International Journal of Operations Research

International Journal of Operations Research Vol. 9, No. 1, 1−11 (2012)

EOQ Model for Deteriorating Items with Linear Time Dependent Demand Rate under Permissible Delay in Payments

R.P. Tripathi*

Department of Mathematics, Graphic Era University, Dehradun, (UK) INDIA

Received October 2011; *Revised February 2012*; *Accepted February 2012*

Abstract— This study presents an inventory model for deteriorating items with linearly time-dependent demand rate under trade credits. Mathematical models have been derived under four different situations i.e. Case 1 : The cycle time T is greater than or equal to M_1, to get a cash discount, Case 2 : The cycle time T is less than M_1, Case 3 : The cycle time is greater than or equal to M_2, and Case 4 : The cycle time is less than M_2. Computational procedures are proposed to obtain optimal cycle time of all four cases. Numerical example and sensitivity analysis shows the applicability of the proposed model.

Keywords— Inventory, deteriorating item, cash-discount, linear time-dependent-demand rate.

1. INTRODUCTION

Large number of research papers / articles has been presented by many authors for controlling the inventory of deteriorating items. Deteriorating items such as fashion goods, blood banks, medicines, volatiles, green vegetable, radioactive material, photographic films, etc. In many inventory systems the product generated have indefinitely long lives. Generally, almost all items deteriorate over time. Often the rate of deterioration is low and there is little need to consider the deterioration for determining the economic lot size. Hence the effect of deterioration cannot be ignored in the decision process of production lot size.

In past few years, great interest has been shown in developing mathematical models in the presence of trade credit. In many cases customers are conditioned to a shipping delay and may be willing to wait for a short time in order to get their first choice. For fashionable items, the length of the waiting time for the next cycle time would determine whether the backlogging will be accepted or not. Thus the backlogging rate should be variable and dependent on the length of the waiting time for the next cycle time. The main objective of inventory management deals with minimization of the inventory carrying cost for which it is required to determine the optimal stock and optimal time of replenishment of inventory to meet the future demand.

In a realistic product life cycle, demand is increasing with time during the growth phase. In classical inventory models the demand rate is assumed to be a constant. In reality demand for physical goods may be time-dependent, stock dependent and price dependent. An inventory system of ameliorating items for price dependent demand rate was considered by Mandal *et al.* (2003). You (2005) developed an inventory model with price and time dependent demand. Hou and Lin (2008) considered an ordering policy with a cost minimization procedure for deteriorating items under trade credit and time discounting. Huang (2007) derived an economic order quantity under conditionally permissible delay in payments. Goyal (1985) derived an EOQ model under the condition of permissible delay in payments. Chang (2004) proposed an inventory model under a situation that the supplier provides the purchaser a permissible delay in payments if the quantity of the purchaser's order is large. Chung and Liao (2004) developed, under the condition of permissible delay in payments by the quantity ordered, a model determining the economic order quantity for exponentially deteriorating items. Chung *et al.* (2005) developed the problem of determining the economic order quantity under the condition of permissible delay in payments by the quantity ordered. An EOQ model for deteriorating items under trade credits is developed by Ouyang *et al.* (2005). Ghare and Schrader (1963) developed a model for an exponentially decaying inventory. Ghare and Schrader's model was extended by Covert and Philip (1973) by considering constant deterioration rate to a two-parameter Weibull distribution. Hariga (1996) generalized the demand pattern to any concave function. Teng *et al.* (1999), Yang *et al.* (2001) and Teng and Yang (2004) further generalized the demand function to include any non-negative, continuous function that fluctuates with time. While determining the optimal ordering policy, the effect of inflation and time value of money cannot be ignored. The research in this direction was done by Buzacott (1975), who developed an EOQ model with inflation subject to different types of pricing policies. Other related articles can be found by Misra (1977) and Roy and Chaudhuri (1997), Liao *et al.* (2000) and Chung and Lin (2001). Hou and Lin (2009) studied a cash flow oriented EOQ model with deteriorating items under permissible delay in payments, and minimum total costs is obtained.

* Corresponding author's email: tripathi_rp0231@rediffmail.com

Recently, Aggrawal *et al.* (2009) developed a model on integrated inventory system with the effect of inflation and credit period. In this paper the demand rate is assumed to be a function of inflation. Tripathi and Misra (2010) developed EOQ model on credit financing in economic ordering policies of non-deteriorating items with time-dependent demand rate in the presence of trade credit using discounted cash flow (DCF) approach. Jaggi *et al.* (2008) developed a model retailer's optimal replenishment decisions with trade credit linked demand under permissible delay in payments. Jaggi *et al.* (2007) developed a model on retailer's optimal ordering policy under two stage trade credit financing. This paper develops an inventory model under two levels of trade credit policy by assuming the demand is a function of credit period offered by the retailer to the customers using discounted cash flow (DCF) approach. Hwang and Shinn (1997) developed retailer's pricing and lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payments. In paper Hwang and Shinn analyzed how a retailer can determine the optimal retail price and lot size simultaneously when the supplier permits delay in payments for an order of a product whose demand rate is represented by a constant price elasticity function. Inventory model with time-dependent demand rate under inflation when supplier credit linked to order quantity is developed by Tripathi (2011). In this paper Tripathi established an inventory model for non- deteriorating items and time- dependent demand rate under inflation when supplier offers a permissible delay to the purchaser, if the order quantity is greater than or equal to a predetermined quantity.

The aim of the paper is to develop an EOQ model for deteriorating items with linear time-dependent demand rate under permissible delay in payment. In this study shortages are not allowed. Mathematical models are derived under four different circumstances i.e. Case 1. M_1 is less than or equal to cycle time; Case 2: Cycle time is less than M_1; Case 3 : Cycle time is greater than or equal to M_2 and Case 4 : Cycle time is less than M_2. The expressions for an inventory systems total relevant costs and derived for these four cases. Finally, we provide numerical example and sensitivity analysis for illustration of the proposed model.

The rest of the paper is organized as follows: In section 2 notation and assumptions are given. In section 3 we develop mathematical formulation for the solution of total relevant cost. Taylor's series expansion is used to find closed form solution of the optimal values of cycle time, Order quantity and total relevant costs with regard to four different cases followed by numerical examples in section 4. We provide sensitivity analysis in section 5 followed by conclusion and future research in the last section 6.

