having negative impact on the quality of the service offered to the users.

Keywords—Telecommunications network design and planning, $\mathcal{K}$-terminal reliability, partition, simulation.

1. INTRODUCTION

Analysis of network reliability is of major importance in communication networks. Because some components of a particular network may be subjected to random failure, we need to compute, as efficiently as possible, the probability that the network is still functional. Such systems are generally modelled either by combinatorial models, or by stochastic processes. This work, considers models known as combinatorial models and is related to the tools that allow modelling and assessing these systems. The main problem is that the system's state size increases in a exponential manner with the details level of the represented system, as in the case of telecommunication networks. The exact methods become, then impractical, in the case of large networks (Cancela, 1996). Alternatively, Monte Carlo type simulation methods may be used. However, failures occur very rarely in the case of highly reliable systems. As a consequence, the reliability of these systems cannot be easily estimated with a reasonable confidence interval. This problem can be solved using techniques based on variance reduction. The study of a network behavior, when some of its components are subject to random failures is treated by reliability theory. We construct a model based on graph concept including the available statistical data, related to properties depending both on the components (terminals and links) and the service that the network has to provide. The reliability evaluation considered here is the probability that, at time t, several sites (terminals) may communicate in a bi-directional network in which the terminals are assumed perfect, and the links are subject to random and independent failures using the $\mathcal{K}$-terminal reliability concept. Two models (discrete and dynamic) for evaluating the reliability of the telecommunications network of Bejaia district are presented. In the first case, a point estimate of the reliability by three methods is calculated. The first one is the exact estimate called the generalized method of Ahmad and the two others are based on simulation. In the dynamic case, an elementary lifespan is associated to each element of the network, in order to evaluate the reliability indices of the network, namely the reliability function and the mean time between failures ($MTBF$). This work is based on the following assumptions: the failures occurrences are independent, the connections have equal probability of correct operations and the use of the concept of $\mathcal{K}$-terminal reliability. The application of the presented methods to the telecommunications network of Bejaia district is described.

2. LITERATURE REVIEW

2.1 Network reliability

Reliability models allow to evaluate the capacity of a given system or product to operate correctly (according to

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specifications). In particular, when the system is made up of a number of individual components, it is necessary to consider their interactions and how their possible failures will affect the operation of the whole system (Lin, 2002).

A telecommunication network is defined as a set of nodes connected by edges, so as to allow the transmission of messages from an extremity to another. When components of a network are subject to random failures, the network may be functional or not after the failure of some components. The probability that the network will function is its reliability (Lucet, 1997).

We distinguish three kinds of reliability:

2-terminal reliability: also called terminal-pair reliability, it is the probability that two given nodes of the graph, called the source and the sink, can communicate (Lucet 1997).

All-terminal reliability: we define the all-terminal reliability as the probability that for every pair of nodes there is at least a path between. This is equivalent to the probability that there is at least one spanning tree in the graph (Hou , 2003).

$\mathcal{K}$-terminal reliability: the $\mathcal{K}$-terminal reliability is the probability that for $\mathcal{K}$ specified target nodes, the graph contains paths between each pair of the $\mathcal{K}$ nodes (Hou , 2003).

Note that 2-terminal reliability and all-terminal reliability are particular cases of the $\mathcal{K}$-terminal reliability.

2.2 Notations

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{K}, r)$ be an undirected, connected, and acyclic network, where $\mathcal{V} = \{1, \cdots, n\}$ is the set of nodes of the network and $\mathcal{E} = \{e_1, \cdots, e_m\}$ the set of links of the network. The set $\mathcal{K} \subseteq \mathcal{V}$ is called the set of terminals of the network. The links of the graph are assumed to be susceptible to break down, the function r being the probability of correct operation of every link. The following notation is used:

- X_i the binary random variable " *state of the link* e_i ", is defined by :

$$X_i = \begin{cases} 1\ If\ the\ link e_i\ works\ correctly \\ 0\ If\ thelink\ e_i\ is\ brokendown, \end{cases}$$

