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A Multi-item Inventory Model for Two-stage Production System with Imperfect Processes Using Differential Evolution and Credibility Measure

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Abstract — A multi-item two stage production inventory system with imperfect production process is formulated. Here, a constraint on the total budget is imposed where the total budget is imprecise in nature. Shortages are allowed and completely backlogged. Stage I (raw materials to semi-finished products) is an automatic process and this process is treated by machines. Stage II (semi-finished products to finished products) is also an automatic process and this process is treated by another machines. It is assumed that the time of transporting items from Stage I to Stage II is negligible. The imperfect items are reworked and assumed that the inspection time and rework time are very short which also can be neglected. The model has been formulated as profit maximization problem in stochastic and fuzzy-stochastic environments by considering inventory costs as imprecise in nature. Credibility theory has been used to transform the fuzzy-stochastic model into an equivalent deterministic one. To solve the problems, Differential Evolution (DE) algorithm has been suitably developed and applied. Finally, to illustrate the model and to show the effectiveness of the proposed approach a numerical example is presented.

Keywords — Two-stage system, inventory, stock-dependent demand, imperfect quality, rework, credibility measure, differential evolution

1. INTRODUCTION

In most of the classical economic production quantity (EPQ) model, it is assumed that items produced are of perfect quality, the quality control of the product generally is not considered. However, in a production system, it is quite natural that a machine cannot produce all items perfectly during whole production period. In most of the production system, a certain portion of defective items are produced and wasted as scraps since they have no recycling or reworking facility. But, in modern manufacturing companies four systems of improving production efficiencies are highly appreciated. These are materials requirement planning (MRP), flexible manufacturing system (FMS), optimized product technology (OPT), and just in time (JIT). The adjustment of production rate with variability in market demand is a major component in FMS. Goyal and Gunasekaran (1995) have developed an integrated production-inventory-marketing model for deteriorating the EPLS and EOQ for raw materials in a multi-stage production system. Bhunia and Maiti (1998) extend the EPLS model by considering the finite production rate depending on on-hand inventory and demand simultaneously.

Two-stage production systems can be found in different applications, like processing and packaging food, extruding and milling plastics, shearing and punching or rolling and cutting metals [cf. Szendrovits (1983)]. Szendrovits (1983) proposed two-stage production/inventoy models in which smaller lots are produced at one stage and one larger lot is produced at the other stage. Kim's (1999) considered a two-stage lot sizing problems with various lot sizing depending on batch transfer and production rates between stages. Hill (2000) extended Kim's (1999) model providing an alternative way of performing the analysis which is easier to understand. Darwish and Ben-Daya (2007) investigated the effect of imperfect production processes involving variable the frequency of preventive maintenance. Recently, Pearn *et al.* (2010) also investigated the effect of imperfect production processes with allowable shortages for two stage production system.

Expensive products, processed or assembled, are not usually scrapped. Consequently, a procedure for recovering the defective items would be beneficial to the company. For example, metal book-shelves and defective filing cabinets are usually repaired in sheet metal industries, and defective alignment of steering wheels is corrected to fix the steering column at a right-angle with the steering wheels in automobile industries. Scrapping many such items is an expensive proposal for any company. Hence, the rejected items are accumulated for a certain number of cycles and reworked while a rework cost is

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assessed for not satisfying the demand and other resource constraints. Hayek and Salameh (2001) derived an optimal operating policy for the finite production model under the effect of reworking of imperfect quality items and assuming that all the defective items are repairable. Chiu (2003) examined an EPQ model with scrap items and the reworking of repairable items. Konstantaras and Papachristos (2007) also extended the salameh and Jaber's (2000) model to the case in which withdrawing of defective units takes place at the end of the planning horizon and minimized directly the mean average cost instead of maximizing the mean average profit.

Till now, algorithms have been developed for solving the inventory problems when inventory parameters like, total floor space and total budget allocation for replenishment, etc. are precisely known. But in real life situation, these parameters may be uncertain in non-stochastic sense. In the competitive market, it is not possible to do the business with predefined fixed budgetary capital. Initially, a decision maker (DM) may start with an amount, but, at a later stage, to meet the sudden increase of demand or to avail the sudden fall in the price of the commodity, he / she is forced to augment some more capital as per demand of the situation. Hence, in this case, budgetary allocation is imprecise. Recently, several researchers such as Roy and Maiti (1997), Kar *et al.* (2000), Roy *et al.* (2008), Das *et al.* (2010, 2011) etc. have developed several fuzzy inventory models.

The purpose of this paper is to study a multi-item, two-stage production inventory cum sale model having imperfect production process with rework under budget constraint. Here the production system with random imperfect items is separated into two stages. In Stage I, semi-finished products are produced by a set of machines and in Stage II, finished products are produced by another set of machines. The model has been defined as a profit maximization problem in stochastic and fuzzy-stochastic nature. In fuzzy-stochastic model, inventory costs and the constraint goal are imprecise in nature. A credibility measure and differential evolution (DE) algorithm are used to solve the model. A numerical example is given for illustration of the theoretical results, and sensitivity analysis for the profit function with respect to some parameters are carried out.

2. ASSUMPTIONS AND NOTATIONS

A multi-item, two-stage production inventory model with rework system is developed on the basis of following assumptions and notations:

2.1 Assumptions

The following assumptions are made:

(i) Inventory system involves two stage and multi-item and is a self production system.
(ii) The time horizon is infinite.
(iii) Shortages are allowed and backlogged.
(iv) Lead time is zero.
(v) Production of Stage I and Stage II starts at same time.
(vi) Machine breakdown does not occur at any production stage and the handling time between processes is assumed to be zero.
(vii) No defective product is scraped.
(viii) No defective items are produced during the rework.
(ix) Set-up time is negligible.
(x) It is well known that the production rate of Stage I is always higher than Stage II.
(xi) The production system is imperfect, and the inspection time and rework time of defective products are very short, which can be neglected.
(xii) Inspection cost is negligible.
(xiii) Transporting time from Stage I to Stage II is ignored.