2. ASSUMPTIONS AND NOTATIONS

The following assumptions are being made throughout the paper:

(1) The demand for the item is linearly time-dependent.
(2) Replenishment is instantaneous.
(3) Shortages are not allowed.
(4) Time horizon is infinite.
(5) If the account is not settled during the time, generated sales revenue is deposited in an interest bearing account. At the end of credit period, the account is settled as well as the buyer pays off all units sold and starts paying for the interest charges on the item in stock. In this case, supplier provides a cash discount if the full payment is paid within M_1 time, otherwise, the full payment is paid within M_2 time. The account is settled when the payment is paid ($M_2 > M_1$).

In addition, the following notations are used throughout the manuscript:

h : the unit holding cost per year excluding interest charges
p : the selling price per unit
c : the unit purchasing cost, with $c < p$
I_c : the interest charged per dollar in stocks per year by the supplier
I_d : the interest earned per dollar per year
s : the ordering cost per order
r : the cash discount rate
θ : the constant deterioration rate, where $0 \le \theta \le 1$
M_1 : the period of cash discount
M_2 : the period of permissible delay in settling account with $M_2 > M_1$
T : the replenishment time interval
D : the demand rate per year i.e. $D = D(t) = a + bt$, $a > 0$, $0 < b < 1$
$I(t)$: the level of inventory at time t, $0 \le \theta \le 1$
Q : the order quantity
$Z_1(T)$: the total relevant cost per year for case 1
$Z_2(T)$: the total relevant cost per year for case 2
$Z_3(T)$: the total relevant cost per year for case 3
$Z_4(T)$: the total relevant cost per year for case 4
$T_1^*, T_2^*, T_3^*, T_4^*$: the optimal cycle times for case 1, case 2, case 3 and case 4 respectively

$Q_1^*(T_1^*), Q_2^*(T_2^*), Q_3^*(T_3^*), Q_4^*(T_4^*)$: the optimal order quantities for case 1, 2, 3 and 4 respectively.

$Z_1^*(T_1^*), Z_2^*(T_2^*), Z_3^*(T_3^*), Z_4^*(T_4^*)$: the optimal total relevant costs per year for case 1, 2, 3 and 4 respectively.

The total relevant cost consists of (1) cost of placing order, (2) cost of deteriorated units, (3) cost of carrying inventory excluding interest charges, (4) cash-discount earned if the payment is made at M_1 (5) cost of interest charges for unsold items after the permissible delay M_1 or M_2 and (6) interest earned for sales revenue during the permissible delay period $[0, M_1]$ or $[0, M_2]$.

3. MATHEMATICAL FORMULATION

The inventory level $I(t)$ at any time 't' generally decreases mainly to meet demand and partially due to deterioration. The variation of inventory with respect to time 't' can be described by the following differential equation:

$$\frac{dI(t)}{dt} + \theta\, I(t) = -D\ (= a + bt),\ 0 \le t \le T \tag{1}$$

With the boundary condition $I(T) = 0$. The solution of equation (1) is given by

$$I(t) = \left(\frac{a}{\theta} - \frac{a}{\theta^2}\right)\left(e^{\theta(T-t)} - 1\right) + \frac{b}{\theta}\left(Te^{\theta(T-t)} - t\right), \quad 0 \le t \le T \tag{2}$$

The order quantity Q is given by

$$Q = \left(\frac{a}{\theta} - \frac{a}{\theta^2}\right)\left(e^{\theta T} - 1\right) + \frac{bT}{\theta}e^{\theta T} \tag{3}$$

The total demand during once cycle is $(aT + \frac{bT^2}{2})$. Thus the number of deteriorating items during a replenishment cycle is

$$\left(\frac{a}{\theta} - \frac{b}{\theta^2}\right)\left(e^{\theta T} - 1\right) + \frac{bTe^{\theta T}}{\theta} - \left(aT + \frac{bT^2}{2}\right) \tag{4}$$

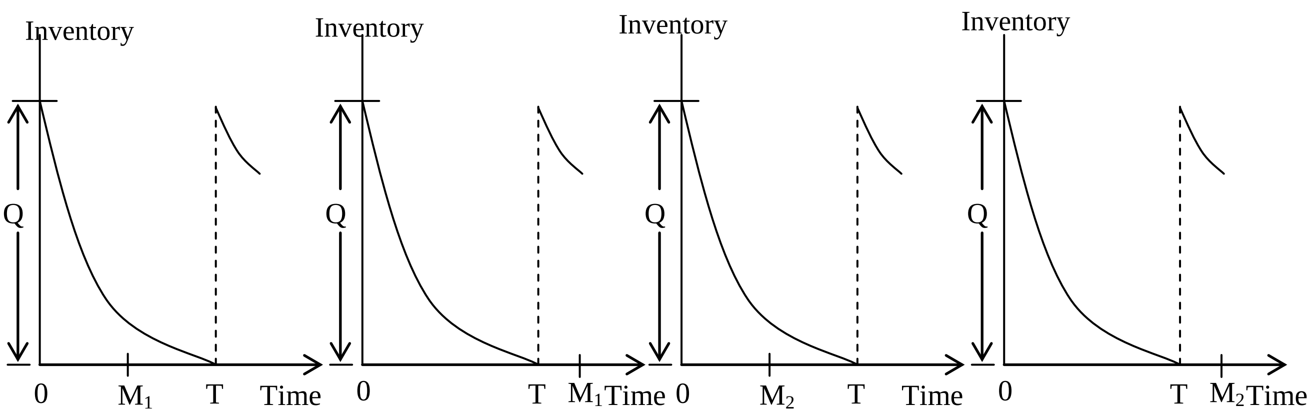
The total relevant cost per year consists of the following elements:

(1) Cost of placing orders $= \dfrac{s}{T}$ (5)

(2) Cost of purchasing units $= \dfrac{cQ}{T} = \dfrac{c}{T}\left\{\left(\dfrac{a}{\theta} - \dfrac{b}{\theta^2}\right)\left(e^{\theta T} - 1\right) + \dfrac{bTe^{\theta T}}{\theta}\right\}$ (6)