- r_{e_i}: the elementary reliability of the link e_i, $r_{e_i} = P\{X_i = 1\}$,
- $X = (X_1, \cdots, X_m)$: The random state vector of the network,
- $\Phi_{\mathcal{G}}^{\mathcal{K}}(X)$: the random variable « state of the network $\mathcal{G}$»;

$$\Phi_{\mathcal{G}}^{\mathcal{K}}(X) = \begin{cases} 1 \quad if\ thegraphde\ duced\ from\ \mathcal{G}\ by\ dropping \\ \quad the\ broken\ links\ in\ X\ is\ connected \\ 0 \quad else. \end{cases}$$

- $R_{\mathcal{K}}(\mathcal{G}) = R_{\mathcal{K}} = P\{\Phi_{\mathcal{G}}^{\mathcal{K}}(X) = 1\}$: the $\mathcal{K}$-terminal reliability of the network $\mathcal{G}$.
- $R(\mathcal{G})$: probability that the graph $\mathcal{G}$ is connected (all-terminals reliability).

2.3 Complexity of network reliability computation

The problem of the evaluation of a network reliability has been treated by a lot of works in the literature, where several methods were developed. The works of Ball, Provan and Valiant (Wood, 1985), (Ramirez-Marquez, 2005), have permitted to classify this problem in the NP-hard class. Consequently, it was conjectured that there is no exact method which can estimate reliability of a network, at a polynomial time according to its size. When the size of a problem is large, we solve it by approximative methods such as Monte Carlo simulation, then to validate the obtained results, we use an exact method which is the most effective possible and easy to implement (Cancela, 1996).

2.4 Methods of resolution

There are two classes for computation of network reliability. The first class is for approximate computation while the second class is concerned with exact computation of network reliability. The existing algorithms in exact computation are in two different categories, the first category deals with the enumeration of all the minimum paths or cuts. In the second one, the algorithms are based on reducing the graph representing the network by removing some of its components. These reductions allow us to compute the reliability in a simpler way. In (Lucet, 1997), the authors distinguished:

2.4.1 Enumerative methods

State enumeration: It consists in enumerating all the possible states of the stochastic graph and keeping those that allow the

network to function $R(\mathcal{G}) = \sum_{G(X) functions} Prob(X)$, where X is a state of the network.
Path enumeration: An other method to compute the network reliability is the enumeration of the minimal paths that provide a working network. The reliability is the probability for the network to have at least a functioning minimal path.
Cut enumeartion: A cut is a set of links whose failures produce network failure. A minimal cut is a cut that does not include another one. After enumerating all the minimal cuts, We obtain a boolean expression that we transform into a probability expression Pr' using the inclusion-exclusion method or the technique of the sum of disjoint product (Lucet, 1997). Then, the reliability is computed by: $R(\mathcal{G}) = 1 - Pr'$. Cut enumeration is essentially used for 2-terminal reliability.

2.4.2 Reduction-Factoring methods

A reduction is a topological method that can be applied to stochastic undirected graphs. we first suppose that the nodes are perfect. The reduction principle consists in reducing the size of the network $\mathcal{G}$ to get a new graph $\mathcal{G}'$, such that $R(\mathcal{G}) = \Omega \times R(\mathcal{G}')$, where Ω is the reduction factor. For this, a part of the network, with a specific topology, is replaced by an other one with new links whose failure probabilities depend on the previous links probabilities. The reduction have to be applied until the obtained graph $\mathcal{G}'$ can not be reduced. When it's not possible to totally reduce a graph with the reduction method, it is often combined with the factoring method (Wood, 1985), that divides the reliability problem into sub-problems on which we can apply some reductions.

2.4.3 Decomposition methods

The decomposition method can solve the reliability problem for some classes of stochastic undirected graphs in linear time. The decomposition principle for reliability problems generalizes the basic decomposition principle of a graph $\mathcal{G}$ into subgraphs H and L separated by an articulation node, which allows the all-terminal reliability to be computed by the formula: $R(\mathcal{G}) = R(H) \times R(L)$.

3. DISCRETE MODEL

The discrete model aims to compute the $\mathcal{K} - reliability$ at discrete moments of time, depending on the elementary reliability of the components of the network.

In this work, we have used two approximative approches: the crude Monte Carlo simulation and the method of Antithetic variates. The exacte method used to validate the results is the generalized method of Ahmad.

