# Is the Trailing-Stop Strategy Always Good for Stock Trading?

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**Abstract** — This paper characterizes the trailing-stop strategy for stock trading and provides a simulation model to evaluate its validity. Based on a discrete time computational model, we perform probabilistic analyses of the risks, rewards and trade-offs of such a trading strategy. Numerical examples using real data from the S&P 500 and the Dow Jones Industrial Average indicate that the trailing-stop strategy may be dominated by the portfolio containing risk-free assets and stocks, if prices are assumed to follow geometric Brownian motion.

Keywords --- Stock trading, trailing-stop, geometric Brownian motion

#### 1. INTRODUCTION

After entering the market, the appropriate time of exiting is a critical decision by traders. Many researchers advocate the trailing-stop strategy in which the exit price is predetermined, and traders exit the market when the market price drops to a stop price lower than the corresponding stage price when price fluctuates (see Iglehart and Stone (1983), Glynn and Iglehart (1995), Acar and Satchell (2002)). Glynn and Iglehart (1995) studied the trailing stop strategy in discrete and continuous-time cases. They assumed that, in the discrete case, the price process is a geometric random walk generated by binomial or double exponential random variables, while in the continuous case the price process follows geometric Brownian motion. They discussed the properties of the optimal strategy, such as the mean and variation of the duration from the current price to stop. Following their work, many different methods have been used to identify the optimal trailing-stop strategy. Yin *et al.* (2008) developed stochastic approximation algorithms to estimate the optimal trailing stop percentage in a continuous case. They verified the derived optimal trailing-stop strategy by looking at real data and comparing it with a moving-average strategy. They found that the average return from a trailing-stop was 71.45%, while the average return from a moving average was only 11.45%; trailing stop therefore outperformed significantly. Abramov *et al.* (2008) described the features of an optimal trailing-stop strategy in a discrete setup by assuming a binomial distribution of prices. They analyzed the relations among important statistics under certain conditions and claimed that there may not exist a trading strategy generating positive discounted gains.

In this paper we are trying to examine trailing stop strategies by following the steps advocated by Warburton and Zhang (2006). In their paper, the authors illustrated an investment strategy from a powerful theoretic computational model, and showed how the terminal distribution can be used to compute a variety of probabilistic risk and reward measures of interest to an investor. We not only enrich the methods of trailing-stop strategies, but also provide a real-data-based simulation of our theoretic model to locate the optimal trailing-stop strategy, and thus to verify its validity.

We consider discrete time wandering processes that move with a "shape" whose form is given by a finite subset  $\Omega$  of the integer grid points in the plane, called a "basic state space" or "virtual state space". Each grid point can be denoted by (x,t). The process starts out at an initial point in  $\Omega$  and moves with time until it hits a "target state" or a "termination state". As soon as it hits a termination state, the process terminates. As soon as it hits a target state, the process begins again in the following sense: if it hits target state (x,t), then the process can now move in the translated region  $(x,t) + \Omega$ ; if it hits a termination state in the translated region, the process stops; if it hits a target state (y,s) in the translated region, the process restarts. It now can move in the region  $(x,t) + (y,s) + \Omega$ , etc. A wandering process is completely determined by the probability distribution of the next-step states and the  $\Omega$ . An example of  $\Omega$  shown in Figure 1 has a trinomial probability distribution of the next-step states and an  $\Omega$  with a horizontal upper target state boundary denoted by K, a

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horizontal lower termination state boundary denoted by L, and a vertical termination boundary denoted by T.<sup>1</sup>

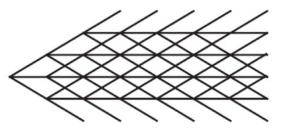


Figure 1. Possible price movements for a case of T = 7, K = 3, L = 2

When the process starts (n = 1), there is only one initial point thus one virtual state space. Since all points on its upper boundary could be the initial points for the next cycle (n = 2), there are multiple virtual state spaces and they are partially overlapped. Therefore Figure 2 shows the wandering process generated by the  $\Omega$  depicted in Figure 1.