(3) Cost of carrying inventory $= \dfrac{h}{T}\displaystyle\int_0^T I(t)dt = \dfrac{h}{T}\left\{\left(\dfrac{a}{\theta} - \dfrac{b}{\theta^2}\right)\left(\dfrac{e^{\theta T} - \theta T - 1}{\theta}\right) + \dfrac{bT}{\theta}\left(\dfrac{e^{\theta T} - 1}{\theta} - \dfrac{T}{2}\right)\right\}$ (7)

Regarding cash discount, interest charges and earned, the four possible cases based on the customer's two choices (i.e. pays at M_1 or M_2) and the length of T. In case 1, the payment is paid at M_1 to get a cash discount and $T \ge M_1$. For case 2, the customer pays in full at M_1 but $T < M_1$. In the same manner, if the payments are paid at time M_2 to get the permissible and $T \ge M_2$, then it is case 3. As to case 4, the customer pays in full at M_2 but $T < M_2$. All four cases shown in figure 1.



Case 1: $T \ge M_1$ Case 2: $T < M_1$ Case 3: $T \ge M_2$ Case 4: $T < M_2$

Figure 1. Graphical representation of four different situations

Case 1: $T \geq M_1$

The discount saving per year by customer is

$$\frac{rcQ}{T} = \frac{rc}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)\left(e^{\theta T}-1\right)+\frac{bTe^{\theta T}}{\theta}\right\} \tag{8}$$

The customer pays off all units ordered at time M_1 to get the cash discount. Thus the items in the stocks have to be financed at interest rate I_c after time M_1, the interest payable per year in this case is

$$\frac{c(1-r)I_c}{T}\int_{M_1}^{T} I(t)dt = \frac{c(1-r)I_c}{T}\left[\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)\left\{\left(\frac{e^{\theta(T-M_1)}-1}{\theta}\right)-\left(T-M_1\right)\right\}+\frac{b}{\theta}\left\{T\left(\frac{e^{\theta(T-M_1)}-1}{\theta}\right)-\frac{\left(T^2-M_1^2\right)}{2}\right\}\right] \tag{9}$$

During $[0, M_1]$, the customer sells products and deposits the revenue into an account that earns I_d per dollar per year. Thus interest earned per year is

$$\frac{pI_d}{T}\int_0^{M_1}(a+bt)tdt = \frac{pI_dM_1^2}{T}\left(\frac{a}{2}+\frac{bM_1}{3}\right) \tag{10}$$

The total relevant cost per year $Z_1(T)$ is given by

$Z_1(T)$ = cost of placing order + cost of purchasing + cost of carrying inventory – discount saving per year + interest payable per year – interest earned per year.

$$= \frac{s}{T}+\frac{c(1-r)}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)\left(e^{\theta T}-1\right)+\frac{bTe^{\theta T}}{\theta}\right\}+\frac{h}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)\left(\frac{e^{\theta T}-\theta T-1}{\theta}\right)+\frac{bT}{\theta}\left(\frac{e^{\theta T}-1}{\theta}-\frac{T}{2}\right)\right\}$$

$$+\frac{c(1-r)I_c}{T}\left[\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)\left\{\left(\frac{e^{\theta(T-M_1)}-1}{\theta}\right)-\left(T-M_1\right)\right\}+\frac{b}{\theta}\left\{T\left(\frac{e^{\theta(T-M_1)}-1}{\theta}\right)-\frac{\left(T^2-M_1^2\right)}{2}\right\}\right]-\frac{pI_dM_1^2}{T}\left(\frac{a}{2}+\frac{bM_1}{3}\right) \tag{11}$$

Case 1: $T < M_1$

In this case the customer sells ($aT + \frac{bT^2}{2}$) units in total at time T, and has c (1–r) ($aT + \frac{bT^2}{2}$) to pay the supplier in full at the time M_1. Thus there is no interest payable while the cash discount is the same as that in case 1. The interest earned per year is

$$pI_d\left\{a\left(M_1-\frac{T}{2}\right)+\frac{bT}{2}\left(M_1-\frac{T}{3}\right)\right\} \tag{12}$$

The total relevant cost per year $Z_2(T)$ is

$$Z_2(T) = \frac{s}{T}+\frac{c(1-r)}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)\left(e^{\theta T}-1\right)+\frac{bTe^{\theta T}}{\theta}\right\}+\frac{h}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)\left(\frac{e^{\theta T}-\theta T-1}{\theta}\right)+\frac{bT}{\theta}\left(\frac{e^{\theta T}-1}{\theta}-\frac{T}{2}\right)\right\}$$

$$-pI_d\left\{a\left(M_1-\frac{T}{2}\right)+\frac{bT}{2}\left(M_1-\frac{T}{3}\right)\right\} \tag{13}$$

Case 3: $T \geq M_2$

In this case, the payment is paid at time M_2, there is no cash discount. The interest payable per year is

$$\frac{cI_c}{T}\left[\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)\left\{\left(\frac{e^{\theta(T-M_2)}-1}{\theta}\right)-\left(T-M_2\right)\right\}+\frac{b}{\theta}\left\{T\left(\frac{e^{\theta(T-M_2)}-1}{\theta}\right)-\frac{\left(T^2-M_2^2\right)}{2}\right\}\right] \tag{14}$$

The interest earned per year is

$$\frac{pI_dM_2^2(3a+2bM_2)}{6T} \tag{15}$$

The total relevant cost per year $Z_3(T)$ is

$$Z_3(T) = \frac{s}{T}+\frac{c}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)\left(e^{\theta T}-1\right)+\frac{bTe^{\theta T}}{\theta}\right\}+\frac{h}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)\left(\frac{e^{\theta T}-\theta T-1}{\theta}\right)+\frac{bT}{\theta}\left(\frac{e^{\theta T}-1}{\theta}-\frac{T}{2}\right)\right\}$$

$$-\frac{cI_c}{T}\left[\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)\left\{\left(\frac{e^{\theta(T-M_2)}-1}{\theta}\right)-\left(T-M_2\right)\right\}+\frac{b}{\theta}\left\{T\left(\frac{e^{\theta(T-M_2)}-1}{\theta}\right)-\frac{\left(T^2-M_2^2\right)}{2}\right\}\right]-\frac{pI_dM_2^2(3a+2bM_2)}{6T} \tag{16}$$