2.2 Notations (for i^{th} item, i=1, 2, ….., n)

The following notations are employed through this paper as to develop the proposed model.

N_{1i} is the total number of machines for i^{th} item in Stage I.

N_{2i} is the total number of machines for i^{th} item in Stage II.

P_{1i} is the production rate per machine for i^{th} item in Stage I.

P_{2i} is the production rate per machine for i^{th} item in Stage II, where $N_{1i}.P_{1i} > N_{2i}.P_{2i}$.

W_{0i} is the maximum shortage amount for i^{th} item.

W_{1i} is the maximum inventory level of semi-finished product for i^{th} item in Stage I.

W_{2i} is the maximum inventory level of finished product for i^{th} item in Stage II.

$q_i(t)$ is the on hand inventory of the item at time t for i^{th} item in Stage I.

$D_i(q_i)$ is the stock-dependent demand rate.

P_{c1i} = The production cost per unit item per unit time in Stage I.

P_{c2i} = The production cost per unit item per unit time in Stage II.

C_{h1i} = The holding cost per unit inventory held per unit of time in Stage I.

C_{h2i} = The holding cost per unit inventory held per unit of time in Stage II.

C_{3i} = The set-up cost per production run.

C_{si} = The shortage cost per unit per unit time.

C_{r1i} = Rework cost per defective item in Stage I.

C_{r2i} = Rework cost per defective item in Stage II.

s_i = Selling price per unit item.

d_{1i} = Percentage of defective semi-finished product produced, a random variable.

d_{2i} = Percentage of defective finished product produced, a random variable.

$f(d_{ji})$ = The probability density function of d_{ji}, (j =1, 2), uniformly distributed with p.d.f as

$$f(d_{ji}) = \frac{1}{a_{ji}}, \qquad 0 \le d_{ji} \le a_{ji}$$
$$= 0, \qquad otherwise$$

m_i = Mark-up of the selling price per unit item.

t_{1i} = Production starts at that time. (decision variable)

t_{2i} = Time at which stock of inventory starts to accumulate of finished product at Stage II.

t_{3i} = Time when the maximum stock of inventory of semi-finished products occur at Stage I. (decision variable)

t_{4i} = Time when the stock of semi-finished products vanish at Stage I and stock of finished products are maximum at Stage II.

T_i = Duration of the cycle.

B = The total budget.

3. MATHEMATICAL FORMULATION

In this model, we have considered the demand rate is dependent on the on-hand inventory i.e.
$D_i(q_i) = \alpha_i + \beta_i q_i(t)$, α_i, β_i >0 are constants.

In the development of the two stage production model, we assume that there exist allowable shortages and the shortages are backlogged and also the cycle starts with shortage at time $t = 0$. The production run begins at $t = t_{1i}$ in both the Stages but production and demand occur simultaneously in Stage II, back-orders are made up to $t = t_{2i}$. Inventory items in Stage I begin to accumulate up to W_{1i} units and inventory items in Stage II begin to accumulate up to W_{2i} units without deterioration. After $t = t_{3i}$ the production in Stage I stops but the production run is continuous up to

$t = t_{4i}$ in Stage II (cf. Fig. 1). At the end of production, at $t = t_{4i}$ the inventory in Stage II would be depleted due to demand and it vanishes at $t = T_i$. This cycle repeats again and again.

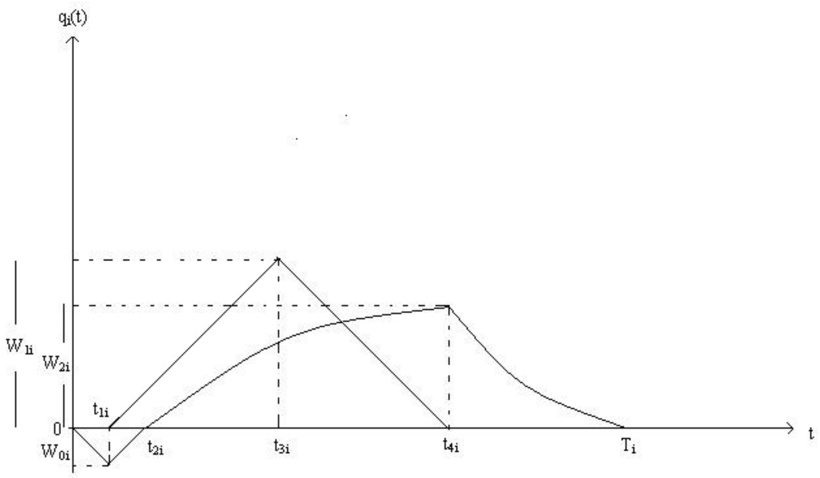

Figure 1. The graph of inventory level during time period $[0, T_i]$

For semi-finished product in Stage I we have the following result:

$(N_{1i}P_{1i} - N_{2i}P_{2i}).(t_{3i} - t_{1i}) = W_{1i}$.

Also

$N_{2i}P_{2i}.(t_{4i} - t_{3i}) = W_{1i}$.