3.1 Exact evaluation by the generalized method of Ahmad

The Ahmad method belongs to the family of partitioning methods for computing the source-terminal reliability. It has been introduced in (Ahmad, 1982), (Ahmad, 1987), and in (Marie, 1988), with more powerful versions, but always within the framework of source-terminal reliability. In (Cancela, 1996), the general $R_{\mathcal{K}}$case is presented.

This method is based on the assumptions of independence between the behavior of the different links, the nodes are supposed to be perfect.

Let us denote by C_k, the event " the graph is k-connected". We have then $R(\mathcal{K}) = P(C_k)$. Let have $\{\pi_1, .., \pi_{nk}\}$, the set of the k-trees and P_j, the event " each link in the j^{th} k −tree is working correctly ".Then, $C_k = \bigcup_j P_j$.

Since, the events P_j are not independent, the formula above is not very useful for computing R_k. Consequently, we define an other partition of C_k, $C_k = \bigcup_j B_j$, where B_j are disjoint events. Hence, we'll have
$R_k = P(C_k) = \sum_j P(B_j)$.

The events B_j are called "*branches*". These branches are identified using alphabetic symbols sequences, each branch moves continuously, until having an element of the partition, that is what we call *a finished branch*. Now, we compute the probability of such event and accumulate it, but without storing the branch, this is why this method requires a very small size of memory.

3.2 Evaluation by simulation

Due to the complexity of evaluating the exact value of the reliability of the network, approximate methods based on simulation are proposed.

3.2.1 Standard Monte Carlo method

Monte Carlo methods provide approximate solutions to a variety of mathematical problems by performing statistical sampling experiments. They can be loosely defined as statistical simulation methods, where statistical simulation is defined in quite general terms to be any method that utilizes sequences of random numbers.

The standard Monte Carlo simulation method is used as a reference for comparing the various methods of estimating the parameter $R_{\mathcal{K}}(\mathcal{G})$. This method is also called Monte Carlo crude or naive method, because it is the most direct method for computing the network reliability by simulation (Cancela, 1996).

Let $\Omega = \{0,1\}^m$ denote the set of the possibilities of the random vector $X = (x_e)_{e\in\mathcal{E}}$. The assumption of independency of the variables x_e implies that for $\tilde{x} = (\tilde{x}_1, \cdots, \tilde{x}_m) \in \Omega$,

$$P\{X = \tilde{x}_e\} = \prod_{\{e:\ \tilde{x}_e=1\}} r_e \cdot \prod_{\{e:\ \tilde{x}_e=0\}} (1 - r_e). \tag{1}$$

The standard estimator of $R_{\mathcal{K}}(\mathcal{G})$ is given by

$$R_{\mathcal{K}}(\mathcal{G}) = \frac{1}{N}\sum_{i=1}^{N} \Phi_{\mathcal{G}}^{\mathcal{K}}(X^{(i)}). \tag{2}$$

where $X^{(1)}, \ldots, X^{(N)}$ is a random sequence of independent vectors having the same distribution as X.

The variance is given by the unbiased estimate (El Khadiri, 1992):

$$\hat{V} = \frac{1}{N-1}\hat{R}_{\mathcal{K}}(\mathcal{G})[1 - \hat{R}_{\mathcal{K}}(\mathcal{G})]. \tag{3}$$

The well known drawback of this method is the high number of iterations required to obtain exact estimates when the network is highly reliable (which can lead to an exponential run time). Several techniques known as variance reduction techniques were proposed to overcome this problem. Among these we cite the antithetic variates method, which offers higher efficiency for the estimate without carrying out changes in the number of tests.

3.2.2 Method of antithetic variates

The theoretical study elaborated in (El Khadiri, 1992), allowed to highlight an algorithm that can solve the problem with higher efficiency for the estimate, and give lower variance than the one given by the standard method, with the same sample.

