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Simultaneous Multi-player Game-solution Identification for Non-cooperative Advertising in Supply Chain Using MOPSO-CD and NSGA II

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Abstract **—** Supply chain members, manufacturers and retailers, usually share common responsibility on national and local advertising for sales promotion. However, how to delineate and execute the non-cooperative advertising strategies, which is one kind of channel coordination mechanism in marketing domain, within a supply chain is a critical issue and must be addressed by both manufacturers and retailers. For analyzing non-cooperative advertising among multiple agents with interdependent objectives, game theory has become a popular tool recently. Hence, in this research, the game theory is used to investigate different gaming structures in a supply chain might influence the manufacturer's subsidy policy and their branding investment in national advertising activities. The primary objective of this research is to construct the mathematic models in different market response functions associated with the gaming structures, and then to identify their equilibrium (or solutions) and also to explore some preference conditions for both supply chain players. In this research, four different cases of market response functions are constructed. There are cases of market response functions that we consider the manufacturers whose polices in long-term branding investments could influence the retailers whose polices in short-term promotion efforts. Thus, this research problems become to solve the systematic non-cooperative advertising problem under different market response functions by using (1) the analytic solution approach to identify both the equilibriums of two-player of manufacturing-retailer (M-R) simultaneous Nash game; it's solution as a reference of the solution using swarm particle optimization-crowding distance (MOPSO-CD) or non-dominated sorting genetic algorithm (NSGA II) and (2) by using MOPSO-CD or NSGA II integrated the Nash game identify the equilibriums of the case of both multi-manufacturer and multi-retailer without considering the polices in long-term branding investments could be influenced each other by manufacturer firms and in short-term promotion efforts could be influenced each other by retailers in the non-cooperative advertising. Finally, this research will implement a real case and their numerical results will demonstrate the feasibility of the equilibrium (or solution) using MOPSO-CD or NSGA II for solving multi-objective and multi-disciplinary optimization problems of the non-cooperative advertising in supply chain.

Keywords **—** Non-cooperative advertising, nash equilibrium, multi-objective problem optimization, multi-player game, particle swarm optimization, genetic algorithms

1. INTRODUCTION

Many studies on supply chain management have emphasized the long-term strategic relationship between a manufacturing firm and its retailer (Maloni and Benton, 1997). The premise underlying this relationship is that such a partnership makes both the manufacturer and the retailer better off than before. That is, the essence of partnership is to take both parties into a win-win situation through coordination. Many works has been carried out on coordination

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mechanisms in the last decade (Malone and Crowston, 1994). Just-in-time (JIT) purchasing, collaborative planning, vendor managed inventory (VMI), third party logistics methods to deal with the inter-organizational coordination. These mechanisms can enhance the partner's quality, reduce the risk and supply cost, promote their communication and share their profits.

Although a lot of literature has focused on supply chain coordination, most of them are just conceptual discussions instead of an analytical analysis. An interesting survey found by Croom *et al.* (2000) suggested the relative lack of theoretical work in this filed when compared to empirically based studies. They set a framework for literature analysis which is categorized according to two dimensions - from theoretical to empirical and prescriptive to descriptive. The survey shows that the literature is dominated by descriptive empirical studies. Very few in the way of theoretical works have been developed. The efforts of this study will make to fit in the theoretical and prescriptive category, which takes up some percentages of the relevant literature.

Among those inter-organization coordination mechanisms, non-cooperative advertising is a famous one in supply chain domain. Generally speaking, there are two types of advertising for any product. National advertising is intended to reinforce the brand image in the eyes of potential consumers and to build their purchasing preference. On the other hand, local advertising is to encourage consumers' buying behaviors and to give consumers more reasons to buy target products in the near future. That is, the emphasis in national advertising is to create favorable product attitudes, whereas local advertising is often price oriented.

Non-cooperative advertising is often defined as an arrangement whereby a manufacturer pays for some or all of the costs of local advertising undertaken by a retailer who is responsible for selling products made by the manufacturer (Bergen and John, 1997). The main purpose for a manufacturer to utilize non-cooperative advertising is to strengthen the image of its own brand and to increase the short-term sales at the retail lever (Hutchins, 1953). Bergen and John (1997) also pointed out that non-cooperative advertising is not a specialized kind of advertising, instead; it is essentially a financial arrangement under which both parties agree how the costs of mutual promotion are to be defrayed. They focused on the most prominent aspect of these plans – the "participation rate", that is, the percentage of the retailers' local advertising expenditures that the manufacturer agrees to pay. In a same vein from Croom *et al.* (2000), Crimmins (1984) also found that much of the extant work on non-cooperative advertising is descriptive, reporting trends in industry practice, legal issues and management problems.

In their seminal work, Jorgensen *et al.* (2000) used differential games to analytically study how a manufacturer can design an inter-temporal advertising support program that is optimally non-coordinated with his own advertising strategies and the retailer's advertising efforts. Besides that, Li *et al.*, (2002) investigated the efficiency of transactions for the non-cooperative advertising in the context of game theory. They developed manufacturer and a retailer in a supply chain. The reason to use game theory is rooted on the fact that many models in supply chains developed from single decision maker's perspective cannot adequately stand for the sophisticated competitive and non-cooperative relationships in supply chains, and game theory, a tool of strategy analysis for conflict and non-cooperation, should be a more desirable approach. Cachon and Netessine (2003) also pointed out that game theory has become an essential tool in the analysis of supply chains consisting of multiple agents with conflicting objectives. The reason is that it is more important to understand the interactions among independent agents within and across firms. They also expect that the application of game theory to supply chain management is still in its infancy; much more progress will be made soon. Therefore, this kind of research is highly demanded.

As indicated in their possible avenues for future research, different sales response function may yield interesting results in the analysis for systematic non-cooperative advertising agreements (Li *et al.*, 2002). Lilien *et al.* (1992) proposed a market response function based on a product was determined by the manufacturer's national brand image investment and the retailer's local advertising expenditures. In the model, the effects of the local advertising expenditures and brand image investments on sales quantity are limited. And only when both efforts are exhausted, the market saturation level will be attained. In addition, the participation rate of local advertising expenditures shared by the manufacturer, which is a decision variable for manufacturer in addition to the national brand image investment. Assuming the interactions between long-term branding investments and short-term promotion efforts can be neglected, and based on a simple and different market response functions, this research will study the non-cooperative advertising problem under simultaneous Nash game.

Nash equilibrium is the solution of a non-cooperative strategy of multi-objective optimization first introduced by Nash (1951). Since it appeared first in Economics, the notion of player is often used in the sequel. Each player is in charge of one objective, has his own strategy set and its own criterion. During the game, each player looks for the best strategy in his search space in order to improving his own criterion while criteria of other players are fixed. The frequency of exchange of strategies ν is called the Nash frequency, generally $\nu = 1$, which means the exchange of best strategies takes place at the end of each generation. When no player can further improve his criterion, the system has reached a state of equilibrium named Nash equilibrium.

In the following a two-player Nash game is considered to present the Nash equilibrium mechanism. Let A denotes the search space for the first player, *B* the search space for the second player, a strategy pair $(x^*, y^*) \in A \times B$ is a two-player Nash equilibrium iff:

$f_A(x^*, y^*) = \inf_{x \in A} f_A(x, y^*)$ or $f_A(x^*, y^*) = \operatorname{Min}_{x \in A} f_A(x, y^*)$; while f_A is the cost response function, or (1)

$f_A(x^*, y^*) = \operatorname{Sup}_{x \in A} f_A(x, y^*)$ or $f_A(x^*, y^*) = \operatorname{Max}_{x \in A} f_A(x, y^*)$; while f_A is the profit response function, (1')

$f_B(x^*, y^*) = \inf_{y \in B} f_B(x^*, y)$ or $f_B(x^*, y^*) = \operatorname{Min}_{y \in B} f_B(x^*, y)$; while f_B is the cost response function, or (2)

$f_B(x^*, y^*) = \operatorname{Sup}_{y \in B} f_B(x^*, y)$ or $f_B(x^*, y^*) = \operatorname{Max}_{y \in B} f_A(x^*, y)$; while f_B is the profit response function, (2')

where f_A could be denoted as the gain or loss for the first player, *A*, f_B could be denoted as the gain or loss for the second player, *B*.