Case 4: $T < M_2$

In this case, there is no interest charged. The interest earned per year is

$$pI_d\left\{a\left(M_2-\frac{T}{2}\right)+\frac{bT}{2}\left(M_2-\frac{T}{3}\right)\right\} \tag{17}$$

Therefore, the total relevant cost per year $Z_4(T)$ is

$$Z_4(T)=\frac{s}{T}+\frac{c}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)\left(e^{\theta T}-1\right)+\frac{bTe^{\theta T}}{\theta}\right\}+\frac{h}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)\left(\frac{e^{\theta T}-\theta T-1}{\theta}\right)+\frac{bT}{\theta}\left(\frac{e^{\theta T}-1}{\theta}-\frac{T}{2}\right)\right\}$$
$$-pI_d\left\{a\left(M_2-\frac{T}{2}\right)+\frac{bT}{2}\left(M_2-\frac{T}{3}\right)\right\} \tag{18}$$

It is difficult to obtain the optimal solution in explicit form for equations (11), (13), (16) and (18). Therefore, the model will be solved approximately by using a truncated Taylor's series for the exponential terms i.e.

$$e^{\theta T}\approx 1+\theta T+\frac{\theta^2T^2}{2},\qquad e^{\theta M_1}\approx 1+\theta M_1+\frac{\theta^2M_1^2}{2}\ \text{ etc.} \tag{19}$$

which is valid approximation for smaller values of θT, θM, etc. With the above approximation, the total relevant cost per year in all four cases are given by

$$\mathrm{Z}_1(\mathrm{T})\approx\frac{s}{T}+c(1-r)\left\{a\left(1+\frac{\theta T}{2}\right)+\frac{bT}{2}(1+\theta T)\right\}+\frac{(a+bT)}{2}\left\{hT+\frac{c(1-r)I_c(T-M_1)^2}{T}\right\}$$
$$-\ \frac{pI_dM_1^2}{6T}\left(3a+2bM_1\right) \tag{20}$$

$$Z_2(T)\approx\frac{s}{T}+c(1-r)\left\{a\left(1+\frac{\theta T}{2}\right)+\frac{bT}{2}(1+\theta T)\right\}+\frac{hT}{2}(a+bT)-pI_d\left\{a\left(M_1-\frac{T}{2}\right)+\frac{bT}{2}\left(M_1-\frac{T}{3}\right)\right\} \tag{21}$$

$$Z_3(T)\approx\frac{s}{T}+c\left\{a\left(1+\frac{\theta T}{2}\right)+\frac{bT}{2}(1+\theta T)\right\}+\frac{(a+bT)}{2}\left\{hT+\frac{cI_c(T-M_2)^2}{T}\right\}-\frac{pI_dM_2^2\left(3a+2bM_2\right)}{6T} \tag{22}$$

$$Z_4(T)\approx\frac{s}{T}+c\left\{a\left(1+\frac{\theta T}{2}\right)+\frac{bT}{2}(1+\theta T)\right\}+\frac{hT}{2}(a+bT)-pI_d\left\{a\left(M_2-\frac{T}{2}\right)+\frac{bT}{2}\left(M_2-\frac{T}{3}\right)\right\} \tag{23}$$

Note that the purpose of this approximation is to obtain the unique closed form solution for the optimal solution. By taking first and second order derivatives of $Z_i(T)$, i = 1, 2, 3, 4, with respect to 'T', we obtain

$$\frac{dZ_1(T)}{dT}=-\frac{s}{T^2}+\frac{c(1-r)}{2}\left\{a\theta+b(1+2\theta T)+\frac{bI_c\left(T-M_1\right)^2}{T}+(a+bT)I_c\left(1-\frac{M_1^2}{T^2}\right)\right\}+\frac{h}{2}(a+2bT)$$
$$+\ \frac{pI_dM_1^2}{6T^2}\left(3a+2bM_1\right) \tag{24}$$

$$\frac{dZ_2(T)}{dT}=-\frac{s}{T^2}+\frac{c(1-r)}{2}\left\{a\theta+b(1+2\theta T)+\right\}+\frac{h}{2}(a+2bT)-\frac{pI_d}{2}\left\{-a+b\left(M_1-\frac{2T}{3}\right)\right\} \tag{25}$$

$$\frac{dZ_3(T)}{dT}=-\frac{s}{T^2}+\frac{c}{2}\left\{a\theta+b(1+2\theta T)\right\}+\frac{h}{2}(a+2bT)+\frac{cI_c\left(T-M_2\right)}{2T}\left\{a\left(1+\frac{M_2}{T}\right)+2bT\right\} \tag{26}$$

$$\frac{dZ_4(T)}{dT}=-\frac{s}{T^2}+\frac{c}{2}\left\{a\theta+b(1+2\theta T)+\right\}+\frac{h}{2}(a+2bT)-\frac{pI_d}{2}\left\{-a+b\left(M_2-\frac{2T}{3}\right)\right\} \tag{27}$$

$$\frac{d^2Z_1(T)}{dT^2}=\frac{2s}{T^3}+\left\{2s+c(1-r)aI_cM_1^2-\frac{pI_dM_1^2(3a+2bM_1)}{3}\right\}+b\left\{\left(h+c(1-r)(\theta+I_c\right)\right\}>0 \tag{28}$$

$$\frac{d^2Z_2(T)}{dT^2}=\frac{2s}{T^3}+b\left\{h+c\theta(1-r)\right\}+\frac{bpI_d}{3}>0 \tag{29}$$

$$\frac{d^2Z_3(T)}{dT^2}=\frac{2s}{T^3}+\left\{2s+cI_caM_2^2-\frac{pI_dM_2^2(3a+2bM_2)}{3}\right\}+b\left(h+c\theta+cI_c\right)>0 \tag{30}$$

$$\frac{d^2Z_4(T)}{dT^2}=\frac{2s}{T^3}+b(h+c\theta)+\frac{bpI_d}{3}\quad>0 \tag{31}$$