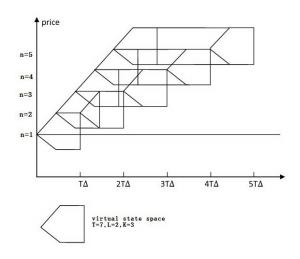


Figure 2. Real state space for a horizon of  $5T\Delta$ 

Our study on the wandering process is motivated by evaluating "trailing-stop" strategies used in stock trading. In these strategies, a stop-loss order is periodically adjusted to lock in profits as the market moves in a favorable direction. For example, in a long position a trader might enter a market and place a stop-loss order 20% below the stock price, which is like the lower absorbing boundary. If the price increases by 10%, which is like the upper target boundary, and the trade has not yet been stopped out, the stop-loss order is raised to 20% below the current price. Finally, if the stock price languishes for too long, perhaps it's "going nowhere" and it is time to sell the asset. This process continues until the trade is finally stopped out.

To evaluate the performance of such a "trailing stop" strategy, we need to obtain the probability distribution of a set of feasible termination and non-termination states for a given planning horizon. In this paper we develop a procedure to compute this probability distribution.

The rest of the paper is organized as follows: section 2 provides the model development and main results; section 3 presents several numerical examples to illustrate use of the procedure; section 4 concludes.

# 2. NOTATION AND MODEL DESCRIPTION

Assume that we are given a basic state space  $\{(x,t): (x,t) \in \Omega\}$  and a probability distribution  $\{P(x,t) | (x,t) \in \Omega\}$ over the states in  $\Omega$ . First define three disjoint subsets of the basic state space. These states are associated with the wandering process.

 $\Omega_1$ : the "target" states.

<sup>&</sup>lt;sup>1</sup> It is also Fig.1 in Warburton and Zhang (2006).

 $\Omega_2$ : the "non target" absorbing or time T states.

 $\Omega_3$ : the "intermediate" states  $\Omega_3 = \Omega \setminus (\Omega_1 \cup \Omega_2)$ .

Then we define the following conditional probabilities for the underlying basic state space  $\Omega$ .

(i)  $P_1(x,t)$ : the probability that the basic T period trinomial process is in a state  $(x,t) \in \Omega_1$  given that the process reaches the target.

(ii)  $P_2(x,t)$ : the probability that the process is in a terminal state  $(x,t) \in \Omega_2$  given that the process does not reach the target.

(iii)  $P_3(x,t)$ : the probability that the process is in an intermediate state  $(x,t) \in \Omega_3$  given that the process does not reach the target.

Note that the probabilities  $P_1(x,t)$  and  $P_2(x,t)$  and  $P_3(x,t)$  can be computed using the procedure presented in Art and Zhang (2003). We now develop some important recursions for computing the probability distributions. Define a complete cycle of the process as the time between two consecutive target re-settings (translations). We know that the process terminates after n cycles according to a geometric distribution with probabilities  $U(n) = (1 - p)(p)^{n-1}, n \ge 1$ , where  $P = P(\Omega_1) = \sum_{(x,t)\in\Omega_1} P(x,t)$ .

We can write separate recursions for terminal and non-terminal states for the wandering process. The form of each recursion is the same. The following describes these two sets of recursions; one for terminal states at which the wandering process terminates at time t (f = 2) the other for states where the wandering process is still in process at time t (f = 3). Let  $P_f^{(n)}(x,t)$  be the probability that the process terminates after n cycles, and  $\Omega_f^{(n)}$  be the set of all price-time possibilities (hereinafter called states) if the wandering process terminates in n cycles.

**Theorem 1.** The probability that the process is in state (x,t), given that the process terminates in n cycles satisfies the recursion

$$P_f^{(n)}(x,t) = \sum_{\substack{(k,i)\in\Omega_1^{(n-1)}\\(x-k,t-i)\in\Omega_f}} P_1^{(n-1)}(k,i)P_f(x-k,t-i)$$
(1)

**Proof.** We start with the first cycle. Suppose the wandering process terminates in one cycle and consider any time  $t \leq T$ . Let  $(x,t) \in \Omega_f$  be given. Then  $P_f(x,t)$  is the probability that the process is in state  $(x,t) \in \Omega_f$  at time given that the wandering process terminates in one cycle.