Wang and Periaux (2001) proposed two optimization strategies based on games and compared to address the problem of multi-objective optimization: a non-symmetric hierarchical approach named a Stackelberg game and a symmetric non-cooperative game based on Nash equilibrium. They combined Stackelberg/GAs (namely S/GAs) and Nash/GAs (namely N/GAs) optimization methods to implement and optimize the position of flap and slat of a high lift system operating simultaneous at taking off and landing conditions. S/GAs provide better solutions if the leader and the follower are correctly chosen according to the physics of the problem. In Wang and Periaux (2001), the main interest of N/GAs is that they are faster and more robust even if the solutions that they find are near optimal rather than optimal. Therefore, in this research the multi-objective techniques combining ideas from game theory with GAs may lead to powerful and robust methods for solving non-cooperative advertising in the manufacturer-retailer supply chain problems. Deb (2001) and Deb *et al.* (2002) have introduced elitism and diversity preservation mechanisms to the non-dominated sorting genetic algorithm (NSGA) (Srinivas and Deb, 1994) to improve the algorithm performance. The revised algorithm, named NSGA-II, has been applied to many theoretical and real-world problems, and is shown to be more efficient than its predecessor. Optimization problems are based on NSGA and NSGA-II. Both NSGA and NSGA-II use non-dominated sorting to determine preliminary fitness values.

A particle swarm optimization (PSO) algorithm is a member of the wide category of swarm intelligence methods (Kennedy and Eberhart, 2001) for solving global optimization (GO) problems. It was originally proposed by J. Kennedy as a simulation of social behavior, and it was initially introduced as an optimization method in 1995 (Eberhart and Kennedy, 1995). PSO algorithm is related with artificial life and specifically to swarming theories, and also with evolutionary computation (EC), especially evolution strategies (ES) and genetic algorithm (GA). PSO algorithm can be easily implemented and it is computationally inexpensive, since its memory and CPU speed requirements are low (Eberhart et. al., 1996). Moreover, it does not require gradient information of the corrective function under consideration, but only its values, and it uses only primitive mathematical operators. PSO algorithm has been proved to be an efficient method for many goal optimization (GO) problems and in some cases it does not suffer the difficulties encountered by other EC techniques (Eberhart and Kennedy, 1995). Later, Clerc and Kennedy (2002) indicated that even though PSO had been shown to perform well, but researchers had not adequately explained how it works. Traditional versions of the PSO had had some dynamical properties that were not considered to be desirable, notably the particles' velocities needed to be limited in order to controlling their trajectories and convergence. They analyzed the particle's trajectory as it moved in discrete time (the algebraic view), then progresses to the view of it in continuous time (the analytical view). Those analyses leaded to a generalized model of the algorithm, containing a set of coefficients to control the system's convergence tendencies. Some results of the particle swarm optimizer suggested methods for altering the original algorithm in ways that eliminated problems and increased the optimization power of the particle swarm.

The performance of different multi-objective algorithms that incorporate such optimization techniques was compared in Coello *et al.*, (2004) using five test functions. These algorithms are NSGA-II, PAES (Knowles and Corne, 2000) and SPEA2 (Zitzler *et al.*, 2000), Micro-GA (Coello and Pulido, 2001) and MOPSO. The results show that MOPSO was able to generate the best set of non-dominated solutions close to the true Pareto front in all test functions except in one function where NSGA-II is superior. In terms of diversity of the non-dominated solutions, NSGA-II produced the best results in all test functions but was not able to cover the entire Pareto front in all test functions. MOPSO was the only algorithm which was able to cover the entire Pareto front.

Multi-objective particle swarm optimization - crowding distance (MOPSO-CD) approach extends the algorithm of the single-objective PSO to handle multi-objective optimization problems. It incorporates the mechanism of crowding distance computation into the algorithm of PSO specifically on global best selection and in the deletion method of an external archive of non-dominated solutions (Raquel and Naval, 2005). The crowding distance mechanism together with a mutation operator maintains the diversity of non-dominated solutions in the external archive. MOPSO-CD also has a constraint handling mechanism for solving constrained optimization problems.

In this research, the Nash/MOPSO-CD (namely (N/MOPSO-CD)) optimization method will be proposed. In N/MOPSO-CD game, the MOPSO-CD algorithm will be used to handle the Nash games by incorporating the mechanism of crowding distance computation, specifically on global best selection and in the deletion method of an external archive of non-dominate d solutions (Raquel and Naval, 2005). The performance in converging of this approach will be evaluated on test functions and metrics from literature.

Based on previous description, the research problem can be addressed as follows.

(1) Comparing and analyzing the equilibriums of two-player of M-R Nash game for the non-cooperative advertising in supply chain by using the analytic solution approach and the identified equilibriums can be the reference of the solution approach by using MOPSO-CD or NSGA II for comparison.

(2) Comparing and analyzing the case of U vs. V equilibriums of multi-player M-R Nash game for the non-cooperative advertising in supply chain using the MOPSO-CD and SNGA II, where U is the number of manufacturers and V is the number of retailers.

2. GAME THEORY FOR MANUFACTURER AND RETAILER NON-COOPERATIVE ADVERTISING IN SUPPLY CHAIN

The main purpose of this section is to identify the equilibriums associated with Nash game for both supply chain players. This game-theoretic model is a systematic move game in which the manufacturer and the leader both players are in parity and engage in a simultaneous move game.

2.1 Two-player Nash Game for Non-cooperative Advertising

Consider a supply chain consisting of a manufacturer and a retailer. Assume that the market response function of the product is mainly determined by the manufacturer's national brand image investment, N, and the retailer's local advertising expenditures, L. The participation rate of local advertising expenditures shared by the manufacturer is θ, which is a decision variable for manufacturer in addition to the national brand image investment, N. The following is the market response function adopted from Lilien *et al.*, (1992).

$$Q(N,\,L) = \alpha - \beta_1 N^{-\delta} - \beta_2 L^{-\gamma},\ \alpha > 0,\ \beta_1 > 0,\ \beta_2 > 0,\ \delta > 0,\ \gamma > 0 \tag{3}$$

where α is the market saturation level, β_1, β_2, δ and γ are positive constants and are the experience parameters. The rationale to use a polynomial format for the sales quantity is based on the following properties. First of all, $Q(N,L)$ is a non-decreasing function of both N and L. Next, the sales quantity $Q(N,L)$ approaches to α when both the local advertising efforts and brand image investments turn to infinity, that is, $\lim_{N\to\infty,\,L\to\infty} Q(N,L) = \alpha$. In other words, the effects of their local advertising expenditures and brand image investments on sales quantity are limited. Only when both efforts are exhausted, the market saturation level will be attained. Finally, because $\partial^2 Q(N,L)/\partial N^2 < 0$ and $\partial^2 Q(N,L)/\partial L^2 < 0a$, there are decreasing returns to scale of efforts on branding investments and local advertising. The higher the value of the influence parameter δ is, the more the impact of brand image investments is on the market response function. In the same logic, the higher the value of another influence parameter γ is, the more the impact of local advertising is on the sales quantity.