Since $\frac{d^2 Z_i(T)}{dT^2} > 0$, i = 1, 2, 3, 4, the optimal (minimum) values of $T = T_i$, i = 1, 2, 3, 4 are obtained on solving $\frac{dZ_i(T)}{dT} = 0$, i = 1, 2, 3, 4 from equations (24), (25), (26) and (27) respectively, we obtain

$$6b\{h + c(\theta + I_c)(1-r)\}T^3 + 3\{ha + c(1-r)(b + a\theta - 2bM_1 I_c + aI_c)\}T^2 - \{6s + 3acI_c(1-r)M_1^2 - pI_d M_1^2(3a + 2bM_1)\} = 0 \tag{32}$$

$$b\{6h + 6c(1-r)\theta + 2pI_d\}T^3 + 3\{ha + c(1-r)(b + a\theta) + pI_d(a - bM_1)\}T^2 - 6s = 0 \tag{33}$$

$$6b(h + c\theta + cI_c)T^3 + 3\{ha + c(b + a\theta) + cI_c(a - 2bM_2)\}T^2 - \{6s + 3acI_c M_2^2 - pI_d M_2^2(3a + 2bM_2)\} = 0 \tag{34}$$

$$b\{6h + 6c\theta + 2pI_d\}T^3 + 3\{ha + c(b + a\theta) + pI_d(a - bM_2)\}T^2 - 6s = 0 \tag{35}$$

Special case 1(a)

If $6s + 3acI_c(1-r)M_1^2 = pI_d M_1^2(3a + 2bM_1)$. From equation (32), we obtain

$$T = T_1^{'*} = \frac{1}{2}\left[\frac{c(1-r)(2bM_1 I_c - aI_c - b - a\theta) - ah}{b\{h + c(\theta + I_c)(1-r)\}}\right] \tag{36}$$

Therefore $Z_1(T) = Z_1^{'*}(T_1^{'*})$ is

$$Z_1^{'*}(T_1^{'*}) = c(1-r)\left\{a(1 - I_c M_1) + \frac{bI_c M_1^2}{2}\right\} - \frac{1}{8b}\frac{\{c(1-r)(2bM_1 I_c - ac - b - a\theta) - ah\}^2}{\{h + c(\theta + I_c)(1-r)\}} \tag{37}$$

Special case 1(b)

If $ha = c\,(1 - r)\,(2bM_1 I_c - ac - b - a\theta)$. From Eq. (32), we obtain

$$T = T_1^{''*} = \left[\frac{\{6s + 3acI_c(1-r)M_1^2 - pI_d M_1^2(3a + 2bM_1)\}}{6b\{h + c(\theta + I_c)(1-r)\}}\right] \tag{38}$$

Therefore $Z_1(T) = Z_1^{''*}(T_1^{''*})$ is

$$Z_1^{''*}(T_1^{''*}) = \frac{3b^{1/3}}{2(6)^{2/3}}\left\{h + c(1-r)(\theta + I_c)\right\}^{1/3}\left\{6s + 3ac(1-r)I_c M_1^2 - pI_d M_1^2(3a + 2bM_1)\right\}^{2/3} + c(1-r)\left\{a(1 - I_c M_1) + \frac{bI_c M_1^2}{2}\right\} \tag{39}$$

Special case 2.

If $pI_d(bM_1 - a) = ha + c(1-r)(b + a\theta)$. From equation (33), we obtain

$$T = T_2^{'*} = \left[\frac{3s}{b\{3h + 3c(1-r)\theta + pI_d\}}\right]^{1/3} \tag{40}$$

Therefore $Z_2(T) = Z_2^{'*}(T_2^{'*})$ is

$$Z_2^{'*}(T_2^{'*}) = \frac{(3s)^{2/3}}{2}\left\{3h + 3c(1-r)\theta + pI_d\right\}^{1/3} + a\left\{c(1-r) - pI_d M_1\right\} \tag{41}$$

Special case 3(a)

If $pI_d M_2^2(3a + 2bM_2) = 6s + 3acI_c M_2^2$. From equation (34), we obtain

$$T = T_3^{'*} = \frac{1}{2}\left\{\frac{2bcI_c M_2 - ah - ac\theta - bc - acI_c}{b(h + c\theta + cI_c)}\right\} \tag{42}$$

Therefore $Z_3(T) = Z_3^{'*}(T_3^{'*})$ is

$$Z_3^{'*}(T_3^{'*}) = c\left\{a(1 - I_c M_2) + \frac{bcI_c M_2^2}{2}\right\} - \frac{1}{8b}\frac{\left(2bcI_c M_2 - ah - ac\theta - bc - acI_c\right)^2}{(h + c\theta + cI_c)} \tag{43}$$

Special case 3(b)

If $2bc\,M_2\,I_c = ha + c\,(b + a\theta) + ac\,I_c$. From equation (34), we obtain

$$T = T_3^{''*} = \left\{\frac{6s + 3acI_cM_2^2 - pI_dM_2^2(3a + 2bM_2)}{6b(h + c\theta + cI_c)}\right\}^{1/3} \tag{44}$$

Therefore $Z_3(T) = Z_3^{''*}(T_3^{''*})$ is

$$Z_3^{''*}(T_3^{''*}) = \frac{3b^{1/3}}{2(6)^{2/3}}\left\{h + c\theta + cI_c\right\}^{1/3}\left\{6s + 3acI_cM_2^2 - pI_dM_2^2(3a + 2bM_2)\right\}^{2/3} + c\left\{a(1 - I_cM_2 + \frac{bcI_cM_2^2}{2}\right\} \tag{45}$$

Special case 4

If $pI_d(bM_2 - a) = ha + bc + ac\theta$. From equation (35), we obtain

$$T = T_4^{'*} = \left\{\frac{3s}{b(3h + 3c\theta + pI_d)}\right\}^{1/3} \tag{46}$$

Therefore $Z_4(T) = Z_4^{'*}(T_4^{'*})$ is

$$Z_4^{'*}(T_4^{'*}) = \frac{(3s)^{2/3}}{2}\left(3h + 3c\theta + pI_d\right)^{1/3} + a(c - pI_dM_2) \tag{47}$$