If the wandering process terminates in 2 cycles, we can interpret the event as the first cycle succeeds and the second cycle fails, and

$$\Omega_{f}^{(n)} = \{(x,t) : (x,t) = (k,i) + (m,j), (k,i) \in \Omega_{1}, (m,j) \in \Omega_{f}\}$$
(2)

$$P_{f}^{(2)}(x,t) = \sum_{\substack{(k,i)\in\Omega_{t}\\(x-k,t-i)\in\Omega_{t}}} P_{1}(k,i)P_{f}(x-k,t-i)$$
(3)

is the probability that the process is in state (x,t) if the process terminates in 2 cycles. Suppose the wandering process terminates in 3 cycles. Then the first two cycles succeed and the third cycle fails. Let

$$\Omega_1^{(2)} = \{(x,t) : t = i + j, x = k + m, (k,i) \in \Omega_1, (m,j) \in \Omega_1\}$$

$$(4)$$

be the set of all price-time possibilities just at the end of 2 successful cycles. Let  $(x,t) \in \Omega_1^{(2)}$  and let  $P_1^{(2)}(x,t)$  be the probability that the process first reaches price level x at time t given that the first two cycles succeed at time (t-i) at level (x-k), so

$$P_1^{(2)}(x,t) = \sum_{\substack{(k,i)\in\Omega_1\\(x-k,t-i)\in\Omega_f}} P_1(k,i)P_1(x-k,t-i)$$
(5)

Let  $\Omega_f^{(3)}$  be the set of all price time possibilities (x,t) if the wandering process terminates in 3 cycles. That is  $\Omega_f^{(3)} = \{(x,t) : (x,t) = (k,i) + (m,j), (k,i) \in \Omega_1^{(2)}, (m,j) \in \Omega_f\}$ 

Let  $(x,t) \in \Omega_1^{(3)}$  and let  $P_f^{(3)}(x,t)$  be the probability that the process is in state (x,t) given that the process terminates in 3 cycles. Then

(6)

$$P_f^{(3)}(x,t) = \sum_{\substack{(k,i)\in\Omega_f^{(2)}\\(x-k,t-i)\in\Omega_f}} P_1^{(2)}(k,i)P_f(x-k,t-i)$$
(7)

Suppose the wandering process terminates in n cycles. Then the first (n-1) cycles succeed and the final cycle fails. Let

$$\Omega_1^{(n-1)} = \{(x,t) : t = i+j, x = k+m, (k,i) \in \Omega_1^{(n-2)}, (m,j) \in \Omega_1\}$$
(8)

be the set of all price time possibilities just at the end of (n-1) successful cycles. Let  $(x,t) \in \Omega_1^{(n-1)}$  and let  $P_1^{(n-1)}(x,t)$  be the probability that the process first reaches price level x at time t given that the first (n-1) cycles succeed. If the total time to the end of the first (n-2) cycles is i, then cycle (n-1) succeeds at time (t-i), so  $P_1^{(n-1)}(x,t) = \sum_{i=1}^{n-2} P_1^{(n-2)}(t,i) P_1(x-t,t-i)$ 

$$P_1^{(n-1)}(x,t) = \sum_{\substack{(k,i)\in\Omega_1^{(n-2)}\\(x-k,t-i)\in\Omega_1}} P_1^{(n-2)}(k,i)P_1(x-k,t-i)$$
(9)

Let  $\Omega_f^{(n)}$  be the set of all price time possibilities (x,t) if the wandering process terminates in n cycles. That is  $\Omega_f^{(n)} = \{(x,t): (x,t) = (k,i) + (m,j), (k,i) \in \Omega_1^{(n-1)}, (m,j) \in \Omega_f\}$ (10)

Let  $(x,t) \in \Omega_f^{(n)}$  and let  $P_f^{(n)}(x,t)$  be the probability that the process is in state (x,t) given that the process terminates in n cycles. Then equation (1) is achieved.  $\Box$ 

Based on this recursion, we can compute the joint probabilities as follows.

For f = 2, 3 we have the following joint probabilities:

 $Prob_{f}$  (process terminates in (n = 1) cycle and is in state  $(x, t) \in \Omega_{f}^{(1)}$ ) =  $U(1)P_{f}^{(1)}(x, t)$ , where  $P_{f}^{(1)}(x, t) = P_{f}(x, t)$ ;

 $Prob_{f}$  (process terminates in (n = 2) cycle and is in state  $(x, t) \in \Omega_{f}^{(2)}$ ) =  $U(2)P_{f}^{(2)}(x, t)$ ;

 $Prob_{_f}$  (process terminates in (n = 3) cycle and is in state  $(x, t) \in \Omega_{_f}^{(3)} = U(3)P_{_f}^{(3)}(x, t)$ ; etc..