The marginal profits of the manufacturer and retailer for each product sold are ρ_m and ρ_r, respectively. The participation rate of local advertising expenditures shared by the manufacturer is θ, which is a decision variable for manufacturer in addition to the national brand image investment. According to the above parameters, let π_m, π_r and π_s be the payoff functions for both parties and whole supply chain, thus, there are the following payoff functions.

$$\pi_m = \rho_m(\alpha - \beta_1 N^{-\delta} - \beta_2 L^{-\gamma}) - \theta\, L - N\,, \tag{4}$$

$$\pi_r = \rho_r(\alpha - \beta_1 N^{-\delta} - \beta_2 L^{-\gamma}) - (1-\theta)L\,, \tag{5}$$

$$\pi_s = \pi_m + \pi_r = (\rho_m + \rho_r)(\alpha - \beta_1 N^{-\delta} - \beta_2 L^{-\gamma}) - L - N \tag{6}$$

The objective of each player is to simultaneously maximize its own payoff function with respect to any possible strategies set by the other players in the supply chain (Fudenberg and Tirole 1991). The objective profit of manufacturer is to maximize Eq. (4), that is

$$\underset{N,\,L,\,\theta}{\text{Max}}\, \pi_m = \rho_m(\alpha - \beta_1 N^{-\delta} - \beta_2 L^{-\gamma}) - \theta L - N \tag{7}$$

s. t. $L = [\gamma\rho_r \beta_2 / (1-\theta)]^{1/(\gamma+1)} \geq 0a$, $\alpha > 0,\ \beta_1 > 0,\ \beta_2 > 0,\ \delta > 0,\ \gamma > 0,\ \rho_m > 0,\ 0 \leq \theta \leq 1 \quad N \geq 0$.

$$\underset{N,\,L,\,\theta}{\text{Max}}\, \pi_r = \rho_r(\alpha - \beta_1 N^{-\delta} - \beta_2 L^{-\gamma}) - (1-\theta)L \tag{8}$$

s. t. $L \geq 0$, $\alpha > 0,\ \beta_1 > 0,\ \beta_2 > 0,\ \delta > 0,\ \gamma > 0,\ \rho_r > 0$.

However, it is obvious that the cooperative participation rate, θ, is zero because of its negative coefficient in the payoff function. Therefore, the first order conditions for both players are reduced to $(\partial\pi_m / \partial N) = 0$ and $(\partial\pi_r / \partial L) = 0$, and

$$\delta\rho_m \beta_1 N^{-(\delta+1)} - 1 = 0, \tag{9}$$

and

$$\gamma\rho_r \beta_2 L^{-(\gamma+1)} - (1-\theta) = 0. \tag{10}$$

Solving system of Eqs. (9) and (10), we obtain that the solution, $(N^{**}, \theta^{**}, L^{**})$, concerning the Nash equilibrium of the simultaneous move game are as follows.

$$N^{**} = [\delta\beta_1\rho_m]^{\frac{1}{\delta+1}}, \tag{11}$$

$$\theta^{**} = 0, \text{ and} \tag{12}$$

$$L^{**} = [\gamma\beta_2\rho_r]^{\frac{1}{\gamma+1}}. \tag{13}$$

This theorem implies several important facts. First of all, no matter what values of the other parameters are, the manufacture is unwilling to share any portion of local marketing effort with the retailer under the simultaneous move game. Secondly, the manufacturer's marginal profit is positively related to the national brand image investment. That is, $\partial N^{**} / \partial\rho_m > 0$. For the manufacturer, a higher marginal profit gives him a string incentive to invest more in its brand reputation. Thirdly, the retailer's marginal profit is positively related to the local advertising efforts. That is, $\partial L^{**} / \partial\rho_r > 0$. In view of the retailer, the higher the marginal profit, the stronger the incentive to increase local advertising budgets even though the manufacturer does not share the cost.

2.2 Multi-player Nash Game for Non-Cooperative Advertising (Case of *U* vs. *V* M-R Nash Game)

Assume that the market response function of the product is also mainly determined by the each manufacturer's national brand image investment, N_i, $i = 1, 2, \ldots, U$, where the U manufacturers whose polices are not influenced by each other and each retailer's local advertising expenditures, L_{ij}, $i = 1, 2, \ldots, U$, $j = 1, 2, \ldots, V$, where the V retailers whose polices are not influenced by each other, and each retailer's local advertising expenditures corresponded to each manufacturer's participation rate. The market response function could be

$$\begin{aligned} &Q(N_i, L_{ij});\ i = 1, 2, \ldots, U,\ j = 1, 2, \ldots, V \\ &= \alpha - \sum_{i=1}^{U} \beta_{1i} N_i^{-\delta_i} - \sum_{i=1}^{U}\sum_{j=1}^{V} \beta_{2ij} L_{ij}^{-\gamma_{ij}} \\ &\alpha > 0,\ \beta_{1i} > 0,\ \beta_{2ij} > 0,\ \delta_i > 0,\ \gamma_{ij} > 0;\quad i = 1, 2, \ldots, U,\quad j = 1, 2, \ldots, V, \end{aligned} \tag{14}$$

where α, is the market saturation level, β_{1i}, $i = 1, 2, \ldots, U$, β_{2ij}, $i = 1, 2, \ldots, U$, $j = 1, 2, \ldots, V$, δ_i, $i = 1, 2, \ldots, U$ and γ_{ij}, $i = 1, 2, \ldots, U$, $j = 1, 2, \ldots, V$ are the positive constants. Similarly, the marginal profits of manufacturers and retailers for each product sold in averaged prices are ρ_m and ρ_r , respectively. Assume every retailer's marginal profit is the same, that is, the product price will be one price from each retailer. In addition, the participation rate of V local advertising expenditures shared by the U manufacturers is θ_{ij}, $i = 1, 2, \ldots, U$, $j = 1, 2, \ldots,$ which is the decision variable for each manufacturer in addition to the national brand image investment, N_i, $i = 1, 2, \ldots, U$.

The objective of the manufacturer is

$$\text{Max}_{N_i,\theta_{ij}}\pi_m = \rho_m(\alpha - \sum_{i=1}^{U}\beta_{1i}N_i^{-\delta_i} - \sum_{i=1}^{U}\sum_{j=1}^{V}\beta_{2ij}L_{ij}^{-\gamma_{ij}}) - \sum_{i=1}^{U}\sum_{j=1}^{V}\theta_{ij}L_{ij} - \sum_{i=1}^{U}N_i \tag{15}$$

s. t. $0 \le \sum_{i=1}^{U}\sum_{j=1}^{V}\theta_{ij} \le 1$, $i = 1, 2, ..., U$; $j = 1, 2, ..., V$, $N_i \ge 0$,

$i = 1, 2, ..., U$, $\alpha > 0$, $\beta_{1i} > 0, \beta_{2ij} > 0, \delta_i > 0, \gamma_{ij} > 0$, $\rho_r > 0$, $L_{ij} \ge 0$, $0 \le \theta_{ij} \le 1$, $j = 1, 2, ..., V$, $N_i \ge 0$, where α is also the market saturation level, β_{1i} , $i = 1, 2, ..., U$, β_{2ij} , $i = 1, 2, ..., U$, $j = 1, 2, ..., V$, δ_i , $i = 1, 2, ..., U$ and γ_{ij} , $i = 1, 2, ..., U$, $j = 1, 2, ..., V$ are the positive constants.

$$\text{Max}_{N_i,L_{ij},\theta_{ij}}\pi_r = \rho_r(\alpha - \sum_{i=1}^{U}\beta_{1i}N_i^{-\delta_i} - \sum_{i=1}^{U}\sum_{j=1}^{V}\beta_{2ij}L_{ij}^{-\gamma_{ij}}) - \sum_{i=1}^{U}\sum_{j=1}^{V}(1-\theta_{ij})L_{ij} \tag{16}$$

s. t. $L_{ij} \ge 0$, $i = 1, 2, ..., U$, $j = 1, 2, ..., V$.

However, it is obvious that the non-cooperative participation rate, θ_{ij} , $i = 1, 2, ..., U$; $j = 1, 2, ..., V$, is zero because of its negative coefficient in the payoff function. Solving system of Eqs. (15) and (16) for N_i , θ_{ij} and L_{ij} , the optimal solution, N_i^{**} , θ_{ij}^{**} and L_{ij}^{**} are not the same as using Eqs. (11), (12) and (13), individually. The analytical solution procedure could be difficult, thus, the MOPSO-CD or SNGA II and can be used to the solution procedures.

The two-stage game can be analyzed by (Fudenberg and Tirole, 1991), S/MOPSO-CD or S/NSGA II (Wang and Periaux, 2001).

3. SOLUTION METHODOLOGY

3.1 The Algorithms of PSO and PSO-CD

PSO represents an optimization method where particles collaborate as a population to reach a collective goal. Each *n*-dimensional particle $\mathbf{x}_i$ is a potential solution to the collective goal, usually to minimize a function, *f*. PSO differs from traditional optimization methods, in that a population of potential solutions is used in the search. The direct fitness information instead of function derivatives or other related knowledge is used to guide the search.