To ensure $T_1^{'*} > M_1$, we substitute (36) into inequality $T_1^{'*} > M_1$, we obtain

$$h(a + 2bM_1) + c(1 - r)(2M_1b + aI_c + b + a\theta) < 0 \tag{48}$$

Since $0 < r < 1$ inequality (48) does not exist. Therefore special case 1(a) does not exist for $6s + 3acI_c(1 - r)M_1^2 = pI_dM_1^2(3a + 2bM_1)$

Again, to ensure $T_1^{''*} > M_1$, we substitute (38) into inequality $T_1^{''*} > M_1$, we obtain

$$6s + 3acI_c(1 - r)M_1^2 > M_1^2\left\{pI_d(3a + 2bM_1) + 6bM_1h + 6bcM_1(\theta + I_c)(1 - r)\right\} \tag{49}$$

In equality (49) is valid, if $ah = c(1 - r)(2bM_1I_c - ac - b - a\theta)$

To ensure $T_2^{'*} > M_1$, we substitute (40) into inequality $T_1^{'*} < M_1$, we obtain

$$3s < bM_1^3\left\{3h + 3c(1 - r)\theta + pI_d\right\} \tag{50}$$

Special case 3(a) does not exists for $6s + 3acI_cM_2^2 = pI_dM_2^2(3a + 2bM_2)$ (as special case 1(a))

To ensure $T_3^{''*} > M_2$, we substitute (44) into inequality $T_3^{''*} > M_2$, we obtain

$$6s + 3acI_cM_2^2 > \left\{pI_d(3a + 2bM_2) + 6bM_2h + 6bcM_2(\theta + I_c)\right\}M_2^2 \tag{51}$$

The inequality (51) is valid if $2bcM_2I_c = ha + c(b + a\theta) + acI_c$

To ensure $T_4^{'*} < M_2$, we substitute (46) into inequality $T_4^{'*} < M_2$, we obtain

$$3s < bM_2^3\left\{3h + 3c\theta + pI_d\right\} \tag{52}$$

The inequality (52) is valid if $pI_d(bM_2 - a) = ha + bc + ac\theta$.

4. NUMERICAL EXAMPLES

Given $a = 500$ units/year, $b = 0.5$ unit, $h = \$\,5$/unit/year, $I_c = \$0.09$/year, $I_d = 0.06$/year, $c = \$\,25$ per unit, $p = \$\,40$ per unit, $r = 0.02$, $\theta = 0.03$, $M_1 = 15$ days $= 0.04109589$ years and $M_2 = 30$ days $= 0.08219178$ years.

Case 1 : $T > M_1$

For $s = 5$, $T = T_1^* = 0.049695$ years, $Q = Q_1^* = 24.866649$, $Z_1^* = \$\,12402.60$

Case 2: $T < M_1$

For $s = 3$, $T = T_2^* = 0.038348$ years, $Q = Q_2^* = 19.185401$, $Z_2^* = \$\,12357.14$

Case 3: $T \geq M_2$

For $s = 14$, $T = T_3^* = 0.082771$ years, $Q = Q_3^* = 41.438641$, $Z_3^* = \$\,12739.68$

Case 3: $T < M_2$

For $s = 5$, $T = T_4^* = 0.049461$ years, $Q = Q_4^* = 24.749469$, $Z_4^* = \$\,12603.5$

5. SENSITIVITY ANALYSIS

We have performed sensitivity analysis by changing s and h and keeping the remaining parameters at their original values. The corresponding variations in the cycle time, economic order quantity and total relevant cost per year are exhibited in Table 1 (Table 1.a, Table 1.b) for case I, Table 2 (Table 2.a, Table 2.b) for case 2, Table 3 (Table 3.a, Table 3.b) for case 3, and Table 4 (Table 4.a, Table 4.b) for case 4 respectively.

Table 1. Case 1

Table 1.a Sensitivity analysis on s (h = 5)

s	Replenishment cycle time T_1^* (in years)	Economic order quantity $Q_1^*(T_1^*)$ units	Total relevant cost $Z_1^*(T_1^*)$ in dollars
5	0.049695	24.866649	12402.60
6	0.054514	27.280044	12421.79
7	0.058940	29.496939	12438.42
8	0.063056	31.558834	12455.82
9	0.066919	33.494230	12471.20
10	0.070572	35.324626	12485.75
11	0.074044	37.064522	12499.59

Table 1.b Sensitivity analysis on h (s = 5)

h	Replenishment cycle time T_1^* (in years)	Economic order quantity $Q_1^*(T_1^*)$ units	Total relevant cost $Z_1^*(T_1^*)$ in dollars
1	0.070436	35.256448	12344.32
2	0.062944	31.502725	12360.94
3	0.057425	28.738072	12375.96
4	0.053143	26.593399	12389.76
5	0.049695	24.866649	12402.60
6	0.046841	23.437512	12414.66
7	0.044429	22.229805	12426.06
8	0.042354	21.190908	12436.30

Table 2. Case 2

Table 2.a Sensitivity analysis on s (h = 5)

s	Replenishment cycle time T_2^* (in years)	Economic order quantity $Q_2^*(T_2^*)$ units	Total relevant cost $Z_2^*(T_2^*)$ in dollars
1.0	0.022141	11.074300	12291.02
1.5	0.27117	13.564200	12311.15
2.0	0.031312	15.679710	12328.43
2.5	0.035007	17.513001	12343.51
3.0	0.038348	19.185401	12357.14

Table 2.b Sensitivity analysis on h (s = 3)

h	Replenishment cycle time T_2^* (in years)	Economic order quantity $Q_2^*(T_2^*)$ units	Total relevant cost $Z_2^*(T_2^*)$ in dollars
5	0.038348	19.185401	12347.14
6	0.036195	18.107657	12366.45
7	0.034367	17.192657	12375.27
8	0.032791	16.403836	12383.66
9	0.031414	15.714651	12391.52
10	0.030197	15.105569	12399.38

Table 3.

