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Managerial Efficiency with Disrupted Production System

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Abstract — Production system may be affected due to labor problem, manufacturing defects, machine breakdowns etc. and those reduce the reliability of system and also affect the goodwill of product. Before a production system disrupts, management needs to study the variation of demand pattern and customer arrival pattern. Many models are developed in literature with constant demand rate. In this paper, we incorporate variable demand rate and the uniform production rate both, and suggest a flexible managerial decision policy for a disrupted production system. The disruption based problem is solved analytically to determine production time before and after disruptions. An attractive feature of the approach is that both increasing and decreasing trends of demand are analyzed for deteriorating items with useful outcomes and results. A graph based simulation study is appended in order to find which of the model parameter is having most significant effect for a disrupted production system.

Keywords — Inventory, disrupted production system, deterioration, shortage

1. INTRODUCTION

Control and maintenance of a production system are challenging jobs for inventory managers and also attention oriented for researchers. There are many reasons who disrupt the production system like machine breakdown, supply chain disruption, unexpected events or acute crises. An oil-drilling company may be disrupted due to electricity supply, failure of drilling machines, labor strikes etc. Whereas oil refining company faces problem of crude oil supply, availability of other raw materials, earthquake and labor strike. So, management needs to make a policy/strategy to cope-up such type of problems. The amount received of input products may differ from the amount ordered, which also creates uncertainty in the system. In beginning the classic EOQ model does not include the chances of disruption in supply. Parlar and Berkin (1991) modeled for the economic ordered quantity under disruption in which demand is deterministic and also when inventory management has no stock and the supplier is down or lost. Berk and Arreola-Risa (1994) showed the cost function used in Parlar and Berkin's model (1991) is incorrect and provided the correct model.

Due to a disrupted production system, management not only fails to achieve the turnover but also loses creditability in the market resulting, that customers may turn to another product. Lin and Kroll (2006) solved the production problem under an imperfect production system subject to random machine breakdowns. They assumed that production rate and deterioration rate are fixed. Under this policy, the production runs is aborted when a breakdown occurs. The time-to-shift and the time-to-breakdown are two random variables follow different exponential distributions. Ma *et al.*, (2010) revisited the same idea with assumption that after a period the process may shift to an out-of-control state at random time, and machine produces defective item, and could not be repaired or reworked. Mishra and Singh (2011) considered Weibull distribution deterioration in disrupted production system and analyzed the model in different situations.

Market of any product depends on customers' responses, and quality of competitor's product. When supply change, company faces problems to fulfill the customer's demand otherwise they may turn to competitors' product. This reduces the market share of company and also reduces the profit. These aspects has been considered by Chen and Zhang (2010) and studied a three-echelon supply chain system which consists of suppliers, one manufacturer and other customers under demand disruption and optimizes the total average cost. They recommended to companies to run the stress test which involves estimating how the company will perform and which supplier should be selected under unusual market moves.

Teng and Chang (2005) presented an economic production quantity model for deteriorating items when the demand rate depends not only on display stock, but also on the selling price per unit of an item. Also demand rate may influence by

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economic policy, political scenario or agriculture productivity. Qi *et al.*, (2004) analyzed the supply chain-coordination with demand disruption in a deterministic scenario. Furthermore, supply shortages for the managerial purpose investigated by Yang *et al.*, (2005) and gave the solution by a greedy method. A number of structural properties of the inventory system are studied analytically by Samanta and Roy (2004) by determination of production cycle time and backlog for deteriorating item, which follows an exponential distribution. A central policy presented by Benjaafar and EIHafsi (2006) specifies a single product assemble-to-order system for components, an end–product to serve to customer classes. They solve the problem by Markov decision process and characterize the structure of an optimal policy. Hendricks and Singhal (2005) studied the average abnormal stock returns of firm's experienced high penalty, and shown that the supply-chain disruption could significantly affect the normal operation and financial health of the company. We refer some useful contribution to the reader, such as Howick and Eden (2001), Shukla *et al.*, (2010), Khedlekar (2012), Kumar and Sharma (2012).

In beginning, demand of computers increased exponentially and a similar story followed for mobile and other telecommunication products. So, demand rate of these products was going up day by day and even if production disrupted then the problem is going out-of-control. The two telecommunication companies, Nokia and Ericsson, which directly the source from Philips, took different remedial actions. The outcomes were drastically different. Nokia gained 3% market share while Ericsson lost market share (Wall Street Journal, 1/29/2011). The classic EOQ model consists of constant demand, however, the dynamic counterparts known as generalized economic order quantity model assuming time varying demand rate. Due to varying demand a motivation is derived to consider time dependent demand for deteriorating item and computed shortages, optimum time of placing an order, and optimal production time before and after getting disruption. We studied the exponential increasing and decreasing aspects both in a single model, also extend the model by obtained rates of change of production time before and after disruptions with respect to deterioration and other parameters.