A particle $\mathbf{x}_i$ has memory of the best solution $\mathbf{y}_i$ that it has found, called its *personal best*; it flies through the search space with a velocity $\mathbf{v}_i$, which is dynamically adjusted according to its personal best and the *global best* solution $\mathbf{y}_i$ by found by the rest of the swarm (called the *gbest* topology). Other topologies for information sharing have also been investigated (Kennedy and Eberhart, 1995; Kennedy *et al.*, 2001; Kennedy and Mendes, 2002).

Let *i* indicates a particle's index in the swarm, such that $\mathbf{S} = \{\mathbf{x}_1, \mathbf{x}_2, \cdots\cdots, \mathbf{x}_s\}$ is a swarm of s particles. During iterations of the PSO algorithm, the personal best $\mathbf{y}_i$ of each particle is compared to its current performance, and set to the better performance. If the objective function to be minimized is defined as $f : \mathbf{R}^n \to \mathbf{R}$ then

$$\mathbf{y}_i^{(t)} = \begin{cases} \mathbf{y}_i^{(t-1)} & if \;\; f(\mathbf{x}_i^{(t)}) \ge f(\mathbf{y}_i^{(t-1)}) \\ \mathbf{x}_i^{(t)} & if \;\; f(\mathbf{x}_i^{(t)}) < f(\mathbf{y}_i^{(t-1)}) \end{cases} . \tag{17}$$

The global best $\mathbf{y}_i$ is updated to the position with the best performance within the swarm, with

$$\hat{\mathbf{y}}^{(t)} \in \{\mathbf{y}_1^{(t)}, \mathbf{y}_2^{(t)}, \cdots\cdots, \mathbf{y}_s^{(t)}\} \mid f(\hat{\mathbf{y}}^{(t)})$$
$$= min\{f(\mathbf{y}_1^{(t)}), f(\mathbf{y}_2^{(t)}), \cdots\cdots, f(\mathbf{y}_s^{(t)})\} . \tag{18}$$

Traditionally, each particle's velocity and position is updated separately for each dimension *j*, with

$$v_{ij}^{(t+1)} = wv_{ij}^{(t)} + c_1 r_{1j}^{(t)}(y_{ij}^{(t)} - x_{ij}^{(t)}) + c_2 r_{2j}^{(t)}(\hat{y}_j^{(t)} - x_{ij}^{(t)}) \; , \tag{19}$$

$$x_{ij}^{(t+1)} = v_{ij}^{(t+1)} + x_{ij}^{(t)} \; . \tag{20}$$

The stochastic nature of the algorithm is determined by $r_{1j}^{(t)}$ and $r_{2j}^{(t)}$, two uniform random numbers between zero and one. These random numbers are scaled by acceleration coefficients c_1 and c_2 , where $0 \le c_1, c_2 \le 2$. The inertia weight w was introduced to improve the convergence rate of the PSO algorithm (Shi and Eberhart, 1998). It is possible to clamp the velocity vectors by specifying upper and lower bounds on $\mathbf{v}_i$, to avoid too rapid movement of particles in the search space.

The standard PSO algorithm is summarized below (Paquet and Engelbrecht, 2003):

Algorithm – Standard Particle Swarm Optimizer

Step 1. Set the iteration number t to zero, and randomly initialize swarm $\boldsymbol{S}$ within the search space.

Step 2. Evaluate the performance $f(\mathbf{x}_i^{(t)})$ of each particle.

Step 3. Compare the personal best of each particle to its current performance, and set $\mathbf{y}_i^{(t)}$ to the better performance, according to Eq. (17).

Step 4. Set the global best $\hat{\mathbf{y}}_i^{(t)}$ to the position of the particle with the best performance within the swarm, according to Eq. (18).

Step 5. Change the velocity vector for each particle, according to Eq. (19).

Step 6. Move each particle to its new position, according to Eq. (20).

Step 7. Let $t := t + 1$.

Step 8. Go to Step 2, and repeat until convergence.

The PSO algorithm described above does not lend itself well to optimizing constrained functions (Paquet and Engelbrecht, 2003).

3.1.1 MOPSO Algorithm

MOPSO-CD algorithm which is proposed by Raquel and Naval (2005) is as follows.

1. For $i = 1$ to M (M is the population size)
   a. Initialize $P[i]$ randomly (P is the population of particles)
   b. Initialize $VEL[i] = 0$ (VEL is the speed of each particle)
   c. Evaluate $P[i]$
   d. Initialize the personal best of each particle pbests[i] = $P[i]$
   e. gbest = Best particle found in $P[i]$
2. End For
3. Initialize the iteration counter $t = 0$
4. Store the non-dominated vectors found in P into A (A is the external archive that stores non-dominated solutions found in P)
5. Repeat
   a. Compute the crowding distance values of each non-dominated solution in the archive A
   b. Sort the non-dominated solutions in A in descending crowding distance values
   c. For $i = 1$ to M
      i. Randomly select the global best guide for $P[i]$ from a specified top portion (e.g. top 10%) of the sorted archive A and store its position to gbest.
      ii. Compute the new velocity:

$$VEL[i] = W \times VEL[i] + R_1 \times (pbests[i] - P[i]) + R_1 \times (A[gbest] - P[i]) \tag{21}$$

(W is the inertia weight equal to 0.4) (R_1 and R_2 are random numbers in the range [0 ...1]) ($pbests[i]$ is the best position that the particle i have reached) ($A[gbest]$ is the global best guide for each non-dominated solution)

      iii. Calculate the new position of $P[i]$: $P[i] = P[i] + VEL[i]$
      iv. If $P[i]$ goes beyond the boundaries, then it is reintegrated by having the decision variable take the value of its corresponding lower or upper boundary and its velocity is multiplied by -1 so that it searches in the opposite direction.
      v. If $(t < (MAXT \times PMUT)$, then perform mutation on $P[i]$. ($MAXT$ is the maximum number of iterations) ($PMUT$ is the probability of mutation)
      vi. Evaluate $P[i]$
   d. End For
   e. Insert all new non-dominated solution in P into A if they are not dominated by any of the stored solutions. All dominated solutions in the archive by the new solution are removed from the archive. If the archive is full, the solution to be replaced is determined by the following steps:
      i. Compute the crowding distance values of each non-dominated solution in the archive A

ii. Sort the non-dominated solutions in A in descending crowding distance values

iii. Randomly select a particle from a specified bottom portion (e.g. lower 10%) which comprise the most crowded particles in the archive then replace it with the new solution

f. Update the personal best solution of each particle in P. If the current pbests dominates the position in memory, the particles position is updated using *pbests* [i] = $P[i]$

g. Increment iteration counter t

6. Until maximum number of iterations is reached

3.1.2 Crowding Distance Computation

The crowding distance value of a solution provides an estimate of the density of solutions surrounding that solution (Deb, 2000). Figure 1 shows the calculation of the crowding distance of point i which is an estimate of the size of the largest cuboid enclosing i without including any other point.

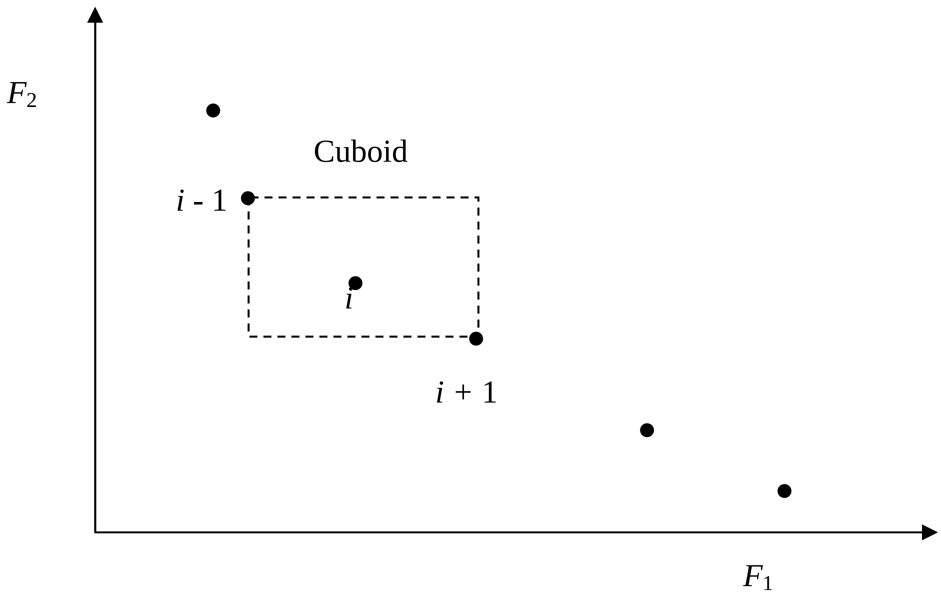

Figure 1 Crowding Distance Computation (Raquel and Naval, 2005)

Crowding distance is calculated by the first sorting the set of solutions in ascending objective function values. The crowding distance value of a particular solution is the average distance of its two neighboring solutions. The boundary solutions which have the lowest and highest objective function values are given an infinite crowding distance values so that they are always selected. This process is done for each objective function. The final crowding distance value of a solution is computed by adding the entire individual crowding distance values in each objective function. The pseudo code of crowding distance computation is shown below.

