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The Entropy-Theoretic Stability Index for Manpower Systems

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Abstract — In this study, we have derived an entropy index called the adaptive entropy estimator (AEE) which welds the relevant ingredients of the plethora of transitions in a manpower system into a coherent framework flexible enough to estimate the stability of a manpower system. Some interesting results on the maximum and minimum entropy conditions for graded manpower systems are obtained from the AEE. Using dataset in a university setting, the AEE is found to give a better picture of stability in the system than the commonly used McClean/Abodunde-type entropy as the inference from the former agrees reasonably well with the chi-square decision.

Keywords — Entropy, manpower system, Markov chain, stability

1. INTRODUCTION

Manpower systems are well-known hierarchical systems in literature that consist of individual stocks and flows (Agrafiotis 1984). The flows in a manpower system include promotion, recruitment and wastage. As a consequence, a unified framework is employed in the study of manpower systems. In modelling manpower systems, most authors (Bartholomew et al. 1991; McClean et al. 1992) rely on Markov chain methodologies as an analytic tool to unify the states of the system with the axiomatic foundation that there is a one-stage dependence of events, i.e. each event depends immediately on the preceding event, but not on the other prior events. It is essential in manpower planning to be able to monitor stability of the transition process of individuals (McClean, 1986). The commonly used stability index is the McClean/Abodunde-type entropy (McClean and Abodunde 1978). Basically, the term entropy refers to a measure of the degree of disorderliness, flexibility, uncertainty or randomness in a system (Tirtiroglu 2005). The use of entropy as a measure of stability in graded manpower system quantifies the degree of uniformity in accessing a grade from other grades in the system. From the way the concept of entropy is used in McClean and Abodunde (1978), maximum entropy indicates a uniform distribution of experience, i.e. the system is in a 'steady-state', while zero entropy occurs when all members of staff are recruits and always leave after their first years' service; lower entropy is an indication of poor stability, and higher entropy implies a high-level of stability. The use of entropy-theoretic methodologies is not restricted to manpower systems alone as they are increasingly been used in various fields. Details can be found in the works of Horowitz (1970), Pulliainen (1970), Thomas (1979), Dinkel and Kochenberger (1979), Freund and Saxena (1984), Rodrigues (1989), Paris and Howitt (1998), Ebrahimi (2000), Mussard *et al.* (2003), Tirtiroglu (2005), Osagiede and Ekhosuehi (2007), Ekhosuehi and Osagiede 2010) and Lee *et al.* (2011). However, we shall primarily limit our scope of study to the use of entropy in manpower systems.

This study is designed to develop an adaptive entropy estimator for hierarchical manpower systems based on the imbedding of discrete-time Markov chains (Tsaklidis 1994). Specifically, we solve the following problem:

P: Resolve the Boltzmann entropy

$$S(t) = k_B \sum_{i=1}^{k} \log_e \frac{\left(\sum_{j=1}^{k} n_{ij}\right)!}{\prod_{j=1}^{k} \left(n_{ij}\right)!},$$

subject to the imbedded flow pattern $\hat{n}_{ij}(t) = n_i(t)\sigma_{ij}^*(t),\ i = 1,2,\ldots,k;\ j = 1,2,\ldots,k$.

By so doing, we are able to estimate the entropy value from the plethora of transitions in the system. The following symbols and nomenclature are used, *inter alia*, in this paper:

k_B : Boltzmann constant.

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$n_{ij}(t)$: the number of individuals who move from grade i to grade j in period t.

$p_{ij}(t)$: the probability of individuals' flow from grade i to grade j in period t.

$p_{i0}(t)$: the probability of individuals leaving grade i in period t.

k: the maximum grade in the system.

p_i : the probability that a member of staff is in tenure class i.

$\boldsymbol{\sigma}^*(t) = (\sigma^*_{ij}(t))$: the imbedded Markov chain with elements $\sigma^*_{ij}(t) = p_{ij}(t) + p_{i0}(t)\,p_{0j}(t)$.