Therefore

$$N_{2i}P_{2i}.(t_{4i} - t_{3i}) = (N_{1i}P_{1i} - N_{2i}P_{2i}).(t_{3i} - t_{1i})$$

$$\Rightarrow t_{4i} = \frac{N_{1i}P_{1i}t_{3i} - (N_{1i}P_{1i} - N_{2i}P_{2i}).t_{1i}}{N_{2i}P_{2i}} \tag{1}$$

Now the change of inventory level in Stage II with respect to time can be described by the following differential equations:

$$\frac{dq_i(t)}{dt} = \begin{cases} -\alpha_i, & 0 \le t \le t_{1i} \\ N_{2i}P_{2i} - \alpha_i, & t_{1i} \le t \le t_{2i} \\ N_{2i}P_{2i} - (\alpha_i + \beta_i\, q_i(t)), & t_{2i} \le t \le t_{4i} \\ -(\alpha_i + \beta_i\, q_i(t)), & t_{4i} \le t \le T_i \end{cases} \tag{2}$$

with the boundary conditions $q_i(0) = 0$, $q_i(t_{1i}) = -W_{0i}$, $q_i(t_{2i}) = 0$, $q_i(t_{4i}) = W_{2i}$, $q_i(T_i) = 0$.

Then the solutions of the differential equations (2) are represented by

$$q_i(t) = \begin{cases} -\alpha_i t, & 0 \le t \le t_{1i} \\ (N_{2i}P_{2i} - \alpha_i).(t - t_{1i}) - \alpha_i t_{1i}, & t_{1i} \le t \le t_{2i} \\ \dfrac{N_{2i}P_{2i} - \alpha_i}{\beta_i}\left\{1 - e^{-\beta_i(t - t_{2i})}\right\}, & t_{2i} \le t \le t_{4i} \\ \dfrac{N_{2i}P_{2i} - \alpha_i}{\beta_i}\left\{1 - e^{-\beta_i(t_{4i} - t_{2i})}\right\}.e^{-\beta_i(t - t_{4i})} - \dfrac{\alpha_i}{\beta_i}.\left\{1 - e^{-\beta_i(t - t_{4i})}\right\}, & t_{4i} \le t \le T_i \end{cases} \tag{3}$$

At $t = t_{2i}$, $q_i(t_{2i}) = 0$ and from (3) we get

$$t_{2i} = \frac{N_{2i}P_{2i}t_{1i}}{N_{2i}P_{2i} - \alpha_i}. \tag{4}$$

At $t = T_i$, $q_i(T_i) = 0$ and from (3) we get

$$T_i = t_{4i} + \frac{1}{\beta_i}\ln\left|1 + \frac{(N_{2i}P_{2i} - \alpha_i).\left\{1 - e^{-\beta_i(t_{4i}-t_{2i})}\right\}}{\alpha_i}\right|. \tag{5}$$

Now the total holding cost (C_{HOLi}) during the period $(0,\ T_i)$ is given by,

$$C_{HOLi} = C_{HOL1i} + C_{HOL2i} \tag{6}$$

where

$$C_{HOL1i} = \frac{1}{2}C_{h1i}(N_{1i}P_{1i} - N_{2i}P_{2i}).(t_{3i} - t_{1i}).(t_{4i} - t_{1i}).$$

And

$$\begin{aligned} C_{HOL2i} &= C_{h2i}\left[\int_{t_{2i}}^{t_{4i}} q_i(t)dt + \int_{t_{4i}}^{T_i} q_i(t)dt\right] \\ &= C_{h2i}\left[\int_{t_{2i}}^{t_{4i}} \frac{N_{2i}P_{2i} - \alpha_i}{\beta_i}\left\{1 - e^{-\beta_i(t-t_{2i})}\right\}dt\right. \\ &\left. + \int_{t_{4i}}^{T_i} \frac{N_{2i}P_{2i} - \alpha_i}{\beta_i}\left\{1 - e^{-\beta_i(t_{4i}-t_{2i})}\right\}.e^{-\beta_i(t-t_{4i})}dt - \int_{t_{4i}}^{T_i} \frac{\alpha_i}{\beta_i}.\left\{1 - e^{-\beta_i(t-t_{4i})}\right\}dt\right] \\ &= C_{h2i}\left[\frac{N_{2i}P_{2i} - \alpha_i}{\beta_i}\left\{(t_{4i} - t_{2i}) - \frac{1}{\beta_i}(1 - e^{-\beta_i(t_{4i}-t_{2i})})\right\}\right. \\ &\left. + \frac{\alpha_i + (N_{2i}P_{2i} - \alpha_i)\{1 - e^{-\beta_i(t_{4i}-t_{2i})}\}}{\beta_i^2}\left(1 - e^{-\beta_i(T_i-t_{4i})}\right) - \frac{\alpha_i}{\beta_i}(T_i - t_{4i})\right]. \end{aligned}$$

Total production cost (PC_i) is given by

$$PC_i = N_{1i}P_{1i}P_{c1i}(t_{3i} - t_{1i}) + N_{2i}P_{2i}P_{c2i}(t_{4i} - t_{1i}). \tag{7}$$

The sales revenue (SR_i) is given by

$$\begin{aligned} SR_i &= s_i N_{2i}P_{2i}(t_{4i} - t_{1i}) \\ &= m_i(P_{c1i} + P_{c2i})N_{2i}P_{2i}(t_{4i} - t_{1i}) \end{aligned} \tag{8}$$

where $m_i > 1$.