1. Get the number of non-dominated solutions in the external repository
   a. $n = | S |$
2. Initialize distance
   a. For i = 0 to MAX
   b. $S[i]$.distance = 0
3. Compute the crowding distance of each solution
   a. For each objective m
   b. Sort using each objective value S = sort(S, m)
   c. For i = 1 to (n - 1)
   d. $S[i]$.distance = $S[i]$.distance + ($S[i+1].m - S[i-1].m$)
   e. Set the maximum distance to the boundary points so that they are always selected $S[0]$.distance = $S[n]$.distance = maximum distance

3.1.3 Global Best Selection

The selection of the global best guide of the particle swarm is a crucial step in a multi-objective-PSO algorithm. It affects both the convergence capability of the algorithm as well as maintaining a good spread of non-dominated solutions. In MOPSO-CD, a bounded external archive stores non-dominated solutions found in previous iteration. It can be noted that any of the non-dominated solutions in the archive can be used as the global best guide of the particles in the swarm. But it is necessary to ensure that the particles in the population move towards the sparse regions of the search space. In MOPSO-CD, the global best guide of the particles is selected from among those non-dominated solutions with the highest crowding distance values. Selecting different guides for each particle in a specified top part of the sorted repository based on a decreasing crowding distance allows the particles in the primary population to move towards those non-dominated solutions in the external repository which are in the least crowded area in the objective space. Also, whenever the archive is full, crowding distance is again used in selecting which solution to replace in the archive. This promotes diversity among the stored solutions in the archive since those solutions which are in the most crowded areas are most likely to be replaced by a new solution.

3.1.4 Mutation

The mutation operator of MOPSO was adapted because of the exploratory capability it could give to the algorithm by initially performing mutation on the entire population then rapidly decreasing its coverage over time (Coello *et al.*, 2004). This is helpful in terms of preventing premature convergence due to existing local Pareto fronts in some optimization problems.

3.1.5 Constraint Handling

In order to handle constrained optimization problem, MOPSO-CD adapted the constraint handling mechanism used by GAs due to its simplicity in using feasibility and non-dominance of solutions when comparing solutions. A solution i is said to constrained-dominate a solution j if any of the following conditions is true:

1. Solution i is feasible and solution j is not.
2. Both solutions i and j are infeasible, but solution i has a smaller overall constraint violation.
3. Both solutions i and j are feasible and solution i dominates solutions j. When comparing two feasible particles, the particle which dominates the other particle is considered a better solution. On the other hand, if both particles are infeasible, the particle with a lesser number of constraint violations is a better solution.

3.2 MOPSO-CD for Solving Nash Game

During a Nash game, each player uses the MOPSO-CD to improve his own criterion along generations constrained by strategies of the other player. In applications, design variables are geometrically split between players who exchange symmetrically their best strategies (or best chromosomes) at each generation. Such a process is continued until no player can further improve its criterion. At this stage, the system has reached the Nash equilibrium. One of the evident properties of N/MOPSO-CD is their inherent parallel structure during evolution. A flow chart of the N/MOPSO-CD is shown in Figure 2. Based on Eqs. (1') and (2'), and both $\text{Max}_{N,L,\theta}\ \pi_m = \rho_m(\alpha - \beta_1 N^{-\delta} - \beta_2 L^{-\gamma}) - \theta L - N$; s. t. $L = [\gamma\rho_r \beta_2 / (1-\theta)]^{1/(\gamma+1)} \geq 0$, $\alpha > 0, \beta_1 > 0,\ \beta_2 > 0,\ \delta > 0,\ \gamma > 0,\ \rho_m > 0,\ 0 \leq \theta \leq 1, N \geq 0$ in Eq. (7) and $\text{Max}_{N,L,\theta}\pi_r = \rho_r(\alpha - \beta_1 N^{-\delta} - \beta_2 L^{-\gamma}) - (1-\theta)L$; s. t. $\alpha > 0,\ \beta_1 > 0,\ \beta_2 > 0,\ \gamma > 0,\ \delta > 0,\ \rho_r > 0,\ N \geq 0,\ L \geq 0$ in Eq. (8) are for 1 vs. 1 M-R Nash game. Both Eqs. (7) and (8) are the string representing the potential solution for a dual objective optimization, where N corresponds to the first criterion, and L and θ to the second one. However, the problem become to optimize $\text{Max}_{N,L,\theta}\ \pi_s(N, L, \theta) = \pi_m(N, L, \theta) + \pi_m(N, L, \theta)$; s. t. $L = [\gamma\rho_r\beta_2 / (1-\theta)]^{1/(\gamma+1)} \geq 0$, $\alpha > 0, \beta_1 > 0,$ $\rho_r > 0$, $0 \leq \theta \leq 1,$ $N \geq 0$, $L \geq 0$, where $\pi_m(N, L, \theta) = \rho_m(\alpha - \beta_1 N^{-\delta} - \beta_2 L^{-\gamma}) - \theta L - N$ and $\pi_r(N, L, \theta) = \rho_r(\alpha - \beta_1 N^{-\delta} - \beta_2 L^{-\gamma}) - (1-\theta)L$. In Figure 2, Player 1 optimizes $\pi_m(N_1, L_0^*, \theta_0^*)$ in generation 1 (Gen. 1), that is, searching all N_1, while L_0^* and θ_0^* are the optimal values by randomly choose N_0^*, L_0^*, θ_0^* for $\pi_s(N_0^*, L_0^*, \theta_0^*)$. Simultaneously, Player 2 optimizes $\pi_r(N_0^*, L_1, \theta_1)$ in generation 1 (Gen. 1), that is, searching both of all L_1 and θ_1, while N_0^* is the optimal values by randomly choose N_0^*, L_0^*, θ_0^* for $\pi_s(N_0^*, L_0^*, \theta_0^*) (= \pi_m(N_0^*, L_0^*, \theta_0^*) + \pi_r(N_0^*, L_0^*, \theta_0^*))$. Smarmily, Player 1 optimizes $\pi_m(N_2, L_1^*, \theta_1^*)$ in Gen. 2, that is, searching all N_2, while both L_1^* and θ_1^* are the optimal values based on the previous generation of $\pi_s(N_1^*, L_1^*, \theta_1^*) (= \pi_m(N_1^*, L_1^*, \theta_1^*) + \pi_r(N_1^*, L_1^*, \theta_1^*))$. At the same time, Player 2 optimizes $\pi_r(N_1^*, L_2, \theta_2)$ in Gen. 2, that is, searching both of all L_2 and θ_2, while N_1^* is the optimal values also based on the previous generation of $\pi_s(N_1^*, L_1^*, \theta_1^*)$. Following this process to Gen. *K*, the result become to $\pi_s(N_k^*, L_k^*, \theta_k^*)$ $= \pi_m(N_k^*, L_k^*, \theta_k^*) + \pi_r(N_k^*, L_k^*, \theta_k^*)$. Here, each player has his own MOPSO-CD algorithm with different population. Nash equilibrium is reached when neither player can further improve its criterion (Eyi *et al.*, 1996). Similarly, the solution procedures in the case of *U vs. V* of M-R with N/MOPSO-CD are setting N, α, β_1, β_2, δ, γ, L and θ be the multi-item, that is, N_i, $i = 1, 2, ..., U$ (or U'), L_{ij}, $i = 1, 2, ..., U$ (or U'), $j = 1, 2, .., V$, β_{1i}, $i = 1, 2, ..., U$, β_{2ij}, $i = 1, 2, ..., U$, $j = 1, 2, ..., V$, δ_{ij}, $i = 1, 2, ..., U$, $j = 1, 2, ..., V$, γ_{ij}, $i = 1, 2, ..., U$, $j = 1, 2, ..., V$ and θ_{ij}, $i = 1, 2, ..., U$, $j = 1, 2, ..., V$. To identify the equilibrium of Eqs. (15) and (16) for the case of 1 *vs.* 1 of M-R with N/MOPSO-CD is shown in Figure 2.