2. THE USE OF ENTROPY IN MANPOWER CONTEXT

In modelling the state-transition process of a manpower system using a Markov chain model, it is essential to measure the stability of the process and thus evade possible trauma. The commonly used measure of stability is the McClean/Abodunde-type entropy. McClean and Abodunde (1978) introduced the entropy stability measure for manpower systems wherein one of the problems associated with Shannon entropy (which is the absence of a fixed maximum value) is resolved. The McClean/Abodunde-type entropy is obtained for a steady-state manpower system by modifying the basic Shannon entropy as

$$H = -\frac{\sum_{i=1}^{k} p_i \log p_i}{\log k} \quad . \tag{1}$$

Steady-state manpower systems had earlier been discussed in McClean (1977). Tyler (1984) reported that the McClean's steady-state model is a method for smoothing out the disturbing effects of short-term fluctuations in manpower systems. McClean and Abodunde (1978) used the transition probabilities generated from the McClean's steady-state model as inputs to the formula in equation (1) so as to measure the stability of the length of service in a system. The entropy measure in equation (1), denoted by H, lies in the closed interval $[0,1]$. $H = 0$ when $p_1 = 1$ and $p_2 = p_3 = \ldots = p_k = 0$ i.e. when all members of staff are recruits and always leave after their first years' service. The upper bound of entropy is achieved when there is an equal distribution of experience throughout the state, i.e. H =1 only when $p_1 = p_2 = \ldots = p_k = 1/k$. For intermediate values, the entropy measures the degree of experience which will be present in steady-state. The work of McClean and Abodunde (1978) was extended by Vassiliou (1984) by finding the probability of a person to be in a specific length of service class as $t \to \infty$; and was refined by McClean (1986) for a manpower system with continuous-time tenure profile. The McClean and Abodunde's entropy measure is rooted on the McClean's steady-state model and Shannon entropy formula. Tyler (1983) had criticized the McClean's steady-state model that it makes no mention of the size of the system. According to Ebrahimi *et al.* (2010), the information methodologies are often developed in isolation, i.e. a particular measure is used without consideration of the larger picture. Since the McClean and Abodunde's measure is based on Shannon (information-theoretic) entropy measure, the results obtained from the McClean and Abodunde's measure for manpower systems is incomprehensive. Omosigho and Osagiede (1999) applied McClean/Abodunde-type entropy to an organisation by assuming that the wastage rate in the system satisfies the log-normal model of Chu and Lin (1994). The entropy measures used by authors such as Omosigho and Osagiede (1999) and Vassiliou (1984) suffer from the same shortcoming of concentrating on one aspect of transitions within the manpower system which is primarily the survival of individuals from one tenure class to another within the system, while neglecting other transitions such as recruitment and wastage flows. This gives a partial picture of the behaviour of the system and a limited interpretation of the results.

Apart from the use of statistical entropy in manpower systems, the thermodynamic entropy has also been applied. Tyler (1989) developed an entropy measure for manpower systems based on the concepts of thermodynamics wherein the size of the manpower system is analogous to the absolute temperature. By considering the size of the tenure class in the McClean's steady-state manpower model, Tyler (1989) evaluated the entropy of a manpower system from the Boltzmann's formula as well as the internal energy of the manpower system given as

$$S = \frac{H_B}{\log c} \quad , \tag{2}$$

where H_B is the Boltzmann's formula expressed as

$$H_B = \log\left\{\frac{N!}{N_1!\,N_2!\ldots N_c!}\right\}, \tag{3}$$

$N_1 = N_1$, $N_k = N_1 \prod_{i=1}^{k-1} p_i$, $k > 1$, is the McCleans' steady-state number of members of the population belonging to each tenure class c, c is the number of tenure classes, $N = \sum_{i=1}^{c} N_i$ is the total size of the population, and p_i is the probability of surviving from one tenure class i to tenure class $i+1$. The thermodynamic entropy measure proposed by Tyler (1989) has a critical limitation emanating from the assumption that size of the manpower system is analogous to the absolute temperature. This assumption is unrealistic as it has no theoretical basis. More so, the mathematical complexities inherent in the application of the entropy measure limit its use only to the mathematically equipped researchers.