The total rework cost (RC_i) is given by

$$RC_i = C_{r1i}d_{1i}N_{1i}P_{1i}(t_{3i} - t_{1i}) + C_{r2i}d_{2i}N_{2i}P_{2i}(t_{4i} - t_{1i}). \tag{9}$$

The total shortage cost (SHC_i) is given by

$$\begin{aligned} SHC_i &= C_{si}\left[\int_0^{t_{1i}} q_i(t)dt + \int_{t_{1i}}^{t_{2i}} q_i(t)dt\right] \\ &= C_{si}\left[\frac{1}{2}\alpha_i t_{1i}^{\ 2} + (t_{2i} - t_{1i})\left\{\frac{1}{2}(N_{2i}P_{2i} - \alpha_i)(t_{2i} + t_{1i}) - N_{2i}P_{2i}t_{1i}\right\}\right] \end{aligned} \tag{10}$$

Hence the average profit (AP_i) during the cycle $(0, T_i)$ is given by

$$
\begin{aligned}
AP_i &= \frac{1}{T_i}\left[SR_i - PC_i - C_{HOLi} - RC_i - SHC_i - C_{3i}\right] \\
&= \frac{1}{T_i}\Big[m_i(P_{c1i} + P_{c2i})N_{2i}P_{2i}(t_{4i} - t_{1i}) - N_{1i}P_{1i}P_{c1i}(t_{3i} - t_{1i}) \\
&- N_{2i}P_{2i}P_{c2i}(t_{4i} - t_{1i}) - \frac{1}{2}C_{h1i}(N_{1i}P_{1i} - N_{2i}P_{2i}).(t_{3i} - t_{1i}).(t_{4i} - t_{1i}) \\
&- C_{h2i}\left\{\frac{N_{2i}P_{2i} - \alpha_i}{\beta_i}\left((t_{4i} - t_{2i}) - \frac{1}{\beta_i}(1 - e^{-\beta_i(t_{4i} - t_{2i})})\right)\right. \\
&\left. + \frac{\alpha_i + (N_{2i}P_{2i} - \alpha_i)\{1 - e^{-\beta_i(t_{4i} - t_{2i})}\}}{\beta_i^2}\left(1 - e^{-\beta_i(T_i - t_{4i})}\right) - \frac{\alpha_i}{\beta_i}(T_i - t_{4i})\right\} \\
&- C_{r1i}d_{1i}N_{1i}P_{1i}(t_{3i} - t_{1i}) - C_{r2i}d_{2i}N_{2i}P_{2i}(t_{4i} - t_{1i}) \\
&- C_{si}\left\{\frac{1}{2}\alpha_i t_{1i}^{\ 2} + (t_{2i} - t_{1i})\left(\frac{1}{2}(N_{2i}P_{2i} - \alpha_i)(t_{2i} + t_{1i}) - N_{2i}P_{2i}t_{1i}\right)\right\} - C_{3i}\Big].
\end{aligned} \tag{11}
$$

Then the expected value of the average profit (EAP_i) is given by

$$
\begin{aligned}
EAP_i &= E\left[\frac{1}{T_i}\left(SR_i - PC_i - C_{HOLi} - RC_i - SHC_i - C_{3i}\right)\right] \\
&= \frac{1}{T_i}\Big[m_i(P_{c1i} + P_{c2i})N_{2i}P_{2i}(t_{4i} - t_{1i}) - N_{1i}P_{1i}P_{c1i}(t_{3i} - t_{1i}) \\
&- N_{2i}P_{2i}P_{c2i}(t_{4i} - t_{1i}) - \frac{1}{2}C_{h1i}(N_{1i}P_{1i} - N_{2i}P_{2i}).(t_{3i} - t_{1i}).(t_{4i} - t_{1i}) \\
&- C_{h2i}\left\{\frac{N_{2i}P_{2i} - \alpha_i}{\beta_i}\left((t_{4i} - t_{2i}) - \frac{1}{\beta_i}(1 - e^{-\beta_i(t_{4i} - t_{2i})})\right)\right. \\
&\left. + \frac{\alpha_i + (N_{2i}P_{2i} - \alpha_i)\{1 - e^{-\beta_i(t_{4i} - t_{2i})}\}}{\beta_i^2}\left(1 - e^{-\beta_i(T_i - t_{4i})}\right) - \frac{\alpha_i}{\beta_i}(T_i - t_{4i})\right\} \\
&- C_{r1i}.E(d_{1i}).N_{1i}P_{1i}(t_{3i} - t_{1i}) - C_{r2i}.E(d_{2i}).N_{2i}P_{2i}(t_{4i} - t_{1i}) \\
&- C_{si}\left\{\frac{1}{2}\alpha_i t_{1i}^{\ 2} + (t_{2i} - t_{1i})\left(\frac{1}{2}(N_{2i}P_{2i} - \alpha_i)(t_{2i} + t_{1i}) - N_{2i}P_{2i}t_{1i}\right)\right\} - C_{3i}\Big].
\end{aligned} \tag{12}
$$

Hence the total expected value of the average profit (EAP) is given by

$$EAP = \sum_{i=1}^{n} EAP_i \tag{13}$$

4. CREDIBILITY MEASURE

To construct the inventory model for two-stage production system in fuzzy environment, we shall first introduce some knowledge of credibility theory. Credibility theory was initialized by Liu and Liu (2002). If $\tilde{\xi}$ is a fuzzy variable with membership function $\mu_{\tilde{\xi}}(x)$, then for any set A of $\Re$, the possibility measure of fuzzy event $\{\tilde{\xi} \in A\}$ is defined as

$$Pos\{\tilde{\xi} \in A\} = \underset{x \in A}{Sup}\, \mu_{\tilde{\xi}}(x).$$

The necessity of this fuzzy event is defined as the impossibility of the opposite event. That is

$$Nec\{\tilde{\xi} \in A\} = 1 - \underset{x \in A^c}{Sup}\, \mu_{\tilde{\xi}}(x).$$

The credibility measure of $\{\tilde{\xi} \in A\}$ is defined as the average of its possibility and necessity measure. Therefore

$$Cr\{\tilde{\xi} \in A\} = \frac{1}{2}\left\{Pos\{\tilde{\xi} \in A\} + Nec\{\tilde{\xi} \in A\}\right\}, \text{ for any } \tilde{A} \in 2^{\Re},$$

where $2^{\Re}$ is the power set of $\Re$.