Table 3.a Sensitivity analysis on *s* (*h* = 5)

s	Replenishment cycle time T_3^* (in years)	Economic order quantity $Q_3^*(T_3^*)$ units	Total relevant cost $Z_3^*(T_3^*)$ in dollars
14	0.082771	41.438641	12739.68
15	0.085728	42.917131	12751.55
16	0.088587	44.354375	12763.02
17	0.091356	45.742742	12774.13
18	0.094044	47.090610	12784.93
19	0.096657	48.400977	12794.91
20	0.099201	49.676845	12805.63

Table 3.b Sensitivity analysis on *h* (*s* = 15)

h	Replenishment cycle time T_3^* (in years)	Economic order quantity $Q_3^*(T_3^*)$ units	Total relevant cost $Z_3^*(T_3^*)$ in dollars
1	0.112542	56.369273	12651.82
2	0.100723	50.440207	12680.48
3	0.091986	46.058638	12706.41
4	0.085188	42.650291	12730.26

Table 4.

Table 4.a Sensitivity analysis on *s* (*h* = 5)

s	Replenishment cycle time T_4^* (in years)	Economic order quantity $Q_4^*(T_4^*)$ units	Total relevant cost $Z_4^*(T_4^*)$ in dollars
1	0.022120	11.063793	12491.79
2	0.031282	15.648586	12529.24
3	0.038312	19.049969	12557.98
4	0.044239	22.134674	12582.20
5	0.049461	24.749469	12603.55
6	0.054181	27.113263	12622.84
7	0.058522	29.287558	12640.59
8	0.062563	31.311854	12657.11
9	0.066357	33.212649	12672.62
10	0.069947	35.011445	12687.29

Table 4.b Sensitivity analysis on *h* (*s* = 5)

h	Replenishment cycle time T_4^* (in years)	Economic order quantity $Q_4^*(T_4^*)$ units	Total relevant cost $Z_4^*(T_4^*)$ in dollars
6	0.046687	23.360401	12615.56
7	0.044334	22.182240	12626.93
8	0.042304	21.165876	12637.75
9	0.040529	20.277235	12648.10
10	0.038961	19.492269	12658.06
11	0.037561	18.791438	12667.60
12	0.036303	18.161718	12676.83
13	0.035163	17.591086	12685.76
14	0.034124	17.071028	12694.42
15	0.033172	16.594531	12702.83

From the above tables the following results have been obtained:

(a) The computational results are shown in Table 1.a, indicates that higher value of ordering cost '*s*' implies higher values of replenishment cycle time T_1^*, order quantity $Q_1^*(T_1^*)$ and total relevant cost $Z_1^*(T_1^*)$.

(b) The computational results are shown in Table 1.b, indicates that higher value of unit holding cost '*h*' implies lower values of replenishment cycle time T_1^*, order quantity $Q_1^*(T_1^*)$ and total relevant cost $Z_1^*(T_1^*)$.

(c) The computational results are shown in Table 2.a, indicates that higher value of ordering cost '*s*' implies higher values of replenishment cycle time T_2^*, order quantity $Q_2^*(T_2^*)$ and total relevant cost $Z_2^*(T_2^*)$.

(d) The computational results are shown in Table 2.b, indicates that higher value of unit holding cost '*h*' implies lower values of replenishment cycle time T_2^*, order quantity $Q_2^*(T_2^*)$ and total relevant cost $Z_2^*(T_2^*)$.

(e) The computational results are shown in Table 3.a, indicates that higher value of ordering cost '*s*' implies higher values of replenishment cycle time T_3^*, order quantity $Q_3^*(T_3^*)$ and total relevant cost $Z_3^*(T_3^*)$.

(f) The computational results are shown in Table 3.b, indicates that higher value of unit holding cost '*h*' implies lower values of replenishment cycle time T_3^*, order quantity $Q_3^*(T_3^*)$ and total relevant cost $Z_3^*(T_3^*)$.

(g) The computational results are shown in Table 4.a, indicates that higher value of ordering cost '*s*' implies higher values of replenishment cycle time T_4^*, order quantity $Q_4^*(T_4^*)$ and total relevant cost $Z_4^*(T_4^*)$.

(h) The computational results are shown in Table 4.b, indicates that higher value of unit holding cost '*h*' implies lower values of replenishment cycle time T_4^*, order quantity $Q_4^*(T_4^*)$ and total relevant cost $Z_4^*(T_4^*)$.

The main difference between Hwang and Seong (1997) paper and this paper is as follows:

(1) In Hwang and Seong (1997) paper demand rate is a function of retail price while in this paper demand rate is a function of time.

(2) In Hwang and Seong (1997) paper two different cases have been considered i.e. case 1. Credit period 't_c' is less than or equal to cycle time and case 2. Credit period 't_c' is greater than cycle time, while in this paper four different cases have been considered i.e. case 1: $T< M_1$, $T \geq M_1$, case 2: $T \geq M_2$, $T< M_2$, where T is the replenishment time interval, M_1 is the period of cash discount and M_2 is the period of permissible delay in settling the account with $M_2 > M_1$.

(3) In Hwang and Seong (1997) paper cash discount is not considered while in this paper cash discount is considered.

(4) In Hwang and Seong (1997) paper maximum annual net profit have been obtained while in this paper minimum total relevant cost per is obtained.

(5) In Hwang and Seong (1997) paper the annual net profit is a concave function of cycle time T, while in this paper total relevant cost is a convex function of cycle time T.

Also the main difference between Tripathi (2011) paper and this paper is as follows:

(1) In R.P.Tripathi (2011) paper, length of planning horizon H = nT have been considered, where n is the number of replenishment and T is an interval of time between replenishments, while is this paper only cycle time T is considered.

(2) In R.P.Tripathi (2011) paper four, different cases have been considered i.e.case1: $0 <T< T_d$, case 2. $T_d < T < m$, case 3. $T_d \leq m \leq T$ and case 4. $m \leq T_d \leq T$, where T_d is the time interval that Q_d units are depleted to zero, Q_d is the minimum order quantity at which the delay in payments is permitted, and m is the permissible delay in settling account, while in this paper four different cases is considered in different ways: i.e. case 1: $T \geq M_1$, case 2: $T < M_1$, case 3: $T \geq M_2$ and case 4: $T< M_2$, where T is replenishment time interval, M_1 is the period of cash discount, M_2 is the period of permissible delay in settling account with $M_2 > M_1$.

(3) In R.P.Tripathi (2011) paper special cases have been not considered, while in this paper four special cases are considered.