So finally, for f = 2, 3 we have  $Prob_f$  (wandering process is in state (x,t)) =  $\sum_{\substack{n \ge 1 \\ (x,t) \in \Omega_f^{(n)}}} (U(n)P_f^{(n)}(x,t))$ .

The overall probability of being in state (x,t) is:

*Prob* (wandering process is in state  $(x,t) = \sum_{f=2}^{3} prob_{f}$  (wandering process is in state  $(x,t) = \sum_{f=2}^{3} prob_{f}$ 

Note that the above summations involve only finitely many n for any given t, if we assume that state transitions are always of the form  $(x,t) \rightarrow (y,t+1)$  since  $\Omega$  is finite.

Remark — summing over all x at a given t won't sum to 1 if it is possible that the wondering process has been terminated before t.

# 3. NUMERICAL EXAMPLE

In this section, numerical examples are presented to illustrate the results of our model and to locate the best trailing-stop strategy. Assume the asset price follows a geometric Brownian motion process and p, q, and r are the probabilities of an up movement, a down movement and a mid movement in a trinomial tree. Then  $p = \left(\frac{e^{\mu\Delta/2} - e^{-\sigma\sqrt{\Delta/2}}}{e^{\sigma\sqrt{\Delta/2}} - e^{-\sigma\sqrt{\Delta/2}}}\right)^2, \quad q = \left(\frac{e^{\sigma\sqrt{\Delta/2}} - e^{\mu\Delta/2}}{e^{\sigma\sqrt{\Delta/2}} - e^{-\sigma\sqrt{\Delta/2}}}\right)^2 \text{ and } r = 1 - p - q \text{ where } \mu \text{ is the mean, } \sigma \text{ is the standard}$ 

deviation and  $\Delta$  is the time step. Based on the S&P 500 data from 1871 to 2010, its mean,  $\mu$ , and standard deviation,  $\sigma$ , are 0.106185 and 0.189392.<sup>2</sup> Therefore the transitional probabilities, p, q and r, are 0.2604868, 0.4997845 and 0.2397286 respectively. In each simulation the stock price process is approximated over a four-week horizon, which is 20 business days,  $H = T \cdot \Delta$  with T = 20 and  $\Delta = 1$  day where T is the termination period. The stock's initial price is \$1. Once stop out, the assets will be invested into a risk-free bond market where they are all terminated at T. The risk-free rate of return,  $r_f$ , is assumed to be 0.02 and interest can accrue day by day. Thus the four-week gross return from a risk-free asset is 1.0015728.

<sup>&</sup>lt;sup>2</sup> The returns include dividends. Data can be found at Online Data Robert Shiller (http://www.econ.yale.edu/~shiller/data.htm).

#### 3.1 A fixed trailing-stop strategy

We start with a fixed trailing-stop strategy in which the absorbing period, TT, is fixed. When the trailing-stop strategy is applied, we first assume the strategy is checked out on a weekly basis. This implies TT is 5 and is the length of each cycle. Assume  $L_t$  is the stop-loss price level in time interval t, and  $K_t$  is the target price level in time interval t. For different values of  $L_t$  and  $K_t$ , the gross rate of return,  $\mu_x$  and the standard deviation,  $\sigma_x$  will be different as well. For example, when  $L_t = 1$  and  $K_t = 1$ ,  $\mu_x$  and  $\sigma_x$  are 1.0029 and 0.0250. When  $L_t = 2$  and  $K_t = 1$ ,  $\mu_x$  and  $\sigma_x$  are 1.0040 and 0.0333. As  $L_t$  and  $K_t$  both could change from 1 to TT, Table 1 reports the gross rates of return and Table 2 reports the standard deviations from all possible combinations of  $L_t$  and  $K_t$ .