It is easy to check that Cr satisfies the following conditions:
(i) $Cr(\varphi)=0$ and $Cr(\Re)=1$;
(ii) $Cr(\tilde{\xi})\le Cr(\tilde{\eta})$ whenever $\tilde{\xi},\tilde{\eta}\in 2^{\Re}$ and $\tilde{\xi}\subset\tilde{\eta}$;

Thus, Cr is also a fuzzy measure defined on $(\Re, 2^{\Re})$. Besides, Cr is self dual, i.e.,
$Cr(\tilde{A})=1-Cr(\tilde{\xi}^c)$ for any $\tilde{\xi}\in 2^{\Re}$.
Credibility measure is defined as the following form:

$$Cr(\tilde{\xi})=[\rho\, Pos(\tilde{\xi})+(1-\rho)Nec(\tilde{\xi})]$$

[cf. Liu and Liu (2002)] for any $\tilde{\xi}\in 2^{\Re}$ and confidence level ρ, $0\le\rho\le 1$. It also satisfies the above conditions.

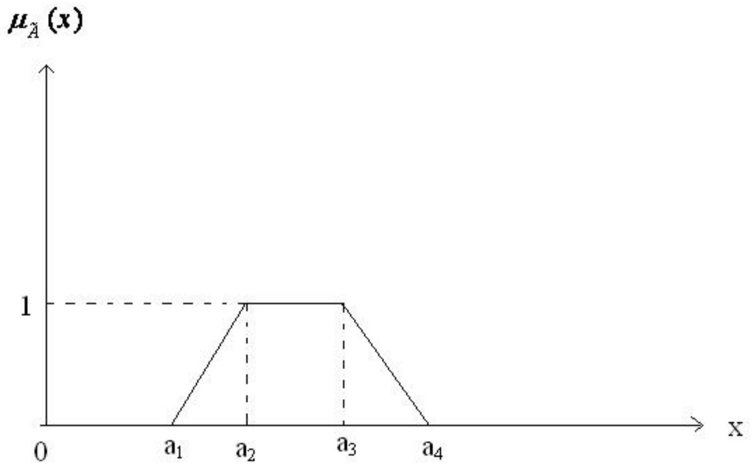

Figure 2. Membership function of a TrFN

Trapezoidal Fuzzy Number: Let $\tilde{A}$ is the trapezoidal fuzzy number (TrFN) with the membership function $\mu_{\tilde{A}}(x)$, a continuous mapping : $\mu_{\tilde{A}}(x):\Re\to[0,1]$

$$\mu_{\tilde{A}}(x)=\begin{cases}0 & for\ -\infty<x\le a_1\\ \dfrac{x-a_1}{a_2-a_1} & for\ a_1\le x\le a_2\\ 1 & for\ a_2\le x\le a_3\\ \dfrac{a_4-x}{a_4-a_3} & for\ a_3\le x\le a_4\\ 0 & for\ a_4\le x<\infty\end{cases}$$

$$Pos(\tilde{A}\ge r)=\begin{cases}1 & if\ r\le a_3\\ \dfrac{a_4-r}{a_4-a_3} & if\ a_3\le r\le a_4\\ 0 & if\ r\ge a_4\end{cases}$$

$$Nec(\tilde{A}\ge r)=\begin{cases}1 & if\ r\le a_1\\ \dfrac{a_2-r}{a_2-a_1} & if\ a_1\le r\le a_2\\ 0 & if\ r\ge a_2\end{cases}$$

The credibility measure for TrFN can be define as

$$Cr(\tilde{A} \geq r) = \begin{cases} 1 & if \;\; r \leq a_1 \\ \dfrac{a_2 - \rho\, a_1}{a_2 - a_1} - \dfrac{(1-\rho)r}{a_2 - a_1} & if \;\; a_1 \leq r \leq a_2 \\ \rho & if \;\; a_2 \leq r \leq a_3 \\ \dfrac{\rho(a_4 - r)}{a_4 - a_3} & if \;\; a_3 \leq r \leq a_4 \\ 0 & if \;\; r \geq a_4 \end{cases}$$

$$Cr(\tilde{A} \leq r) = \begin{cases} 0 & if \;\; r \leq a_1 \\ \dfrac{\rho(r - a_1)}{a_2 - a_1} & if \;\; a_1 \leq r \leq a_2 \\ \rho & if \;\; a_2 \leq r \leq a_3 \\ \dfrac{\rho\, a_4 - a_3 + r(1-\rho)}{a_4 - a_3} & if \;\; a_3 \leq r \leq a_4 \\ 1 & if \;\; r \geq a_4 \end{cases}$$

Based on the credibility measure, Liu and Liu (2002) presented the expected value operator of a fuzzy variable as follows:

Let $\tilde{X}$ be a normalized fuzzy variable, then the expected value of the fuzzy variable $\tilde{X}$ is defined by

$$E[\tilde{X}] = \int_0^{\infty} Cr(\tilde{X} \geq r)\, dr - \int_{-\infty}^{0} Cr(\tilde{X} \leq r)\, dr. \tag{14}$$

When the right side of (14) is of form $\infty - \infty$, the expected value is not defined. Also, the expected value operation has been proved to be linear for bounded fuzzy variables, i.e., for any two bounded fuzzy variables $\tilde{X}$ and $\tilde{Y}$, we have $E[a\tilde{X} + b\tilde{Y}] = aE[\tilde{X}] + bE[\tilde{Y}]$ for any real numbers a and b.