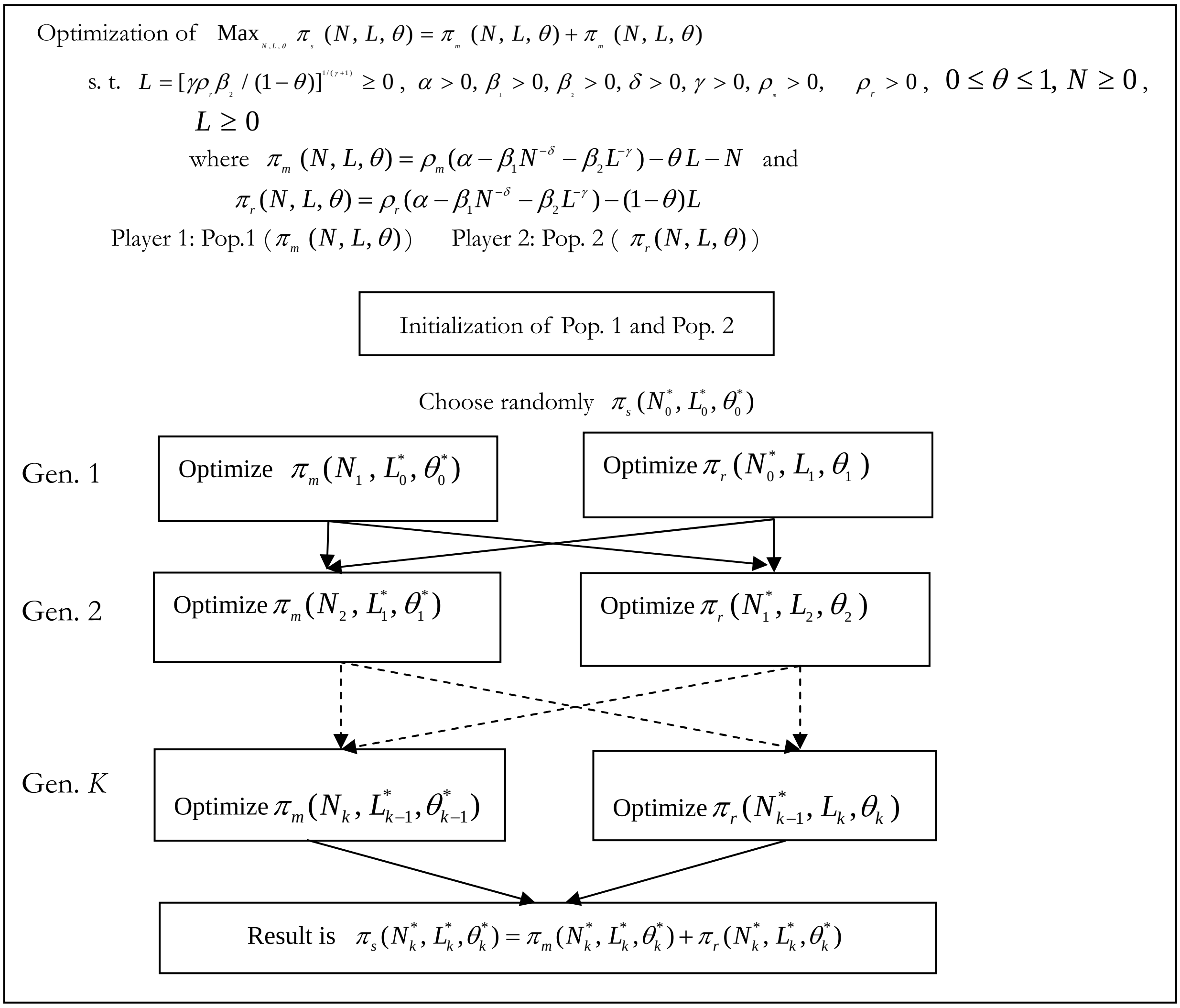

Figure 2 N/MOPSO-CD Flowchart for π_m, π_r Objective Functions (Wang and Periaux, 2001)

3.3 NSGA II for Solving Nash Game

There are three core mechanisms introduced in NSGA and NSGA-II are briefly summarized below. More details can be found in Deb (2001) and Deb *et al.*, (2002).

3.3.1 Non-dominated Sorting

Non-dominated sorting proceeds as follows. First of all, non-dominated individuals in the current population are identified. The non-dominated individuals are those who are not inferior to any other individuals in the population with respect to every objective. The same fitness value is assigned to all the non-dominated individuals. The individuals are then ignored temporarily, and the rest of the population is processed in the same way to identify a new set of non-dominated individuals. A fitness value that is smaller than the previous one is assigned to all the individuals belonging to the second non-dominated front. This process continues until the whole population is classified into non-dominated fronts with different fitness values.

3.2.2 Elitism

Unlike its predecessors, NSGA-II allows the parents to compete with offspring. In each generation, an offspring population of size *N* is generated from a parent population of the same size. The two populations are combined, and the first *N* best-fit individuals from the combined population are chosen to be part of the next generation population. The main purpose of this mechanism is to prevent fit individuals found in earlier generations from being lost easily.

3.3.3 Diversity-Preservation

The original NSGA uses the well-known fitness-sharing approach to preserve the diversity among the Pareto-optimal solutions. Although the fitness-sharing approach is found to maintain diversity in a population, the performance largely depends on its associated parameter. To avoid this sensitivity, Deb *et al.* (2002), have introduced a "crowding distance comparison" approach. First of all, the crowding distance surrounding a particular solution is measured. The crowding distance is given by the perimeter of the cuboids formed by using the nearest neighbors in the same non-dominated front as the vertices. Secondly, the crowding distance is used to break a tie when two solutions have the same fitness, i.e. they belong to the same non-dominated front. A solution with a higher crowding distance becomes a winner. By preferring the solution with a higher crowding distance, this mechanism encourages population diversity.

NSGA II (Deb, 2001; Deb *et al.*, 2002) also possess robustness for capturing the global solution of multi-modal optimization problems. Therefore, NSGA II can also be like MOPSO-CD used to this study.

3.4 Settings of MOPSO-CD or NSGA II for Solving Nash Game

In this research, all variables, N, L, and θ in Eqs. (7) and (8) are the decision variables that will be determined as solutions of problems and they are set as generated random numbers to check various combinations for the near-optimal solution. The objective functions and constraints in Eqs. (7) and (8) are framed clearly, the decision variables, in this research, are randomly generated to check the validity of performing the iterations of MOPSO-CD or NSGA II.

In MOPSO-CD or NSGA II computations, the objective functions of two-player M-R Nash game with two parameters vector $[\rho_m, \alpha, \beta_1, \beta_2, \delta, \gamma, N, L, \theta]$ for a manufacturer and $[\rho_r, \alpha, \beta_1, \beta_2, \delta, \gamma]$ for retailers, and the variables vector in vector, $[N, L, \theta]$can be represented as π_m ($[\rho_m, \alpha, \beta_1, \beta_2, \delta, \gamma, N, L, \theta]$) for manufacturer and π_r ($[\rho_r, \alpha, \beta_1, \beta_2, \delta, \gamma, N, L, \theta]$) for retailer. The terms of objective functions π_m ($[\rho_m, \alpha, \beta_1, \beta_2, \delta, \gamma, N, L, \theta]$) and π_r ($[\rho_r, \alpha, \beta_1, \beta_2, \delta, \gamma, N, L, \theta]$) should be adequately designed for the maximal optimization because it is the major source for fitness evaluation in the MOPSO-CD or NSGA II.