Let U be a random variable uniformly distributed in the interval $[0,1]$. Then, the random variable $(1 - U)$ is also uniformly distributed in the same interval. Let's consider two samples of N vectors of independent states and having the same distribution as X: $X^{(1)1}, X^{(1)2}, \ldots, X^{(1)N}$ and $X^{(2)1}, X^{(2)2}, \ldots, X^{(2)N}$. The two vectors $X^{(1)j}, X^{(2)j} (j = 1, \cdots, N)$ are generated as follow:

for each link $e_i \in \mathcal{E}(i = 1, \ldots, m)$, u is generated according to the uniform distribution in the interval $[0,1]$,

$$X_i^{(1)j} = \begin{cases} 1 & : u < r_{e_i} \\ 0 & : u \geq r_{e_i} \end{cases} \quad \text{and} \quad X_i^{(2)j} = \begin{cases} 1 & : 1-u < r_{e_i} \\ 0 & : 1-u \geq r_{e_i} \end{cases}.$$

If we denote

$$\hat{R}_j = \frac{1}{2}[\Phi_{\mathcal{G}}^{\mathcal{K}}(X^{(1)j}) + \Phi_{\mathcal{G}}^{\mathcal{K}}(X^{(2)j})] \ , \tag{4}$$

the reliability $R_{\mathcal{K}}(\mathcal{G})$ of the graph $\mathcal{G}$ will be estimated by :

$$\hat{R}_{\mathcal{K}}(\mathcal{G}) = \frac{1}{N}\sum_{j=1}^{N} \hat{R}_j. \tag{5}$$

The variance of $\hat{R}_{\mathcal{K}}(\mathcal{G})$ is given by the following unbiased estimate (El Khadiri, 1992):

$$\hat{V} = \frac{1}{(N-1)}\left(\frac{1}{N}\sum_{j=1}^{N} \hat{R}_j^2 - [\hat{R}_{\mathcal{K}}(\mathcal{G})]^2\right). \tag{6}$$

4. DYNAMIC MODEL

4.1 Description of the model

Let $X_i(t)$ be the random variable defining the state of the link $e_i(i = 1, \ldots, m)$ at time t.

$$X_i(t) = \begin{cases} 1 & if\ T_i > t \\ 0 & else, \end{cases}$$, where T_i is the time to failure of the link $e_i(i = 1, \ldots, m)$.

Let $\Phi_{\mathcal{G}}^{\mathcal{K}}(t)$ be the state of the graph $\mathcal{G}$ defined by

$$\Phi_{\mathcal{G}}^{\mathcal{K}}(t) = \begin{cases} 1 & if\ the graph\ deduced\ from\ \mathcal{G}\ by\ the\ suppression\ of \\ & the\ failing\ links\ is connected\ at\ time\ t \\ 0 & otherwise. \end{cases}$$

$$\Phi_{\mathcal{G}}^{\mathcal{K}}(t) = \Phi_{\mathcal{G}}^{\mathcal{K}}(X_1(t), \cdots, X_m(t)) = \begin{cases} 1 & if\ T > t; \\ 0 & otherwise, \end{cases}$$ where T is the time to failure of the graph $\mathcal{G}$.

The $\mathcal{K}$-terminal reliability of the graph $\mathcal{G}$ is given by:

$$R_{\mathcal{G}}^{\mathcal{K}}(t) = P(\Phi_{\mathcal{G}}^{\mathcal{K}}(t) = 1) = P(T > t). \tag{7}$$

4.2 Evaluation by the discrete method

In order to plot the curve of the function $R_{\mathcal{G}}^{\mathcal{K}}(t)$ of the graph $\mathcal{G}$ in the interval $[0, a]$, this interval is subdivided into sub-intervals of a very small length h. Then the time instants $t_0 = 0, t_1, \cdots, t_k = a$ are defined such that $h = t_j - t_{j-1}, j = 1,2, \cdots, k$ and one of the above mentioned procedures is called to compute at every time t the reliability $R_{\mathcal{G}}^{\mathcal{K}}(t)$. So a numerical method based on the generalized method of Ahmad can be used to have an approximation of $R_{\mathcal{G}}^{\mathcal{K}}(t)$ for each t.

4.3 Method based on discrete events simulation

This method consists of generating a sample $Y_1, Y_2, \dots, Y_N$ representing the breakdowns times of the graphs $\mathcal{G}_1, \mathcal{G}_2, \dots, \mathcal{G}_N$ identical to the graph $\mathcal{G}$. This allows building the empirical function of distribution F_N as follows:

$$F_N(t) = \frac{1}{N} \sum_{i=1}^{N} \mathbf{1}_{\{Y_i \le t\}} = \frac{N(t)}{N}, \tag{8}$$

where $\mathbf{1}_{\{Y_i \le t\}} = \begin{cases} 1 & if\ Y_i \le t \\ 0 & otherwise, \end{cases}$ and $N(t)$ stands for the number of broken graphs at time t.