The expected value of trapezoidal fuzzy variable $\tilde{X} = [a_1, a_2, a_3, a_4]$, $\;\; 0 \leq \rho \leq 1$ is defined as

$$E[\tilde{X}] = \frac{1}{2}\left[(1-\rho)(a_1 + a_2) + \rho(a_3 + a_4)\right].$$

5. PROBLEM FORMULATION

5.1 Stochastic Model

So, the equivalent deterministic form of the above stochastic model with budget constraint can be expressed as,

$$\begin{aligned} &Maximize \quad EAP(t_{1i}, t_{3i}) \\ &subject \;\; to, \\ &N_{1i}P_{1i} > N_{2i}P_{2i} > D_i(q) \\ &\sum_{i=1}^{n} [PC_i + RC_i] \leq B. \end{aligned} \tag{15}$$

5.2 Fuzzy-Stochastic Model

As in this model, inventory costs C_{h1i}, C_{h2i}, C_{3i} and available budget B are imprecise, C_{h1i}, C_{h2i} and C_{3i} in (12) are replaced by C_{h1i}, C_{h2i}, C_{3i} and B in constraint of (15) is replaced by $\tilde{B}$ and the expected value of the average profit is represented by $E\tilde{A}P$.

Credibility Approach (CrA)

In this paper we consider C_{h1i}, C_{h2i}, C_{3i} and $\tilde{B}$ as trapezoidal fuzzy number i.e. $\tilde{C}_{h1i} = (C_{h1i1}, C_{h1i2}, C_{h1i3}, C_{h1i4})$, $\tilde{C}_{h2i} = (C_{h2i1}, C_{h2i2}, C_{h2i3}, C_{h2i4})$, $\tilde{C}_{3i} = (C_{3i1}, C_{3i2}, C_{3i3}, C_{3i4})$ and $\tilde{B} = (B_1, B_2, B_3, B_4)$. Since optimization of a fuzzy objective is not well defined, so instead of $E\tilde{A}P$ one can optimize its equivalent optimistic and pessimistic return as stated in section 4. So, the problem can be represented in following way.

When decision maker likes to optimize the optimistic and pessimistic equivalent of $E\tilde{A}P$ with $\tilde{C}_{h1i}$, $\tilde{C}_{h2i}$, $\tilde{C}_{3i}$ and $\tilde{B}$ then, the problem reduces to,

$$\begin{aligned} &Maximize \quad E\tilde{A}P(t_{1i}, t_{3i}) \\ &subject \ \ to, \\ &N_{1i}P_{1i} > N_{2i}P_{2i} > D_i(q) \\ &\sum_{i=1}^{n}[PC_i + RC_i] \le \tilde{B}, \end{aligned} \tag{16}$$

where

$$E\tilde{A}P = \sum_{i=1}^{n} E\tilde{A}P_i .$$

$$\begin{aligned} E\tilde{A}P_i \ =& \frac{1}{T_i}\Big[m_i(P_{c1i} + P_{c2i})N_{2i}P_{2i}(t_{4i} - t_{1i}) - N_{1i}P_{1i}P_{c1i}(t_{3i} - t_{1i}) \\ & - N_{2i}P_{2i}P_{c2i}(t_{4i} - t_{1i}) - \frac{1}{4}\{(1-\rho)(C_{h1i1} + C_{h1i2}) + \rho(C_{h1i3} + C_{h1i4})\}.(N_{1i}P_{1i} \\ & - N_{2i}P_{2i}).(t_{3i} - t_{1i}).(t_{4i} - t_{1i}) - \frac{1}{2}\Big\{(1-\rho)(C_{h2i1} + C_{h2i2}) \\ & + \rho(C_{h2i3} + C_{h2i4})\Big\}.\left\{\frac{N_{2i}P_{2i} - \alpha_i}{\beta_i}\left((t_{4i} - t_{2i}) - \frac{1}{\beta_i}(1 - e^{-\beta_i(t_{4i} - t_{2i})})\right)\right. \\ & \left. + \frac{\alpha_i + (N_{2i}P_{2i} - \alpha_i)\{1 - e^{-\beta_i(t_{4i} - t_{2i})}\}}{\beta_i^2}\left(1 - e^{-\beta_i(T_i - t_{4i})}\right) - \frac{\alpha_i}{\beta_i}(T_i - t_{4i})\right\} \\ & - C_{r1i}.E(d_{1i}).N_{1i}P_{1i}(t_{3i} - t_{1i}) - C_{r2i}.E(d_{2i}).N_{2i}P_{2i}(t_{4i} - t_{1i}) \\ & - C_{si}\left\{\frac{1}{2}\alpha_i t_{1i}^{\ 2} + (t_{2i} - t_{1i})\left(\frac{1}{2}(N_{2i}P_{2i} - \alpha_i)(t_{2i} + t_{1i}) - N_{2i}P_{2i}t_{1i}\right)\right\} \\ & - \frac{1}{2}\{(1-\rho)(C_{3i1} + C_{3i2}) + \rho(C_{3i3} + C_{3i4})\}, \end{aligned} \tag{17}$$

and

$$\tilde{B} = \frac{1}{2}\{(1-\rho)(B_1 + B_2) + \rho(B_3 + B_4)\},\ 0 \le \rho \le 1.$$