3. THE ADAPTIVE ENTROPY ESTIMATOR FOR MANPOWER SYSTEMS

In order to quantify the degree of stability in the modelled system, we solve problem **P**. The problem **P** is a modification of the Boltzmann distribution of energy (Kneen *et al.* 1972; and Toda *et al.* 1978) so as to capture the possible number of ways of selecting individuals moving from one state i to another state j in period t of a $k-$state space system as

$$W_i(t) = \frac{\left(\sum_{j=1}^{k} n_{ij}(t)\right)!}{\prod_{j=1}^{k}\left(n_{ij}(t)\right)!}. \tag{4}$$

For convenience, we suppress the time variable t in equation (4). Consider the contribution of state i to entropy of the system, denoted as S_i, i.e.

$$S_i = k_B \log_e \frac{\left(\sum_{j=1}^{k} n_{ij}\right)!}{\prod_{j=1}^{k}\left(n_{ij}\right)!}.$$

By performing some algebra with the factorial, it is easy to see that

$$S_i = k_B \left\{ \log_e \prod_{r=1}^{n_i} r - \sum_{j=1}^{k} \log_e \prod_{r=1}^{n_{ij}} r \right\}, \text{ where } n_i = \sum_{j=1}^{k} n_{ij}. \text{ Thus}$$

$$S_i = k_B \left\{ \sum_{r=1}^{n_i} \log_e r - \sum_{j=1}^{k} \left(\sum_{r=1}^{n_{ij}} \log_e r \right) \right\}. \tag{5}$$

Applying the Euler summation formula (see Lang 1993), we have for $f(k) = \log_e k$:

$$\sum_{k=1}^{n} \log_e k = \int_1^n \log_e t dt + \frac{1}{2}\log_e n + \int_1^n \frac{t - \left[\, t \,\right] - \frac{1}{2}}{t} dt, \tag{6}$$

where $\left[\, t \,\right]$ is the largest integer $\le t$, which has unit jumps at the integers $1,\ 2,\ 3, \ldots, n$. By considering the function $\left(t - \left[\, t \,\right] - \frac{1}{2}\right)$ which is the sawtooth function $P_1(t) = t - \left[\, t \,\right] - \frac{1}{2}$, in each of the sub-intervals $\left[1,2\right), \left[2,3\right), \ldots, \left[n-1, n\right)$, for $P_1(1) = -\frac{1}{2}$, $P_1(n) = \frac{1}{2}$ and $n \ge 15$, equation (6) is approximated as follows

$$\sum_{k=1}^{n} \log_e k \approx n \log_e n - \int_1^n dt + \frac{1}{2}\log_e n + \lim_{\varepsilon \to 0^+} \sum_{k=1}^{n} \int_{k-1+\varepsilon}^{k-\varepsilon} \frac{\left(t - \frac{1}{2}\right) - \left(k-1\right)}{t} dt \sim n \log_e n - \sum_{k=1}^{n} \left(k-1\right)\left(\sum_{r=1}^{\infty} \frac{1}{k^r}\right).$$

Therefore

$$\sum_{k=1}^{n} \log_e k \sim n \log_e n - n, \tag{7}$$

where the twiddle sign ~ means that $\sum_{k=1}^{n} \log_e k$ in relation (7) approaches the value on the right-hand side. From the result in relation (7), we rewrite equation (5) as $S_i \sim k_B \left\{ n_i \log_e n_i - \sum_{j=1}^{k} n_{ij} \log_e n_{ij} \right\}$. Notice that we have earlier suppressed the time variable t in n_{ij}. Introducing the time variable, we obtain

$$S_i \sim k_B \left\{ n_i(t) \log_e n_i(t) - \sum_{j=1}^{k} n_{ij}(t) \log_e n_{ij}(t) \right\}. \tag{8}$$

By substituting the constraint $\hat{n}_{ij}(t) = n_i(t)\sigma_{ij}^*(t),\ i = 1,2,\ldots,k;\ j = 1,2,\ldots,k$, in relation (8), we have

$$S_i \sim -k_B\, n_i(t) \sum_{j=1}^{k} \sigma_{ij}^*(t) \log_e \sigma_{ij}^*(t). \tag{9}$$

The entropy therefore for the entire system is $S \sim -k_B \sum_{i=1}^{k} n_i(t) \sum_{j=1}^{k} \sigma_{ij}^*(t) \log_e \sigma_{ij}^*(t)$. Taking $\left\langle \Lambda(t) \right\rangle$ as the value on the right-hand side, we obtain

$$\left\langle \Lambda(t) \right\rangle = -k_B \sum_{i=1}^{k} n_i(t) \sum_{j=1}^{k} \sigma_{ij}^*(t) \log_e \sigma_{ij}^*(t). \tag{10}$$