The mean time between failures of the network $\mathcal{G}$ is estimated by the empirical mean value of the variables $Y_1, Y_2, \dots, Y_N$:

$$\widehat{MTBF} = \frac{1}{N} \sum_{i=1}^{N} Y_i. \tag{9}$$

To generate the random variables $Y_1, \dots, Y_N$, a sample of N random vectors $T^{(i)} = (T_1^{(i)}, \dots, T_m^{(i)})$, $(i = 1, \dots, N)$ is generated, corresponding to the graphs $\mathcal{G}_1, \dots, \mathcal{G}_N$, where $T_j^{(i)} (j = 1, \dots, m)$ is the random variable equal to the time of the breakdown of the link e_j of the graph $\mathcal{G}_i$. It is generated according to the distribution of T_j corresponding to the distribution of the time to failure of the link e_j of the graph $\mathcal{G}_i$.

For each vector $T^{(i)} (i = 1, \dots, N)$, its elements $T_1^{(i)}, \dots, T_m^{(i)}$ are ordered increasingly. An ordered random vector $T^{*(i)} = (T_1^{*(i)}, \dots, T_m^{*(i)})$ is thus obtained, where time instants $T_1^{*(i)}, \dots, T_m^{*(i)}$ correspond to the links $e_1, e_2, \dots, e_m$ respectively.

When a breakdown event (of a link of some graph) occurs, a new graph is built by omitting the broken link. During the time instants $T_1^{*(i)}, \dots, T_m^{*(i)}$ corresponding to time instants of breakdowns of the links $e_1, e_2, \dots, e_m$ of the graph $\mathcal{G}_i$, $(i = 1, \dots, N)$, the graphs $\mathcal{G}_i^{*(1)}, \dots, \mathcal{G}_i^{*(k)}$, $(1km)$ are built one by one, by testing their connectedness, until a graph $\mathcal{G}_i^{*(k)}$ that is not connected is obtained. This leads to the realization of the breakdown event of the graph $\mathcal{G}_i$.

4.3.1 Test of connectedness of a graph

To test whether some nodes of a graph $\mathcal{G}$ are connected, we perform a Depth First Search (DFS) on the graph $\mathcal{G}$, using the procedure $DFS($ Appendix). The function presented below is boolean. It is true if the selected nodes of $\mathcal{G}$ are connected and false otherwise.

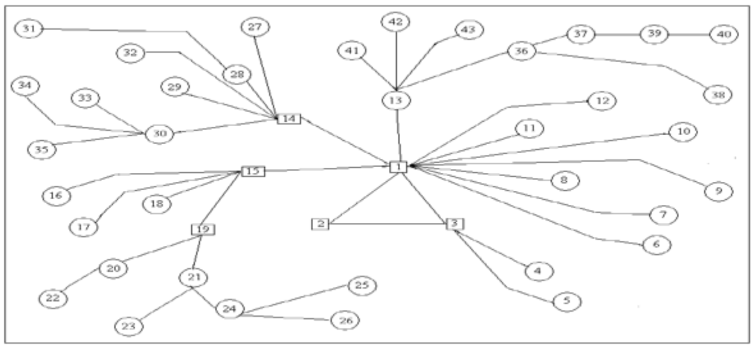
5. APPLICATION TO THE TELECOMMUNICATIONS NETWORK OF BEJAIA DISTRICT

5.1 Modelling

The first stage of the modelling process consists of substituting the studied system by a mathematical model that can be solved either by analytical methods or by simulation.

The modelling problem is considered by referring to an undirected graph. A set $\mathcal{V}$ of n nodes and a set $\mathcal{E}$ of m links are considered. The telecommunication network of Bejaia district (Figure 1.) consists of 43 terminals ($|\mathcal{V}| = 43$) and 43 transmission links ($|\mathcal{E}| = 43$).