6. SOLUTION PROCEDURE

6.1. Differential Evolution (DE)

Many heuristic algorithms have been proposed for global optimization of nonlinear non-convex and non-differentiable functions. These methods are more flexible than classical one as they do not require differentiability, continuity or other restrictive properties which are usually required for the objective function to be optimized. Some of such methods are genetic algorithm, evolutionary strategies, colony optimization, particle swarm optimization and differential evolution (DE). Differential Evolution (DE)[cf. Storn and Price (1997)] is a novel population based stochastic direct search optimization algorithm that is fairly fast and reasonably robust. DE resembles the structure of an evolutionary algorithm but differs from classical evolutionary algorithms in its generation of new candidate solutions and by its use of a `greedy' selection scheme. The key difference is that mutation in DE algorithm is an arithmetic combination of individuals whereas in traditional evolutionary algorithms, it is the result of small perturbations to the genes of an individual. Moreover, in DE, the trial solutions are generated by adding weighted difference vectors to the target vector followed by a recombination (or crossover) step to produce an offspring which is only accepted if it improves the fitness of the parent individual.

DE automatically adapts the mutation increments (i.e. search step) to the best value based on the stage of the evolutionary process. In GA Mutation is caused by small alterations of genes, whereas in DE Mutation is provided by arithmetical combinations of individuals. The core of this operation is the formation of a difference vector which makes mutate an individual.

The basic operators of DE are described in the following sections:

Initial Population

An N dimension parameter optimization problem can be represented as an N-dimensional vector

$$x_i = \left\{x_{j,i}\right\} = \left\{x_{1,i}, x_{2,i}, \ldots\ldots\ldots, x_{N,i}\right\}^T .$$

If there is no preliminary knowledge about the optimization, the first population solutions can be generated randomly as:

$$x_{i,G} = x_{i,(L)} + R_i[x_{i,(H)} - x_{i,(L)}],$$

where $x_{i,(L)}$ and $x_{i,(H)}$ are the lower and higher boundaries of the vector x_i and $R_i \in (0,1)$, drawn uniformly for each i.

Mutation

Mutation operator is employed to expand the search space. By the combination of vectors randomly chosen from the current population at generation G, a mutant vector $v_{i,\,G+1}$ is generated for each target vector $x_{i,\,G}$ as

$$v_{i,G+1} = x_{r_1,G} + F(x_{r_2,G} - x_{r_3,G}), \tag{18}$$

where i, r_1, r_2 and r_3 are reciprocally different random integers less than or equal to population size of solution vectors. $F \in (0,2)$ is a real constant positive weighting factor which controls the amplification of the differential variation.

Crossover

To increase the diversity of the population, DE utilizes crossover operation that integrates successful solutions from the previous generation.

The trial vector $u_{i,G+1}$ is found from its parents $x_{i,G}$ and $v_{i,G+1}$ using the following crossover rule:

$$u^j_{i,G+1} = \begin{cases} v^j_{i,G+1} & if \;\; R^j \le C_R \; or \; j = I_i, \\ x^j_{i,G} & if \;\; R^j > C_R \; and \; j \ne I_i \end{cases} \tag{19}$$

where $i = 1, 2, \ldots\ldots, N$ and C_R is crossover parameter. I_i is an integer randomly chosen with replacement from the set I, i.e., $I_i \in I = \{1, 2, \ldots\ldots, N\}$; the superscript j represents the j^{th} component of respective vectors; $R_j \in (0,1)$, drawn uniformly for each j.

Selection

To decide whether or not to include the trial vector in the population of the next generation $G+1$, it is compared with the target vector using greedy criterion.

If the value of the objective function for the trial vector $u_{i,G+1}$ is better than or equal to the value obtained for the target vector $x_{i,G}$; the latter is replaced by the former otherwise the latter is retained in the population of the next generation.

$$x_{i,G+1} = \begin{cases} u_{i,G+1} & if \quad f(u_{i,G+1}) \le f(x_{i,G}) \\ x_{i,G} & otherwise \end{cases} \tag{20}$$

where $i = 1, 2, \ldots\ldots pop.size$, $f : \Omega \to \Re$ (Ω is assumed to be the feasible search space of the problem) is a continuous real valued function and $x \in \Omega$ is continuous variable vector with domain $\Omega \subseteq \Re^n$.

DE Procedure:

Step-1: Initialize control parameters:

Set the values of the DE control parameters (N, F, C_R).

Step-2: Determine the initial population x_i

$$x_i = \left\{x_{1,i}, x_{2,i}, x_{3,i}, \ldots\ldots\ldots\ldots x_{N,i}\right\}^T$$

where the components of each point $x_{j,i}$ $(j = 1, 2, \ldots\ldots\ldots, N)$ are floating point numbers randomly chosen within the range (0, 1).

Step-3: Generate the solutions of the next population

for *i*=1 to *N* do

Mutation phase:

Generate a mutant vector $v_{i,G+1}$ using equation (18).

Crossover phase:

Generate a trial vector $u_{i,G+1}$ by its parents $x_{i,G}$ and $v_{i,G+1}$ using equation (19).

Accepted phase:

Acceptance of the total expected average profit (EAP) function is occurred with the equation (20).

Endfor

Step-4: Population statistics determine the population best solution if termination condition is not satisfied then go to step 3.

Return

The models (15) and (16) are solved by using differential evolution algorithm approach and credibility measure, discussed in subsection-6.1 and section-4 respectively. Our DE consists of some parameters, the size N of the population, scaling parameter F in its mutation scheme (18) and the controlling parameter C_R in the crossover scheme (19). Here we consider $N = 100$, $C_R = 0.5$ and $F = 0.5$.

7. NUMERICAL ILLUSTRATION

In this section, both the stochastic model and fuzzy-stochastic model are illustrated and solve by DE algorithm with a numerical example.