$K_t \setminus L_t$	1	2	3	4	5
1	1.0029362	1.0040007	1.0042296	1.0042640	1.0042665
2	1.0029750	1.0035351	1.0036641	1.0036838	1.0036852
3	1.0028532	1.0032901	1.0033946	1.0034105	1.0034117
4	1.0027938	1.0032012	1.0032990	10033139	1.0033150
5	1.0027790	1.0031810	1.0032775	1.0032922	1.0032932

Table 1. Gross rates of return when  $\mu_r$  and  $\sigma_r$  are 0.106185 and 0.189392

Table 2. Standard deviations when  $\mu_x$  and  $\sigma_x$  are 0.106185 and 0.189392

$K_t \setminus L_t$	1	2	3	4	5
1	0.0249818	0.0332872	0.0346098	0.0347805	0.0347912
2	0.0251996	0.0294865	0.0302759	0.0303801	0.0303866
3	0.0238279	0.0272193	0.0278815	0.0279686	0.0279740
4	0.0231080	0.0263055	0.0269330	0.0270154	0.0270205
5	0.0229142	0.0260779	0.0266985	0.0267798	0.0267849

These two tables show the positive correlation between the gross rates of return and the standard deviations. The outcomes indicate that the optimal (K, L) = (1,5), which implies that the investors should stay in the stock market as possible as they can. This is the case when the returns on stock are more favorable than on risk-free asset. The outcomes of gross returns and standard deviations are reported in Table 3. If we only consider the S&P 500 data from 2000 to 2008, its mean and standard deviation are -0.01672 and 0.203870 respectively. The outcomes of gross returns and standard deviations are reported in Table 3. If we only consider the S&P 500 data from 2000 to 2008, its mean and standard deviation are -0.01672 and 0.203870 respectively. The outcomes of gross returns and standard deviations are reported in Table 4 given the same values of T and TT (which are 20 and 5 respectively).

$K_t \setminus L_t$	1	2	3	4	5
1	1.0010350	1.0006129	1.0005144	1.0004985	1.0004974
2	1.0010163	1.0007835	1.0007259	1.0007165	1.0007158
3	1.0010586	1.0008717	1.0008241	1.0008163	1.0081570
4	1.0010788	1.0009026	1.0008576	1.0008503	1.0008497
5	1.0010836	1.0009093	1.0008674	1.0008575	1.0008570

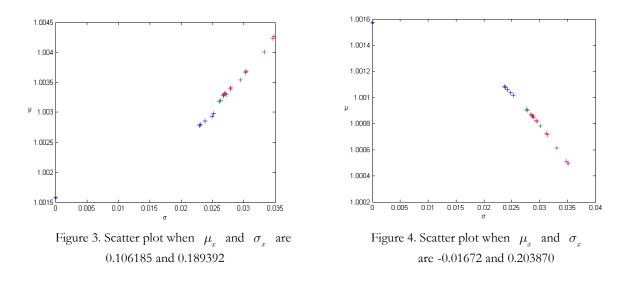
Table 3. Gross rates of return when  $\mu_r$  and  $\sigma_r$  are -0.010672 and 0.203870

$K_t \setminus L_t$	1	2	3	4	5
1	0.0247754	0.0331018	0.0347903	0.0350588	0.0350790
2	0.0252193	0.0300817	0.0311849	0.0313625	0.0313763
3	0.0242761	0.0283996	0.0293726	0.0295294	0.0295416
4	0.0238153	0.0277923	0.0287344	0.0288863	0.0288981
5	0.0237062	0.0276607	0.0285977	0.0287486	0.0287604

Table 4. Standard deviations when  $\mu_x$  and  $\sigma_x$  are -0.010672 and 0.203870

When mean and standard deviation are -0.01672 and 0.203870 respectively, the optimal (K,L)=(1,5), which implies that the investors should keep away from the stock market as much as they can. When K(L) increases given the values of L(K), it is more difficult to move to the next stage, which increases the probability of stopping out; at the same time, it is more difficult to stop out in the current stage. From Table 4, we notice that the return is generally increasing with K(L)given the value of L(K). This is not true for K=1 and 2 when L=1.

This relationship is displayed in Figure 3 and Figure 4 with different  $\mu_x$  and  $\sigma_x$ . The star on the vertical axis indicates the risk-free asset. Crosses are the outcomes from different  $L_t - K_t$  combinations.