Figure 1. Topology of the Bejaia district telecommunication network



5.1.1 Data fitting

Having data on times of the inter-failures of the cables of transmission, we have selected the Weibull model with parameters $\beta = 0.77$ and $\gamma = 79.71$, this choice was validated by the Kolmogorov-Smirnov test and the Chi-square test, with a level of significance $\alpha = 0.05$. Consequently, the cumulative distribution function of the inter-failures of the transmission wires is given by

$$F(t) = 1 - R(t) = 1 - \exp - (\frac{t}{79.71})^{0.77}, t > 0. \quad (10)$$

5.2 Reliability indices evaluation

The previously presented methods are applied to the transmission network of Bejaia district. In the discrete case, we estimate $R_{\mathcal{K}}(\mathcal{G})$, in the dynamic case we estimate the mean time between failures ($MTBF$) and $R_{\mathcal{G}}^{\mathcal{K}}(t)$.

5.2.1 Discrete model

The three most studied cases in the literature have been considered:

- Source-terminal reliability (2-terminal) $R_{st}(\mathcal{G})$ when $\mathcal{K} = \{s, t\}$,
- $\mathcal{K}$-terminal reliability,
- All-terminal reliability when $\mathcal{K} = \mathcal{V}$.

The number of simulation N is fixed at 200 (to agree with the standard of Monte Carlo simulation results) for various values of r such that $r_{e_i} = r, (i = 1, \cdots, m)$. Our results are presented in the Table 1.

Table 1. Estimation results of $R_{\mathcal{K}}(\mathcal{G})$

Elementary reliability r		0.90	0.95	0.98	0.99	0.995	0.999
$\mathcal{K} = \{4,6\}$	Crude	0.8000	0.9050	0.9600	0.9800	0.9900	0.9950
	Antithetic	0.7970	0.8975	0.9590	0.9805	0.9905	0.9975
	Ahmad Method	0.7946	0.8981	0.9596	0.9799	0.9899	0.9979
$\mathcal{K} = \{4,5,14,15\}$	Crude	0.6450	0.7900	0.9450	0.9450	0.9850	0.9950
	Antithetic	0.6350	0.8100	0.9225	0.9600	0.9800	1
	Ahmad Method	0.6436	0.8105	0.9216	0.9604	0.9801	0.9960
$\mathcal{K} = \mathcal{V}$	Crude	0.020	0.1400	0.4650	0.7100	0.8000	0.9595
	Antithetic	0.0150	0.1275	0.4425	0.6700	0.8175	0.9600
	Ahmad Method	0.0143	0.1275	0.4451	0.6687	0.8182	0.9607

Figures 2 to 4 represent the evolution of reliability $R_{\mathcal{K}}(\mathcal{G})$ versus the elementary reliability r, evaluated by the three methods (exact, crude simulation and antithetic variates) according to the three previously treated cases: $\mathcal{K} = \{4,6\}$, $\mathcal{K} = \{4,5,14,15\}$, $\mathcal{K} = \mathcal{V}$ respectively.