7.1 Stochastic Model

To illustrate the proposed two stage stochastic production inventory model, let us consider the total budget $B = 50000$ and the input data for following two items are shown in Table 1.

Table 1. Some data for above inventory model

Item	C_{3i}	C_{h1i}	C_{h2i}	α_i	N_{1i}	N_{2i}	C_{si}	C_{r1i}	C_{r2i}	β_i	m_i	P_{1i}	P_{2i}	P_{c1i}	P_{c2i}
Item-1	25	2.5	3.0	150	7	5	1.2	1.2	1.1	.35	1.93	310	250	3.5	2.5
Item-2	24	2.3	2.8	140	5	5	1.1	1.15	1.05	.33	1.91	290	245	3.4	2.3

Table 2. Expected value of defective items

Item	α_{1i}	α_{2i}	$E(d_{1i})$	$E(d_{2i})$
Item-1	0.04	0.05	0.02	0.025
Item-2	0.045	0.055	0.0225	0.0275

The computational result is shown in Table 3.

Table 3. Optimal solutions for illustrated example with allowable shortage

t_{11}	t_{31}	t_{12}	t_{32}	EAP
1.764	2.030	1.792	2.048	1271.6718

Sensitivity Analysis:

For the given numerical example mentioned in section 7.1, sensitivity analyses are performed to study the effect of changes of different values of the demand parameters α_1, α_2, β_1 and β_2 on maximum expected average profit of the system. It is observed that for different values of α_1 as β_1 increases when α_2 and β_2 are fixed, expected value of the average profit increases and also for different values of α_2 as β_2 increases when α_1 and β_1 are fixed, expected value of the average profit also increases. All these observations agree with the reality.

Tables 4 and 5 show that sensitivity analysis of the demand parameter β_1 for different values of α_1 when α_2 = 140 and β_2 = 0.33 and sensitivity analysis of the demand parameter β_2 for different values of α_2 when α_1 = 150 and β_1 = 0.35 respectively.

Table 4. Sensitivity analysis for β_1

α_1	β_1	t_{11}	t_{31}	EAP
100	0.30	1.764	2.030	970.1006
	0.35	1.764	2.030	996.0732
	0.40	1.764	2.030	1021.8962
150	0.30	1.764	2.030	1258.4014
	0.35	1.764	2.030	1271.6718
	0.40	1.764	2.030	1285.9692
200	0.30	1.786	2.132	1483.8086
	0.35	1.786	2.132	1497.2811
	0.40	1.872	2.237	1511.7812

Table 5. Sensitivity analysis for β_2

α_2	β_2	t_{12}	t_{32}	EAP
90	0.30	1.872	2.048	1042.9662
	0.33	1.884	2.063	1043.7886
	0.36	1.876	2.063	1045.2946
140	0.30	1.798	2.048	1270.1179
	0.33	1.792	2.048	1271.6718
	0.36	1.792	2.063	1273.1778
190	0.30	1.732	2.063	1503.4259
	0.33	1.688	2.018	1507.5662
	0.36	1.688	2.048	1508.7366

7.2 Fuzzy-Stochastic Model

To illustrate the two stage fuzzy-stochastic production inventory model numerically, the input data are taken as follows.

$\tilde{C}_{h11} = (2.2,\ 2.4,\ 2.5,\ 2.75)$, $\tilde{C}_{h12} = (2.1,\ 2.3,\ 2.5,\ 2.75)$, $\tilde{C}_{h21} = (2.5,\ 2.75,\ 3.25,\ 3.7)$,

$\tilde{C}_{h22} = (2.6,\ 2.8,\ 3.0,\ 3.5)$, $\tilde{C}_{31} = (20,\ 25,\ 29,\ 35)$ and $\tilde{C}_{32} = (20,\ 24,\ 29,\ 34)$,

$\tilde{B} = (41000,\ 45000,\ 50000,\ 54000)$ and the other data are same as in stochastic model.

Table 6 shows the results for different values of the confidence level ρ.

Table 6. Results using credibility approach

ρ	t_{11}	t_{31}	t_{12}	t_{32}	$E\tilde{A}P$
0.0	1.764	2.030	1.748	2.018	1297.1885
0.1	1.764	2.030	1.790	2.048	1289.4669
0.2	1.764	2.030	1.790	2.048	1283.7149
0.3	1.764	2.030	1.798	2.063	1277.4276
0.4	1.764	2.030	1.760	2.018	1273.6228
0.5	1.764	2.030	1.760	2.018	1267.8199
0.6	1.764	2.030	1.802	2.048	1260.1317
0.7	1.764	2.030	1.802	2.048	1254.5364
0.8	1.764	2.030	1.806	2.048	1248.5729
0.9	1.764	2.030	1.810	2.063	1242.8642
1.0	1.872	2.153	1.876	2.162	1232.5559

8. CONCLUSION AND FUTURE SCOPE

In this study a two-stage production inventory model for multi-item with budgetary constraint has been presented. Here we have analyzed an inventory system where the demand can be satisfied by the products of stage II production, assuming that imperfect products are reworked. Here it is also assumed that defective items are produced in both the stages in a random fashion. For fuzzy-stochastic model, holding cost, setup cost and total available budget are imprecise. For the first time, random production of defective units with rework have considered in a two-stage production system. Credibility theory approach has been introduced for an imprecise inventory system. A differential evolution (DE) algorithm has been designed for numerical illustration of the proposed model.

Finally, a future study will incorporate more realistic assumptions in the proposed model, such as variable production rate, uncertain/imprecise nature of demand finite or random planning horizon.In this section, both the stochastic model and fuzzy-stochastic model are illustrated and solve by DE algorithm with a numerical example.

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