2. ASSUMPTIONS AND NOTATIONS

Suppose that a deteriorating item manufactured by a single manufacturer and then sold to customers. Demand of product is to be assumed exponentially at rate μe^{ct} ($p - \mu e^{ct} \geq 0$), where c is demand rate parameter, and is a real number. If c is positive demand rate is increasing and if c is negative demand rate is decreasing. The production rate is constant at a rate p in each cycle, thus the on-hand inventory accumulated at a rate $p - \mu e^{ct}$. If the production stopped at the time (Tp) and then inventory depicted due to the demand and deterioration. During production disruption, if shortages occur, then it ordered from the spot market once in a cycle. Notations bearing the concepts utilized in the discussion are given as under:

- H : Time horizon.
- p : Production rate ($p \geq 0$).
- Δp : Change in production rate.
- μe^{ct} : Demand function of item ($p - \mu e^{ct} \geq 0$). If c is demand rate parameter, if it is positive then demand will be increasing and for negative demand will be decreasing.
- θ : Rate of deterioration.
- μ : Initial demand of item.
- T_p : Production time without disruption.
- T_d : Production disruption time when system get disruptions.
- T_p^d : New production time after system get disruptions.
- T_r : Time of placing the order when shortages occur.
- Q_r : Order quantity (shortages) for placing the order when shortage occurs.

3. MODEL WITHOUT DISRUPTION

First of all, management optimizes the production system run without disruption with the production rate p (per unit time), and stopped the production at time T_p and thereafter till time H. Inventory depicted due to demand rate (μe^{ct}) and deterioration rate (θ) of an item (Fig. 1). The interpretations of production system in differential equations for two periods [0, Tp] and [Tp, H] satisfies the following two Equations (1) and (2).

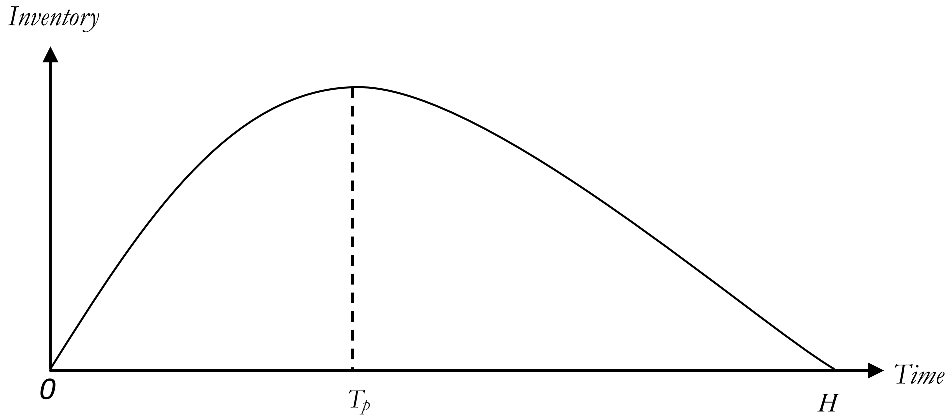

Figure 1. Normal production system without disruption

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = p - \mu e^{ct}, 0 \le t \le T_p, \text{ boundary condition } \quad I_1(0) = 0 \tag{1}$$

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -\mu e^{ct}, T_p \le t \le H, \text{ boundary condition } \quad I_2(H) = 0 \tag{2}$$

On solving Equations (1) and (2) with boundary conditions we get

$$I_1(t) = \frac{p}{\theta}\left(1 - e^{-\theta t}\right) - \frac{\mu}{c+\theta}\left(e^{ct} - e^{-\theta t}\right)$$

$$I_2(t) = \frac{\mu}{c+\theta}\left(e^{(c\ +\ \theta)H - \theta t} - e^{ct}\right) \tag{4}$$

As per Fig. 1 inventory level $I_1(t)$ and $I_2(t)$ are equal at time T_p, i.e. $I_1(Tp) = I_2(Tp)$ yields

$$T_p = \frac{1}{\theta}\log\frac{p\ \ c+\theta p\ \ -\theta\mu+\theta\mu e^{cH+\theta H}}{p\ \ c+p\ \ \theta} \tag{5}$$