In the above two cases TT is predetermined to be 5, though there is no evidence to show that 5 is the optimal value for TT.

# 3.2 A flexible trailing-stop strategy

Now we assume that only the termination period, T, is fixed and TT ranges from 1 to T. For any value of TT, by using the method from the previous subsection we are able to find different values of  $\mu_x$  and  $\sigma_x$  for different  $K_t$  and  $L_t$  as shown in Table 1 and 2 when TT is 5. Because agents are always better off with a higher rate of return and lower risk, we assume  $\mu_x$  is their first concern. Thus for any value of TT they choose the outcome with highest  $\mu_x$ . Table 5 and Figure 5 present the highest values of  $\mu_x$  with corresponding  $\sigma_x$  when TT changes from 1 to 20.  $\sigma_x$  is the number in the table multiplied by  $10^{-4}$ .

Table 5. $\mu_x$ and $\sigma_x$ with different <b>11</b>						
TT	1	2	3	4	5	
$\mu_{_x}$	1.0017827	1.0020179	1.0022690	1.0025294	1.0027898	
$\sigma_{_x}$	0.0054279	0.0079626	0.0100226	0.0118055	0.0133500	
TT	6	7	8	9	10	
$\sigma_{_x}$	1.0030402	1.0032731	1.0034843	1.0036728	1.0038388	
$\sigma_{_x}$	0.0146641	0.0157639	0.0166743	0.0174243	0.0180400	
TT	11	12	13	14	15	
$\sigma_{_x}$	1.0039826	1.0041073	1.0042156	1.0043098	1.0043915	
$\sigma_{_x}$	0.0185419	0.0189534	0.0192931	0.0195744	0.0198071	
TT	16	17	18	19	20	
$\mu_{x}$	1.0044620	1.0045222	1.0045727	1.0046135	1.0046438	
$\sigma_{_x}$	0.0199999	0.0201547	0.0202793	0.0203748	0.0204415	

Table 5.  $\mu_{-}$  and  $\sigma_{-}$  with different *TT* 

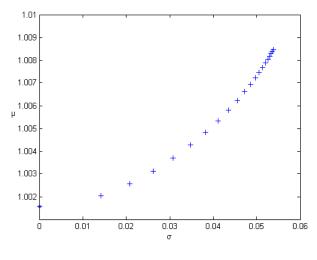


Figure 5.  $\sigma_x$  and  $\mu_x$  with different TT

From Table 5, we can see that the values of  $\mu_x$  and  $\sigma_x$  are monotonically increasing with TT. Figure 6 and 7 visualize the stop out probabilities when T is 5 and 20. When TT is 5, the pike of stop out probability appears at TT and then wanes until the termination period. When TT equals T, the asset will only be stopped out at the termination period. This implies that the longer the agents stay in the stock market, the higher return they can earn, though with higher risk.

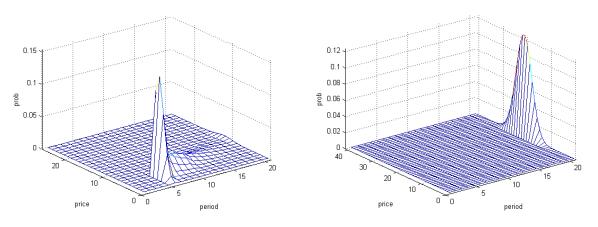
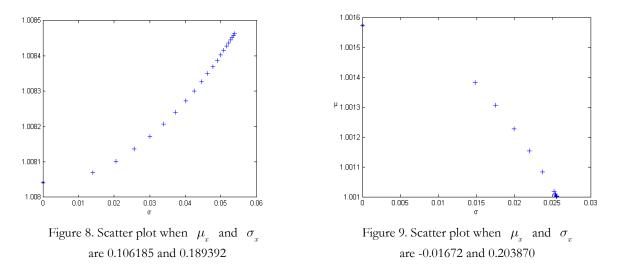


Figure 6. Stop out probabilities when TT is 5

Figure 7. Stop out probabilities when TT is 20

The above outcomes appear when the return from the stock market is favorable, that is,  $\mu$  is greater than  $r_f$ . If not, we may get different pattern of results. Figure 8 presents the outcomes when  $r_f = \mu_x$  while Figure 9 presents the outcomes when  $r_f$  remains at 0.02 but  $\mu_x$  and  $\sigma_x$  are -0.01672 and 0.203870 respectively.