Figure 2. Evolution of $R_{\{4,6\}}(\mathcal{G})$ versus r_{e_i}

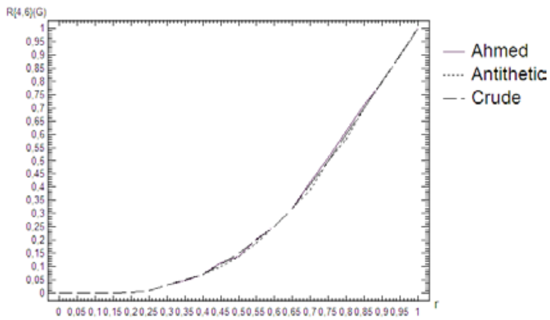


Figure 3. Evolution of $R_{\{4,5,14,15\}}(\mathcal{G})$ versus r_{e_i}

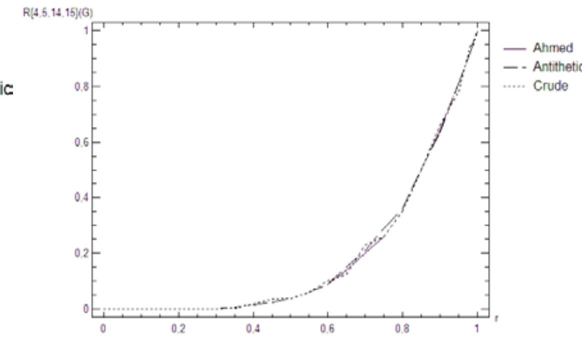
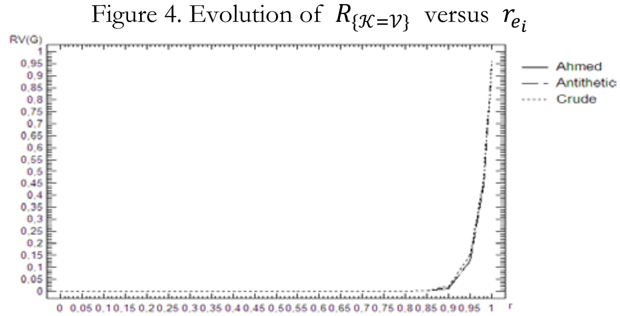


Figure 4. Evolution of $R_{\{\mathcal{K}=\mathcal{V}\}}$ versus r_{e_i}



It is observed that the reliability curves related to the three methods show an increasing function of elementary reliability r_{e_i}. It is also to be noted that the results obtained by using the two methods of crude simulation and antithetic variates agree with those derived using the generalized exact method of Ahmad, confirming the results of the simulation methods. Furthermore, we confirm that the antithetic variates method allows to obtain a higher performance compared to the Monte Carlo method in terms of variance reduction.

Variance of the estimates. Table 2 summarizes the variance of the estimators for the reliability measurement $R_{\mathcal{K}}(\mathcal{G})$ for the graph $\mathcal{G}$, obtained by employing the two simulation methods, and this for the previous values of r.

Table 2. Variance estimates of $R_{\mathcal{K}}(\mathcal{G})$ in the simulation methods

Elementary reliability r		0.90	0.95	0.98	0.99	0.995	0.999
$\mathcal{K} = \{4,6\}$	Crude	0.00080	0.00043	0.00019	0.00009	0.00004	0.00002
	Antithetic	0.00035	0.00021	0.00009	0.00004	0.00002	0.00000
$\mathcal{K} = \{4,5,14,15\}$	Crude	0.00115	0.00083	0.00026	0.00026	0.00007	0.00002
	Antithetic	0.00053	0.00037	0.00017	0.00010	0.00004	0.00000
$\mathcal{K} = \mathcal{V}$	Crude	0.00009	0.00060	0.00125	0.00103	0.00080	0.00019
	Antithetic	0.00003	0.00028	0.00060	0.00058	0.00034	0.00009

We easily remark that the variance estimates obtained by the use of antithetic variates simulation are smaller than those obtained by the crude simulation.

Sensitivity analysis of simulation methods. The run time of the two types of simulation methods in the case where $\mathcal{K} = \mathcal{V}$ as function of the iteration number of simulation is presented. The variance of the estimates for the simulation methods as function of the run time is also given. The elementary reliability is fixed at $r_{e_i} = r = 0.9$, the methods were implemented on a 2.00MHz Pentium(R) processor. Table 3. summarizes the obtained results.

Table 3. Run time and variance of the estimates for the Simulation methods function of the number of simulations N, where $\mathcal{K} = \mathcal{V}$

	Run time	(second)	Variance of	the estimates
Number of iterations	crude	Antithetic	Crude	Antithetic
100	1	2	0.0014	2.5×10^{-5}
200	2	5	9×10^{-5}	7.2×10^{-6}
300	4	8	7.62×10^{-6}	6.4×10^{-6}
400	5	11	5.6×10^{-6}	4.56×10^{-6}
500	7	14	5.4×10^{-6}	3.9×10^{-7}
600	8	17	2.41×10^{-6}	1.11×10^{-7}
700	9	20	2.11×10^{-6}	9.96×10^{-8}
800	11	23	1.07×10^{-6}	9.78×10^{-8}
900	12	25	8.8×10^{-7}	7.6×10^{-8}
1000	13	28	8.1×10^{-7}	6.48×10^{-8}
1500	20	43	8.6×10^{-7}	7.4×10^{-8}
2000	27	57	6.7×10^{-7}	4.98×10^{-8}