Using the idea of McClean and Abodunde (1978), we have $0 \le -\dfrac{\sum_{j=1}^{k} \sigma_{ij}^*(t) \log_e \sigma_{ij}^*(t)}{\log_e k} \le 1$. Multiplying through by $n_i(t)$ and then summing over all i we get $0 \le -\dfrac{\sum_{i=1}^{k} n_i(t) \sum_{j=1}^{k} \sigma_{ij}^*(t) \log_e \sigma_{ij}^*(t)}{\log_e k} \le N(t)$. Since k_B is an arbitrary constant (Kneen et al. 1972), we define k_B in equation (10) for the manpower system as $k_B = \dfrac{1}{N(t) \log_e k}$, where $N(t) = \sum_{i=1}^{k} n_i(t) < \infty$, so as to constrain $\left\langle \Lambda(t) \right\rangle$ in the range $[0,1]$. Hence,

$$\left\langle \Lambda(t) \right\rangle = -\sum_{i=1}^{k} \frac{n_i(t)}{N(t) \log_e k} \sum_{j=1}^{k} \sigma_{ij}^*(t) \log_e \sigma_{ij}^*(t). \tag{11}$$

We refer to equation (11) as the adaptive entropy estimator (AEE) for a $k-$state space system. AEE is a refinement of the entropy rate for Markov processes in Ciuperca and Girardin (2005) as the steady-state probabilities are now being replaced by $\dfrac{n_i(t)}{N(t) \log_e k}$.

We make propositions about the upper and lower bounds of the AEE and provide the required proofs.

Proposition 1: *At maximum entropy, an individual can access every state of the system with equal probability.*

Proof

We shall maximise the constrained discrete countable contribution of state i to entropy as follows:

$$\left. \begin{array}{l} \text{Maximize} \\ \qquad -\dfrac{1}{\log_e k} \displaystyle\sum_{j=1}^{k} \sigma_{ij}^*(t)\ \log_e \sigma_{ij}^*(t) \\ \text{subject to} \\ \qquad \displaystyle\sum_{j=1}^{k} \sigma_{ij}^*(t)\ = 1 \end{array} \right\}. \tag{12}$$

Applying the Lagrangian method, we obtain

$$\lambda = -\frac{1}{\log_e k}\left(1+\log_e \sigma^*_{ij}(t)\right), \text{ for each } j, \tag{13}$$

where $\sum_{j=1}^{k}\sigma^*_{ij}(t) = 1$, and λ is the Lagrange multiplier. The consequence of equation (13) is $\sigma^*_{i1}(t) = \sigma^*_{i2}(t) = \dots = \sigma^*_{ik}(t) = \tau$, say.

Thus, from the constraint in problem (12), we get

$$\sum_{j=1}^{k}\tau = 1 \Rightarrow \tau = \frac{1}{k}.$$

So,

$$\sigma^*_{ij}(t) = \frac{1}{k}, \text{ for each } j. \tag{14}$$

Now, for each i, we have from equation (14) that $\sigma^*_{1j}(t) = \frac{1}{k}, \sigma^*_{2j}(t) = \frac{1}{k}, \dots, \sigma^*_{kj}(t) = \frac{1}{k}$. Summing over all i yields

$$\sum_{i=1}^{k}\sigma^*_{ij}(t) = 1. \tag{15}$$

From the constraint in problem (12) and the results obtained so far, we conclude that an individual can reach every state of the system with equal probability when entropy is at its maximum.

Proposition 2: *In a $k-$state space manpower system which strives for long-term survival, minimum entropy occurs when the flows are stagnated (i.e., $p_{ii}(t) = 1$) and no wastage occurs.*