(4) In R.P.Tripathi (2011) paper EOQ model have been developed for non – deteriorating items, while in this paper EOQ model is developed for deteriorating items.

(5) In R.P.Tripathi (2011) paper demand rate is time dependent , while in this paper the demand rate is linearly time dependent (which is more useful in real life)

6. CONCLUSION AND FUTURE RESEARCH

We developed EOQ model for deteriorating items and time dependent demand rate to find the optimal ordering policy when the supplier provides a cash discount and (or) trade credit. We use Taylor's series approximation to obtain the explicit closed-form solution of the optimal replenishment cycle time. We also characterize the effect of the value of parameters on the optimal replenishment cycle time. Numerical example and sensitivity analysis is given to illustrate the model. Numerical technique method is applied to obtain optimal cycle time.

The proposed model can be extended in several ways. For instance, we may extend the demand rate to a quadratic time-dependent demand rate. We could also consider the demand rate as a function of quantity, selling price, product quality and others. Finally we could generalize the model to allow for shortages, quantity discount and time-dependent deterioration rate, etc.

7. ACKNOWLEDGEMENTS

The author deeply appreciates Managing Editor Ming-Jong Yao for his encouragement and patience. I also thank two anonymous referees for improving the quality of the paper.

REFERENCES

1. Aggarwal, R., Rajput, D. and Varshney, N.K. (2009). Integrated inventory system with the effect of inflation and credit period. *International Journal of Applied Engineering Research*, 4(11): 2234-2248.
2. Buzacott, J.A. (1975). Economic order qualities with inflation. *Operational Research Quarterly*, 26: 553-558.
3. Chang, C.T. (2004). An EOQ model with deteriorating items under inflation when supplier credits linked to order quantity. *International Journal of Production Economics*, 88: 307-316.
4. Chung, K.J., Goyal, S.K. and Huang, Y.F. (2005). The optimal inventory policies under permissible delay in payments depending on the ordering quantity. *International Journal of Production Economics*, 95: 203-213.
5. Chung, K.J. and Liao, J.J. (2004). Lot sizing decisions under trade credit depending on the ordering quantity. *Computers and Operations Research*, 31: 909-928.
6. Chung, K.J. and Lin, C.N. (2001). Optimal inventory replenishment models for deteriorating items taking account of time discounting. *Computer and Operations Research*, 28: 67-83.
7. Covert, R.B. and Philip, G.S. (1973). An EOQ model with Weibull distribution deterioration. *AIIE Transactions*, 5: 323-326.
8. Ghare, P.M. and Schrader, G.P. (1963). A model for an exponentially decaying inventory. *Journal of Industrial Engineering,* 14: 238-243.
9. Goyal, S.K. (1985). Economic order quantity under conditions of permissible delay in payments. *Journal of the Operational Research Society*, 36: 335-338.
10. Hariga, M.A. (1996). Optimal EOQ models for deteriorating items with time-varying demand. *Journal of Operational Research Society*, 47: 1228-1246.
11. Hou, K.L. and Lin, L.C. (2008). An ordering policy with a cost minimization procedure for deteriorating items under trade credit and time discounting. *Journal of Statistics and Management Systems*, 11(6): 1181-1194.
12. Hou, K.L. and Lin, L.C. (2009). A cash flow oriented EOQ model with deteriorating items under permissible delay in payments. *Journal of Applied Sciences*, 9(9): 1791-1794.
13. Huang, H.F. (2007). Economic order quantity under conditionally permissible delay in payments. *European Journal of Operational Research,* 176: 911-924.
14. Hwang, H. and Shinn, S.W. (1997). Retailer's pricing and lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payments. *Computers and operations Research*, 24 (9): 539-547.
15. Jaggi, C.K., Aggarwal, K.K. and Goel, S.K. (2007). Retailer's optimal ordering policy under two stage trade credit financing. *Advanced Modeling and Optimization*, 9(1): 67-80.
16. Jaggi, C.K., Goyal, S.K. and Goel, S.K. (2008). Retailer's optimal replenishment decisions with credit-linked demand under permissible delay in payments. *European Journal of Operational Research*, 1990: 130-135.
17. Liao, H.C., Tsai, C.H. and Su, C.T. (2000). An inventory model with deteriorating items under inflation when a delay in payments is permissible. *International Journal of Production Economics*, 63: 207-214.
18. Misra, R.B. (1977). A note on optimal inventory management under inflation. *Naval Research Logistics*, 26: 161-165.
19. Mondal, B., Bhunia, A.K. and Maiti, M. (2003). An inventory system of ameliorating items for price dependent demand rate. *Computers and Industrial Engineering*, 45(3): 443-456.
20. Ouyang, L.Y., Chang, C.T. and Teng, J.T. (2005). An EOQ model for deteriorating items under trade credits. *Journal of the Operational Research Society*, 56: 719-726.
21. Roy, J. and Chaudhuri, K.S. (1997). An EOQ model with stock dependent demand, shortage, inflation and time-discounting. *International Journal of Production Economics*, 53: 171-180.
22. Teng, J.T., Chern, M.S., Yang, H.L. and Wang, Y.J. (1999). Deterministic lot-size inventory models with shortages and deterioration for fluctuating demand. *Operations Research Letters*, 24: 65-72.
23. Teng, J.T. and Yang, H.L. (2004). Deterministic economic order quantity models with partial backlogging when demand and cost are fluctuating with time. *Journal of Operational Research Society*, 55: 495-503.
24. Tripathi, R.P. (2011). Inventory model with time–dependent demand rate under inflation when supplier credit linked to order quantity. *International Journal of Business & Information Technology,* 1(3): 174- 183.
25. Tripathi, R.P. and Misra, S.S. (2010). Credit financing in economic ordering policies on non-deteriorating items with time-dependent demand rate. *International Review of Business and Finance*, 2(1): 47-55.
26. Yang, H.L., Teng, J.T. and Chern, M.S (2001). Deterministic inventory lot-size model under inflation with shortages and deterioration for fluctuating demand. *Naval Research Logistics*, 48: 148-158.
27. You, S.P. (2005). Inventory policy for products with price and time-dependent demands. *Journal of the Operational Research Society,* 56: 870-873.