Even though Figure 8 presents an upward sloping line, its slope at any given  $\sigma_x$  is much smaller than the one in Figure 5. An agent must take much more risk to pursue the same extra return from the stock market, making the trade-off line in Figure 5 is preferable. It also implies that even though stock market returns are favorable, a higher risk-free rate will flatter the positive trade-off line between  $\sigma_x$  and  $\mu_x$ , and the slope would turn negative after the situation changes, as shown in Figure 9. When stock market returns are not favorable, the best thing the agent can do is to keep away from it.

In the above simulation we changed the values of risk-free rates while keeping the return from the stock market constant. We can also do the reverse: change the values of  $\sigma_x$  and  $\mu_x$  by picking up different stocks but keeping the risk-free rate constant. We would end up with similar results.

#### 3.3 The optimal trailing-stop strategy

To locate the optimal strategy, we need introduce preference curves. These curves express the risk-return trade-off for investors in two-dimensional space. We are not only able to identify the optimal trailing-stop strategy given the knowledge of one stock's prices but also can use it to find the best stock to help the agent achieve the highest utility.

The optimal strategy is determined by the investors' preferences for higher returns to lower returns and less risk to more risk. All investors are assumed to be risk averse and to prefer more to less. Some representative curves for three

different types of investors are presented in Figure 10: more risk-averse, moderately risk-averse, and less risk-averse. The whole set of nested curves is omitted to keep the picture simple.

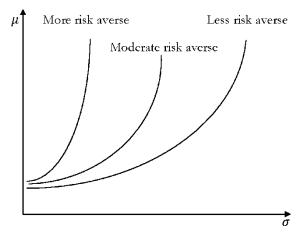


Figure 10. Preference over mean and standard deviation

Different functional forms are used to represent this mean-standard deviation preference. The essential feature of the function is that it must allow people to demand ever-increasing levels of return for assuming more risk. For the purpose of illustration, here we use the function derived by Mclaren (2009) for a much more realistic example of lognormally distributed assets, and constant relative risk aversion (CRRA) preferences.<sup>3</sup>

$$U(\sigma,\mu) = \frac{1}{1-\gamma} e^{(1-\gamma)\ln(\mu) - \frac{1}{2}\gamma(1-\gamma)\ln(1+\frac{\sigma^2}{\mu^2})}$$
(11)

in which  $\gamma$  is a positive parameter measuring the degree of risk aversion. The smaller is  $\gamma$ , the less risk averse the agent is. We assume  $\gamma$  for a more risk averse agent is 8, for a less risk-averse agent is 4. Graphically the optimal strategy is the point with  $(\sigma_x, \mu_x)$  which touches the highest indifference curve in Figure 5. Figure 11 shows the different optimal strategies for different type of investors. Because of the high returns of S&P 500 relative to risk-free assets, investors do not need to be very risk averse to be involved in the stock market ( $\gamma < 5$ ).

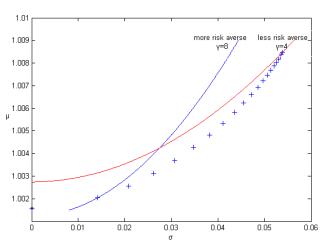


Figure 11. Optimal trailing-stop strategies

We only use one index in the above example, while in the real world there are many portfolios we can choose from. For example, we can use another famous index: the Dow Jones Industrial Average. Based on data of the adjusted close prices from 1928 to 2011, its mean,  $\mu$ , and standard deviation,  $\sigma$ , are 0.064273 and 0.194874. Intuitively, the S&P 500 yields

<sup>&</sup>lt;sup>3</sup> Please see MaLaren (2009) for detailed proof.

higher returns at almost the same risk. Applying the trailing-stop strategy on it, the graph of trade-off lines is shown in Figure 12.