Proof

We prove this proposition by considering the imbedded Markov chain, $\boldsymbol{\sigma}^*(t) = (\sigma^*_{ij}(t))$ (see Tsaklidis 1994). Suppose the flows are stagnated and no departure occurs. Then $p_{ii}(t) + \sum_{i\neq j=1}^{k} p_{ij}(t) + p_{i0}(t)\sum_{j=1}^{k} p_{0j}(t) = 1$, and $\sum_{j=1}^{k} p_{0j}(t) = 1$, implies that: $p_{ii}(t) = 1$ and $p_{i0}(t) = p_{ij}(t) = 0$ for $i \neq j$. Therefore $p_{ii}(t) = 1$ forms a stochastic matrix whose states are absorbing. Thus, the entropy value is minimum. Suppose $p_{ii}(t) \neq 1$. Then $p_{i0}(t)p_{0i}(t) \leq 1$. If $p_{i0}(t)p_{0i}(t) = 1$, then we obtain the assertion for minimum entropy as in McClean and Abodunde (1978). But $p_{i0}(t)$ and $p_{0i}(t)$ cannot be simultaneously equal to one because this would mean that $p_{ij}(t) = 0$, for all i and j, which is unrealistic as manpower system strives to attain long-term survival. If $p_{i0}(t) \neq 0$ and $p_{0i}(t) \neq 0$, then wastage occurs so that recruitment is done. This is a contradiction. Hence, $p_{ii}(t) = 1$.

4. UTILITY OF THE AEE

In this section, we demonstrate the utility of the AEE with Matlab R2007b. We achieve this by collating and tabulating enrolment data from the Senate approved results for each session of a part-time undergraduate programme in the University of Benin, Nigeria as flows. The sessions from 2003/2004 to 2008/2009 are chosen for data (Table I). From Table I, there are new entrants only into level 1 and level 2 of the programme and some figures are in parenthesis. The figures in parenthesis denote the number of graduates.

Table 1. Enrolment data from 2003/2004-2008/2009 at the end of each session

Session	$i\,/\,j$	1	2	3	4	5	6	n_{i0}	$n_i(t)$
2003/2004 Session	n_{0j}	112	4	0	0	0	0	—	116
	1	0	112	0	0	0	0	0	112
	2	0	0	53	0	0	0	0	53
	3	0	0	0	56	0	0	0	56
	4	0	0	0	0	30	0	0	30
	5	0	0	0	0	0	35	0	35
	6	0	0	0	0	0	8	10 (10)	18
2004/2005 session	n_{0j}	110	—	0	0	0	0	—	110
	1	0	106	0	0	0	0	4	110
	2	0	0	90	0	0	0	22	112
	3	0	0	0	45	0	0	8	53
	4	0	0	0	0	48	0	8	56
	5	0	0	0	0	0	26	4	30
	6	0	0	0	0	0	8	35 (28)	43
2005/2006 session	n_{0j}	236	—	0	0	0	0	—	236
	1	0	234	0	0	0	0	2	236
	2	0	0	78	0	0	0	28	106
	3	0	0	0	87	0	0	3	90
	4	0	0	0	0	45	0	0	45
	5	0	0	0	0	0	43	5	48
	6	0	0	0	0	0	13	21 (19)	34
2006/2007 session	n_{0j}	353	—	0	0	0	0	—	353
	1	0	346	0	0	0	0	7	353
	2	0	0	226	0	0	0	8	234
	3	0	0	0	78	0	0	0	78
	4	0	0	0	0	87	0	0	87
	5	0	0	0	0	0	43	2	45
	6	0	0	0	0	0	20	36 (27)	56
2007/2008 session	n_{0j}	471	180	0	0	0	0	—	651
	1	0	470	0	0	0	0	1	471
	2	0	0	404	0	0	0	2	406
	3	0	0	0	211	0	0	15	226
	4	0	0	0	0	78	0	0	78
	5	0	0	0	0	0	80	7	87
	6	0	0	0	0	0	35	28 (27)	63
2008/2009 session	n_{0j}	181	22	0	0	0	0	—	203
	1	0	179	0	0	0	0	2	181
	2	0	0	489	0	0	0	3	492
	3	0	0	0	397	0	0	7	404
	4	0	0	0	0	205	0	6	211
	5	0	0	0	0	0	67	11	78
	6	0	0	0	0	0	44	71 (65)	115

Source: Approved Results by Senate of University of Benin Nigeria for B.Sc. Statistics with Computer Science, Department of Mathematics.

Working from Table I, we code the sessions $t = 2003\,/\,2004$, $\ldots, 2008\,/\,2009$ as $t = 1, \ldots, 6$, and compute the imbedded transition matrix for each session of the system. The block structure of transition matrices is made up of two matrices the direct transition between levels, and the part of wastage flow that goes back into the system as new entrants.

The main diagonal elements of the matrices are either zero or relatively small, while the upper off-diagonal elements ('promotion' probabilities) are large. The reasons for this are that there is no repetition in the system except in the final year, and there is a normal progression to the next higher level while the few students not promoted drop-out of the programme. Entries in columns 1 and 2 represent the part of wastage flow that goes back into the system as new entrants. Zero entries represent absence of transition.