If θ <<1, then production time without disruption is

$$T_p = \frac{\mu e^{cH}\left(1+\theta H\right) - \mu}{p\ \ c + p\theta - \mu\theta + \mu e^{cH}} \tag{6}$$

Corollary 1.
If θ <<1 then T_p is in increasing in θ.
By Equation (6) one can write

$$\begin{aligned}\frac{dT_p}{d\theta} &= \frac{pc\,\mu\,He^{cH} + \mu He^{2cH} - \mu\left(e^{cH} - 1\right)\left(p - \mu\right)}{\left(pc + p\theta - \mu\theta + \mu e^{cH}\right)^2} \\ &\ge \frac{pc\,\mu\,He^{cH} + \mu He^{2cH} - \mu pe^{cH}}{\left(pc + p\theta - \mu\theta + \mu e^{cH}\right)^2} \qquad \mathit{for} \quad p > p - H \\ &\ge \frac{p\,\mu\,e^{cH}\left(Hc - 1\right) + \mu He^{2cH}}{\left(pc + p\theta - \mu\theta + \mu e^{cH}\right)^2} \ge 0\end{aligned} \tag{7}$$

This proved the corollary*

As θ increases optimal production time T_p increases that is more products required to producing. One can conclude that to keep low deterioration is an effective way to keep the lower costs of production of items.

4. MODEL WITH DISRUPTION

In the section 3, production rate remains unchanged but in practice, production system is always disruption due to uncertainty and unplanned events, and thus we consider the production system little changed by, Δp and disruption time is T_d. If $\Delta p < 0$, then the production rate decreases and shortages occurs in the system. If $\Delta p > 0$, then the production rate increases and there is a surplus stock available in the system.

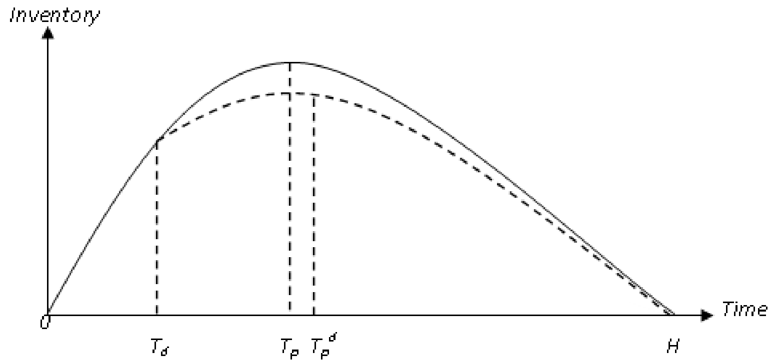

Figure 2. Disrupted production system

Lemma 1.

If $\Delta p \geq \left(p\left(c+\theta\right)e^{-\theta H} - p\left(c+\theta\right) + \mu\theta\left(e^{cH} - e^{-H\theta}\right)\right) / \left(c+\theta\right)\left(1 - e^{\theta T_d - H\theta}\right)$, then manufacturing system still satisfies the exponential demand even production system has been disrupted.

Otherwise if $-p \leq \Delta p \geq \left(p\left(c+\theta\right)e^{-\theta H} - p\left(c+\theta\right) + \mu\theta\left(e^{cH} - e^{-H\theta}\right)\right) / \left(c+\theta\right)\left(1 - e^{\theta T_d - H\theta}\right)$, then production system fails to satisfy the exponential demand that is there will be shortages due to production disruption.

Proof:

Suppose the production system disrupted at time T_d (see Fig. 2) and thus new production rate is $p + \Delta p$.
Presentations of two differential equations for time intervals [0, T_d] and [T_d, H] are given below

$$\frac{dI_1\left(t\right)}{dt} + \theta I_1\left(t\right) = p - \mu e^{ct}, 0 \leq t \leq T_d \text{, boundary condition } I_1\left(0\right) = 0 \tag{8}$$

$$\frac{dI_2\left(t\right)}{dt} + \theta I_2\left(t\right) = p + \Delta p - \mu e^{ct}, T_d \leq t \leq H \tag{9}$$

with boundary condition $I_1\left(T_d\right) = I_2\left(T_d\right) = \frac{p}{\theta}\left(1 - e^{-\theta T_d}\right) - \frac{\mu}{c+\theta}\left(e^{cT_d} - e^{-\theta T_d}\right)$