3.5 Objective Functions for N/MOPSO-CD and N/NSGA II

The objective functions, *f*, of two-player M-R Nash game with two parameters vector $[\rho_m, \alpha, \beta_1, \beta_2, \delta, \gamma]$ for manufacturers and $[\rho_r, \alpha, \beta_1, \beta_2, \delta, \gamma]$ for retailers, and the variables vector in vector, $[N, L, \theta]$ can be represented as $\pi_m([\rho_m, \alpha, \beta_1, \beta_2, \delta, \gamma, N, L, \theta])$ for manufacturers and $\pi_r([\rho_r, \alpha, \beta_1, \beta_2, \delta, \gamma, N, L, \theta])$ for retailers. The terms of objective functions $\pi_m([\rho_m, \alpha, \beta_1, \beta_2, \delta, \gamma, N, L, \theta])$ and $\pi_r([\rho_r, \alpha, \beta_1, \beta_2, \delta, \gamma, N, L, \theta])$ should be adequately designed for the maximal optimization because of it is the major source for fitness evaluation in the MOPSO-CD or NSGA II. In U vs. V M-R Nash game, for example, the variables of N, L and θ denote the solution vectors with summation of U variable (or numbers of manufacturers) and V variables (or numbers of retailers) for the example of case of *U vs. V* of M-R Nash game and they can be expressed as a set of $\{N_i, L_{ij}, \theta_{ij}; i = 1, 2, \ldots, U, j = 1, 2, \ldots, V\}$. In the operations of MOPSO-CD and N/NSGA II, for example, each variable N_i represents an individual's chromosome and variable vector $[N_i, L_{ij}, \theta_{ij}; i = 1, 2, \ldots, U, j = 1, 2, \ldots, V]$ represents a complete individual, and each individual is assigned a fitness value according to the evaluation from functions $\pi_m([\rho_m, \alpha, \beta_{1i}, \beta_{2ij}, \delta_i, \gamma_{ij}, N_i, L_{ij}, \theta_{ij}, i = 1, 2, \ldots, U, j = 1, 2, \ldots, V,])$ and $\pi_r([\rho_r, \alpha, \beta_{1i}, \beta_{2ij}, \delta_i, \gamma_{ij}, N_i, L_{ij}, \theta_{ij}, i = 1, 2, \ldots, U; j = 1, 2, \ldots, V])$, where N_i, L_{ij}, and θ_{ij}; $i = 1, 2, \ldots, U$ and $j = 1, 2, \ldots, V$ are the particles to solve N/MOPSO-CD or genes in N/NSGA II algorithms.

3.6 Algorithm Procedure of Nash Game with MOPSO-CD and NSGA II

The algorithm procedure of Nash Game with MOPSO-CD and NSGA II was proposed. The pseudo-code is shown in the following:

```
  Consider case of U vs. V
  For each manufacturer
      Random generate N_i, 1 ≤ i ≤ U ,
  For each retailer
     Random generate L_ij and θ_ij, 1 ≤ i ≤ U, 1 ≤ j ≤ V
     Initialize sets R[1] and R[2]
     For each P[0] in manufacturer and retailer //* P[0] is initial population *//
     If P[0] is Manufacturer
          Set P[0]'s N_i as variable and fix others N_i' and all L_ij and θ_ij
     Else if P[0] is Retailer
          Set P[0]'s L_ij, θ_ij and as variables and fix other N_i and all L_ij' and θ_ij'
     Optimize for largest π_s by MOPSO-CD/ NSGAII
     Save optimized N_i, L_ij and θ_ij into R[1]
While best_P[i]'s is not converged
     For each P[i] in manufacturer and retailer
          For each set r in P[i]
               If P[i] is Manufacturer
                    N_i as variable, all others N_i', L_ij and θ_ij in r are fixed
               Else if P[i] is Retailer
                    L_ij and θ_ij as variables, all other N_i, L_ij' and θ_ij' in r are fixed
        Optimize for largest π_s by MOPSO-CD/ NSGAII
        Save the largest_π_s and its N_i, L_ij and θ_ij
        Select the largest_π_s π_s as the decision made by π_m and π_r
        Save related N_i, L_ij and θ_ij into R[2]
        Select best π_s in R[2] as Best_π_s
        Clear R
```

Move all elements in $R[2]$ *to* $R[1]$
The lastest N_i , L_{ij} *and* θ_{ij} *are the solution.*

4. CASE STUDY FOR VALIDATION OF NON-COOPERATIVE ADVERTISING TWO-PLAYER AND MULTI-PLAYER GAME-SOLUTION IN SUPPLY CHAIN USING MOPSO-CD OR NSGA II

4.1 Optimum Solution Using N/MOPSO-CD or N/NSGA II for the Case of 1 vs. 1 M-R Nash Game

One model of digital camera from a camera manufacturer which is located in the northern part of Taiwan has been considered to conduct the non-cooperative advertising with its 1 retailer. The following data are collected for validating this study. The structure of case study is as follows.
1. Number of manufacturer is 1,
2. Number of retailer, V = 1,

The MOPSO-CD program used conditions are:
1. Population size: 100,
2. Maximum generation: 100,
3. Archiving size: 500,
4. Inertia weight w = 0.4,
5. Acceleration coefficients c_1 = 1.0,
6. Acceleration coefficients c_2 = 1.0
7. Probability of mutation (P_m) is 0.5, and
8. Terminate at $f(x) \le 10^{-6}$.

The SNGA II program used conditions are:
1. Population size: 100,
2. Maximum generation: 200,
3. Probability of crossover (P_c): 0.9,
4. Probability of mutation (P_m): 1.00 and
5. Terminate at $f(x) \le 10^{-6}$.

The input parameters for both case studies of two-player (or 1 vs. 1 M-R) Nash game is given in Table 1. Table 2 is the lower-bound and upper-bound of variables, N , L and θ for the Cases of 1 vs. 1 M-R Nash game using N/MOPSO-CD or N/SNGA II.

These parameters are also empirically collected by the case companies and these data obtained by using the regression model of statistics.

The optimal solution and results, for the case study of two-player (or 1 vs. 1 M-R) Nash game is obtained using analytic approach, N/MOPSO-CD or N/NSGA II are given in Tables 3 In Table 3, the optimum solutions using analytic approach, N/MOPSO-CD or N/NSGA II for the Case of 1 vs. 1 M-R Nash game are very close, that is $N_{\text{Analytic Approach}}$ = 347,425.865, and $N_{\text{N/MOPSO-CD}}$ = 347,121.293 and $N_{\text{N/NSGA II}}$ = 347,193.003. Similarly, $L_{\text{Analytic Approach}}$ = 26,452.325, $L_{\text{N/MOPSO-CD}}$ = 26,022.719, $L_{\text{N/NSGA II}}$ = 26,011.209, $\theta_{\text{Analytic Approach}} = 0$ %, $\theta_{\text{N/MOPSO-CD}} = 0$ % and $\theta_{\text{N/NSGA II}} = 0$ %, this can make sure that both N/MOPSO-CD or N/NSGA II C# programs can be used to the Case multi-player M-R Nash game. Consequently, optimum solutions using N/MOPSO-CD or N/NSGA II for the Case of 1 vs. 1 M-R Nash game are shown in Table 4 and their objective values, π_m , π_r and π_s are also very close.