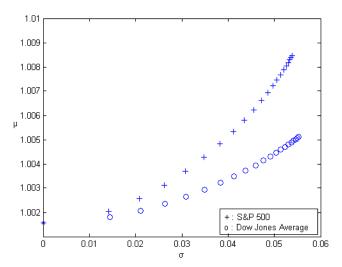


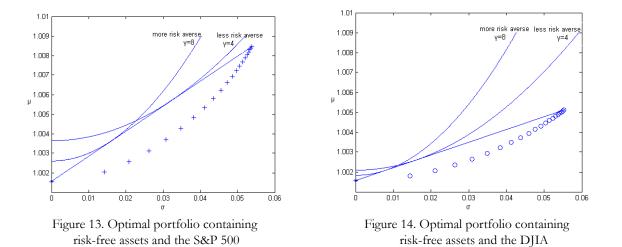
Figure 12. The trade-off  $(\sigma_x, \mu_x)$  of the S&P 500 and the Dow Jones Industrial Average

From Figure 12, the trade-off line of the S&P 500 is steeper than the one for the Dow Jones Industrial Average, which implies the S&P 500 is better for most investors on average. More specifically, because the trade-off line for the Dow Jones Industrial Average is under that for the S&P 500, investment in the Dow Jones Industrial Average is dominated by investment in the S&P 500, since the indifference curves which could be achieved facing the trade-off line of the S&P 500 are all higher than those of the Dow Jones Industrial Average.

#### 3.4 Efficiency of the trailing-stop strategy

Now we know how to locate an optimal trailing-stop strategy when the investor faces the option of entering the stock market or investing in risk-free assets. But the above analysis seems redundant, given the concave trade-off curve between expected value and standard deviation, as this trailing stop strategy is dominated by partially investing in risk-free assets and partially investing in the stock market. Therefore, if we assume that price follows geometric Brownian motion, the trailing-stop strategy may not be good for stock trading.

When partially investing in risk-free assets and partially investing in the S&P 500,  $(\mu, \sigma)$  of this portfolio must lie on the linear segment between the two extreme points, and the standard deviation of the rate of return on the risk-free asset is  $\theta$ . Assuming the same preferences as before, the optimal share of each asset can be found by setting the indifference curves tangent to the trade-off line, as shown in Figure 13. Not surprisingly, if investor is more risk averse  $(\gamma = 8)$ , the optimal portfolio contains 70.1% risk-free assets and 29.9% stock assets; if she is less risk averse  $(\gamma = 4)$ , her portfolio shall contain 39.9% risk-free assets and 60.1% stock assets. Both dominate the outcome from trailing stop strategy. Figure 14 shows the optimal portfolio of risk-free assets and the Dow Jones Industrial Average. If  $(\gamma = 8)$ , the optimal portfolio contains 85.5% risk-free assets and 14.5% stock assets; if  $(\gamma = 4)$ , the portfolio shall contain 70.9% risk-free assets and 29.1% stock assets. This is consistent with the intuition that the Dow Jones Industrial Average yields lower returns than the S&P 500, with similar risk. It is more risky to invest in the Dow Jones Industrial Average, thus investors tend to increase the share of risk-free assets in their portfolios.



## 4. CONCLUSION

Trailing-stop strategies are commonly used for stock trading, and many investors believe it is a good strategy to balance return and risk. Even though different assumptions lead to different optimal trailing-stop strategies for stock trading, the basic frameworks are very similar. This paper examines the validity of the trailing-stop strategy proposed by Warburton and Zhang (2006) in which they assume the trinomial tree of price moving which is a discretized description of geometric Brownian motion. After characterizing their model, we provide some numerical examples by using data from the S&P 500 and the Dow Jones Industrial Average to identify the optimal trailing-stop strategy under the same assumptions and trading rules. By fixing the duration to absorbing period and further relaxing this constraint, we locate the optimal trailing-stop strategies, we show that the trade-off line with maximum returns, given different absorbing duration from a trailing-stop strategy, is upward sloping when the return on the S&P 500 is more preferable than that on risk-free assets. Otherwise it is downward sloping, which implies that the best strategy is to keep away from the stock market. But the most interesting finding is that the upward trade-off line is concave, implying that no single trailing-stop strategy is optimal when compared to a portfolio containing both risk-free assets and S&P 500 stocks without any trailing stop. Therefore, under the framework of Warburton and Zhang (2006) and the assumption of geometric Brownian motion, the trailing-stop strategy is not ideal for stock trading.

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