Next, we calculate the entropy values for each session using the AEE and the McClean/Abodunde's formula. The results are depicted in fig. 1. From fig. 1, the results of the McClean/Abodunde's formula indicate a very high stability for the data.

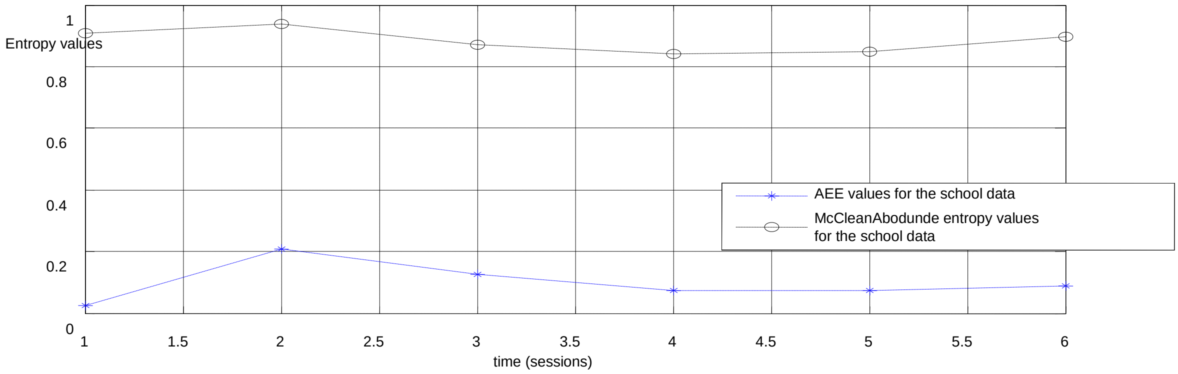

Figure 1. Graph of the AEE and Mc CleanAbodunde entropy values for the school data

The implication of this result is that the part-time programme is near a steady-state so that the transition processes tend to be uniform and stationary. This is misleading as the transition matrices are not only sparse, but they show a high progression rates with variable degrees of fluctuations. To see this, we test the constancy of the transition matrices using the chi-square test statistic as in Zanakis and Maret (1980). We obtain the calculated chi-square value as 393.4455. Since the number of time periods is six sessions for the six-year-graded system, the number of degrees of freedom for the test statistic is 150. This value (150 degrees of freedom) is large so the critical value for α percentile is computed using

$$\chi^2_\alpha = \frac{1}{2}\left(z_\alpha + \sqrt{2k-1}\right)^2 \quad , \quad k > 30\,,$$

where k is the number of degrees of freedom and z_α is the corresponding percentile of the standard normal distribution (Lindgren, 1993). We obtain the critical value at the 5% significance level as $\chi^2_{0.95} = 179.\ 2958$. The calculated chi-square value is greater than the critical value, so we conclude that the transition matrix is not stationary over the period of investigation at 5% significance level. In this light, the system is yet to achieve stability. This is a contradiction to the result of McClean/Abodunde entropy formula. However, the AEE values from fig. 1 indicate low stability in the level of accessibility during the period as the values are less than 0.5. From the foregoing, the AEE gives a better picture of the system in that the inference from it agrees well with the chi-square decision.

5. CONCLUSION

In this paper, we have considered the stability problem for the individual stocks and flows in graded manpower systems. The main contributions of this study include the construction of an adaptive entropy estimator (AEE) for manpower systems and the additional information provided on the conditions for zero entropy in graded systems (Proposition 2). Unlike the traditional McClean/Abodunde-type entropy measure which concentrates on one aspect of the transition process, the AEE explores various aspects of transitions that cannot be ignored in any realistic description of how a hierarchical manpower system works. In the numerical illustration, it is found that the AEE gives a better picture of the system than the McClean/Abodunde-type entropy as the inference from it agrees well with the chi-square decision. Although a case has been studied in this paper, we are optimistic that more interesting features of the AEE will be obtained when it is applied to large manpower systems. Notwithstanding the AEE has a fundamental limitation arising from the underlying assumption of the imbedding formulation. The limitation is that the AEE is only applicable to systems where recruitment is done to replace wastage and to achieve the desired growth.

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