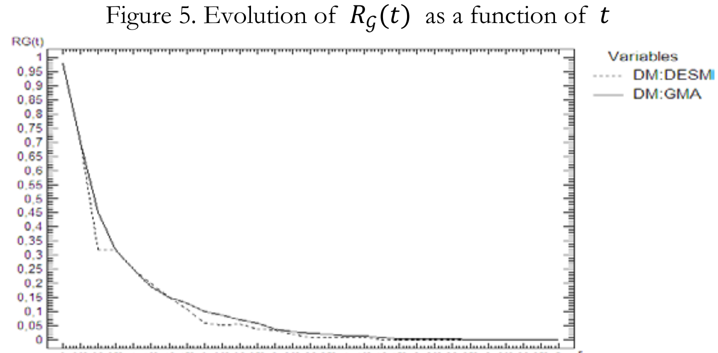
It can easily be seen from these results, that the run time for each method is as a linear function of the number of simulations, although there exists a small difference in favor of the crude Monte Carlo simulation method. But looking at the variance of the estimates, the antithetic variates method is more advantageous because it provides a best estimate.

5.2.2 Dynamic model

In order to plot the curve of the function $R_{\mathcal{G}}^{\mathcal{K}}(t)$, we quantify the reliability of the equipment for different instants, by determining the distribution of the inter-failures of the transmission cables.

To this end, the methods previously explained are implemented. Let us recall that the first is the generalized method of Ahmad, while the second is a discrete events simulation procedure. By specifying the values of the parameters of the methods for which the interval of time, the moving step and sample size of simulation are $I = [0,100]$, $h = 0.05$ and $N = 100$ respectively, as well as the distribution of the inter-failures of the transmission cables, the curve of the function $R_{\mathcal{G}}^{\mathcal{K}}(t)$ is plotted with the restriction to the case of the evaluation of all-terminals reliability $R_{\mathcal{G}}(t)$ of the network. Figure 5. (where DM: G.M.A means dynamic model: generalized method of Ahmad and DM: D.E.S.M means dynamic model: discrete event simulation methods) shows the variation of reliability in terms of time, evaluated by the two above mentioned methods.

Figure 5. Evolution of $R_{\mathcal{G}}(t)$ as a function of t



By observing the evolution of the two curves in $R_{\mathcal{G}}(t)$ of the Fig.5, we remarque that the curve obtained by simulation is close to that derived using the numerical method, confirming, thus, the simulation results. The plots exhibit a rapidly decreasing shape with time. That means, that the network is in a phase of ageing.

The value of the mean time between failures of the network $(MTBF)$ is also estimated by using the formula: $\widehat{MTBF} = \frac{1}{N}\sum_{i=1}^{N} Y_i = 1.018\ days,$ where Y_i are given by the generated sample procedure.

6. CONCLUSION

The Monte Carlo evaluation of the network reliability measures appears to be very useful in the analysis of communication systems since the exact computation of these measures is extremely time consuming. Even relatively small networks (say, with less than one hundred lines) are often impossible to evaluate exactly.

In this paper, two techniques for evaluating usual network reliability measures are presented. A feature of these methods is that their time complexity is sensitive to the number of simulations done, and the variance of the estimates $R_{\mathcal{k}}(G)$ is influenced by the reliability of the component of the network.

The application of these methods shows clearly the convergence of the results. The simulation techniques give solutions often close to the exact method results. These constitute then an alternative to the exact approach. This is the result reached in the case of the discrete and dynamic model. These results show the susceptibility of the network to experience failures having consequences penalizing the subscribers (Quality of Service) and the firm (economic).

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APPENDIX

Procedure Depth First Search

The procedure DFS is defined as follows :

Procedure $\boldsymbol{DFS(\mathcal{G}, s, Mark)}$
Begin $Z := Nil$;
Initialize $Mark$ to false;
$Mark[s] := \boldsymbol{true}$;
push(Z, s);
While $Z <> \boldsymbol{Nil}$ **do**
$x := Top(Z)$; " $Top(Z)$ is the element at the top of the stack Z "
Let y be an unspecified node adjacent to x non marked.
"$y = nil$ if there is no adjacent node to x non marked"
($y \neq nil$) **then**
push(Z, y);
$\boldsymbol{Mark}[y] := true$; **else** pop($Z, x$); **End if.**