Table 1 Input Parameters for the Case of 1 vs. 1 M-R Nash Game

ρ_m (US$)	ρ_r (US$)	α (Pieces)	β_1	β_2
150	300	1,254,000	30,000	2,000
δ	γ	-	-	-
0.122	0.18	-	-	-

Table 2 The Lower-bound and Upper-bound of Variables N, L and θ for the Case of 1 vs. 1 M-R Using N/MOPSO-CD or N/NSGA II

Range / Variable	Lower-bound	Upper-bound
N (US$)	2,80,000	400,000
L (US$)	10,000	30,000
θ (%)	0	100

Table 3 Optimum Solutions Using Analytic Approach, N/MOPSO-CD or N/NSGA II for the Case 1 vs. 1 M-R Nash Game

	Optimum Solution		
	Analytic Approach	N/MOPSO-CD	N/NSGA II
N (US$)	347,425.865	347,121.293	347,193.003
L (US$)	26,452.325	26,022.719	26,011.209
θ (%)	0.000	0.000	0.000

Table 4 Optimal Objective Values of the Case of+ 1 vs. 1

	Analytic Approach	N/MOPSO-CD	N/NSGA II
π_m	186,114,750.241	186,729,715.844	186,729,678.768
π_r	374,955,135.251	374,205,719.711	374,205,756.836
π_s	560,069,885.491	560,935,435.554	560,935,435.604

4.2 Optimum Solution Using N/MOPSO-CD or N/SNGA II for the Case U vs. V M-R Nash Game

Two models of digital camera from two camera manufacturers which are located in the northern and middle parts of Taiwan have been considered to conduct the cooperative advertising with 3 retailers. The following data are collected for validating this study. The structure of case study is as follows.

1. Number of manufacturers, U = 2,
2. Number of retailers, V = 3,

The MOPSO-CD program used the conditions are:

1. Population size: 100,
2. Maximum generation: 100,
3. Archiving size: 500,
4. Inertia weight w = 0.4,
5. Acceleration coefficients c_1 = 1.0
6. Acceleration coefficients c_2 = 1.0
7. Probability of mutation (P_m) is 0.5 and
8. Terminate at $f(x) \le 10^{-6}$.

The SNGA II program used conditions are:

1. Population size: 100,
2. Maximum generation: 200,
3. Probability of crossover (P_c): 0.9,
4. Probability of mutation (P_m): 1.00 and
5. Terminate at $f(x) \le 10^{-6}$.

The input parameters for this case study are given in Table 5. These parameters are empirically collected by the case companies and these data obtained by using the regression model of statistics. Table 6 is the lower-bound and upper-bound of variables $N_i, L_{ij},, \theta_{ij}$; $i = 1, 2, \ldots, U$, $j = 1, 2, \ldots, V$ for the Cases of 2 vs. 3 M-R Nash game using N/MOPSO-CD or N/NSGA II.

The optimal solution and results for the case study of multi-player (or 2 vs. 3) M-R Nash games are obtained using N/MOPSO-CD or N/NSGA II is shown in Table 7. In Table 8, the optimum objective values of $\pi_{m,\text{ N/MOPSO−CD}} \cong \pi_{m,\text{ N/NSGA II}}$, $\pi_{r,\text{ N/MOPSO−CD}} \cong \pi_{r,\text{ N/NSGA II}}$, $\pi_{s,\text{ N/MOPSO−CD}} \cong \pi_{s,\text{ N/NSGA II}}$,individually, are very closed. This can make sure that the N/MOPSO-CD or N/NSGA II C# programs can be used to the Case of 2 vs. 3 M-R Nash games.

Table 5 Input Parameters in the Case of 2 vs. 3 M-R Nash Game

ρ_m (US$)	ρ_r (US$)	α (Pieces)	β_{11}	β_{21}	β_{22}
150	300	1,254,000	30,000	2,000	1,850
β_{23}	δ_1	δ_2	γ_{11}	γ_{12}	γ_{13}
1,100	0.122	0.122	0.18	0.16	0.15
γ_{21}	γ_{22}	γ_{23}			
0.18	0.16	0.15			

Table 7 Optimum Solution Using N/MOPSO-CD or N/NSGA II for the Case of 2 vs. 3 M-R Nash Game

	Optimum Solution	
	N/MOPSO-CD	N/NSGA II
N_1 (US$)	347,359.796	346,769.894
N_2 (US$)	398,632.313	398,094.484
L_{11} (US$)	25,702.718	25,948.587
L_{12} (US$)	26,121.067	26,163.152
L_{13} (US$)	17,088.039	16,758.232
L_{21} (US$)	26,619.941	26,167.339
L_{22} (US$)	26,601.677	26,167.340
L_{23} (US$)	17,678.832	17,228.387
θ_{11} (%)	0.231	0.000
θ_{12} (%)	0.122	0.353
θ_{13} (%)	0.644	0.656
θ_{21} (%)	11.582	0.483
θ_{22} (%)	0.365	0.337
θ_{23} (%)	0.371	0.180

Table 8 Optimal Objective Values of the Case 2 vs. 3

	N/MOPSO-CD	N/NSGA II
π_m	18,4991,649.646	184,990,451.987
π_r	371,465,776.694	371,466,996.559
π_s	556,457,426.341	556,457,448.546

5. CONCLUSIONS

Supply chain management has emphasized the long-term strategic relationship between a manufacturing firm and its retailer. This relationship is that such a partnership makes both the manufacturer and the retailer better off than before and take both parties into a win-win situation through coordination. Most of works, such as: JIT purchasing, collaborative planning, VMI, third party logistics methods and so on to deal with the inter-organizational coordination, have been carried out on coordination mechanisms in the last decade. Among those inter-organization coordination mechanisms, non-cooperative advertising is a famous one in supply chain domain. Non-cooperative advertising is often defined as an arrangement whereby a manufacturer pays for some or all of the costs of local advertising undertaken by a retailer who is responsible for selling products made by the manufacturer. The main purpose for a manufacturer to utilize non-cooperative advertising is to strengthen the image of its own brand and to increase the short-term sales at the retail lever. However, non-cooperative advertising is not a specialized kind of advertising, instead; it is essentially a financial arrangement under which both parties agree how the costs of mutual promotion are to be defrayed. They focused on the most prominent aspect of these plans – the "participation rate", that is, the percentage of the retailers' local advertising expenditures that the manufacturer agrees to pay. The differential games can be used to investigate how a manufacturer can design an inter-temporal advertising support program that is optimally non-coordinated with his own advertising strategies and the retailer's advertising efforts. The reason to use game theory is rooted on the fact that many models in supply chains developed from single decision maker's perspective cannot adequately represent the sophisticated competitive and non-cooperative relationships in supply chains, and game theory, a tool of strategy analysis for conflict and non-cooperation, should be a more desirable approach. In addition, game theory has an essential tool in the analysis of supply chains consisting of multiple agents with conflicting objectives. However, the application of game theory to supply chain management is still in its infancy; much more progress will be made soon. Therefore, this kind of research is highly demanded.

In the different sales response function may yield interesting results in the analysis for systematic non-cooperative advertising agreements. A market response function based on a product could be determined by the manufacturer's national brand image investment and the retailer's local advertising expenditures. However, the effects of the local advertising expenditures and brand image investments on sales quantity are limited. And only when both efforts are exhausted, the market saturation level will be attained. In addition, the participation rate of local advertising expenditures shared by the manufacturer, which is a decision variable for manufacturer in addition to the national brand image investment. In this study, assuming the interactions between long-term branding investments and short-term promotion efforts can be neglected, and based on a simple and different market response functions, this study investigated the non-cooperative advertising problem under simultaneous Nash game. Since the analytical solution procedure for multi-player M-R Nash game for non-cooperative advertising could be difficult, thus, the MOPSO-CD or SNGA II and was used to the solution procedures. Therefore, the major work of this research is to construct the mathematic models in different market response functions associated with the gaming structures, and then employs the N/MOPSO-CD or N/NSGA II to identify the cooperative advertising two-player and multi-player M-R Nash game-solution in the Supply Chain. First of all, we employ two-player analytic approach and N/MOPSO-CD or N/NSGA II for identifying the M-R Nash game-solution. And then we construct U vs. V multi-player M-R models based on and the two-player market response function with the Nash game-solution procedure.

Three solution approaches are: (1) analytic approach for 1 vs. 1 M-R Nash game problem, (2) the N/MOPSO-CD and (3) N/NSGA II for the Case of U vs. V M-R Nash game problems. The input parameters of case study are empirically collected by the examples companies and those data obtained by using the regression models of statistics. The solution and objective values (see Tables 7 and 8) in this study using N/MOPSO-CD or N/NSGA II with C# programs obtained the expected results.

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