On solving Equation (9) with boundary condition we get on hand inventory

$$I_2\left(t\right) = \frac{p}{\theta}\left(1 - e^{-\theta t}\right) + \frac{\Delta p}{\theta}\left(1 - e^{\theta T_d - \theta t}\right) + \frac{\mu}{c+\theta}\left(e^{-\theta t} - e^{ct}\right) \tag{10}$$

If $I_2\left(H\right) \geq 0$, this means production system satisfy the exponential demand of items,

that is $\Delta p \geq \dfrac{p\left(c+\theta\right)e^{-\theta H} - p\left(c+\theta\right) + \mu\theta\left(e^{cH} - e^{-H\theta}\right)}{\left(c+\theta\right)\left(1 - e^{\theta T_d - H\theta}\right)}$, then still satisfy the demand.

If $I_2\left(H\right) < 0$, this means production system does not satisfy the exponential demand of items that is

$-p \leq \Delta p < \dfrac{p\left(c+\theta\right)e^{-\theta H} - p\left(c+\theta\right) + \mu\theta\left(e^{cH} - e^{-\theta H}\right)}{\left(c+\theta\right)\left(1 - e^{\theta T_d - H\theta}\right)}$, then there will be shortages in the system.

This proved the lemma.

Again if $I_2(H) \geq 0$, then we find optimal production time (with disruption) T_p^d such that at time *H* entire stock will be sold-out and inventory level would be zero.

If $I_2(H) < 0$, there will be shortages in the system and in this situation, we will find the optimum time *Tr* of placing the order (shortage) and respective order quantity will be *Qr*.

Lemma 2.

If $I_2(H) \geq 0$, then production time with disruption T_p^d is obtained by

$$e^{\theta T_p^d} = \frac{pc + p\theta - \mu\theta + \Delta p(c+\theta)e^{\theta T_d} + \mu\theta e^{c\theta + H\theta}}{(p+\Delta p)(c+\theta)} \tag{11}$$

Proof:

If $I_2(H) \geq 0$, that management has on hand inventory, or

$$\Delta p \geq \frac{p(c+\theta)e^{-\theta H} - p(c+\theta) + \mu\theta\left(e^{cH} - e^{-\theta H}\right)}{(c+\theta)\left(1 - e^{\theta T_d - H\theta}\right)} \tag{12}$$

Then, we will find out the optimal time T_p^d (see Fig. 3) when we stopped the production after disruption in such a manner that stock remains zero at time *H*. The presentations of two differential equations for intervals $[T_d, T_p^d]$ and $[T_p^d, H]$ are

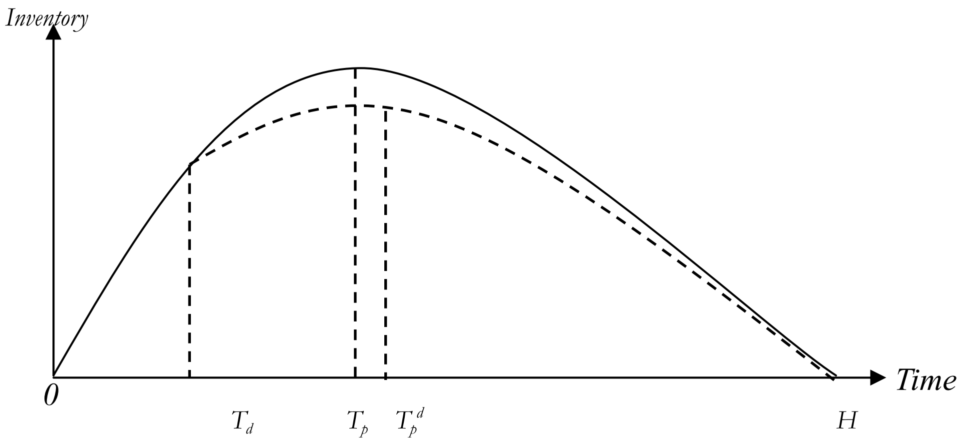

Figure 3. Production system after disruption, $0 \leq T_d \leq T_p \leq T_p^d \leq H$

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = p + \Delta p - \mu e^{ct}, \; T_d \leq t \leq T_p^d \tag{13}$$

Boundary condition $I_1(T_d) = I_2(T_d) = \frac{p}{\theta}\left(1 - e^{-\theta T_d}\right) - \frac{\mu}{c+\theta}\left(e^{cT_d} - e^{-\theta T_d}\right)$

$$\frac{dI_3(t)}{dt} + \theta I_3(t) = -\mu e^{ct}, T_d \leq t \leq H \quad \text{boundary condition } I_3(H) = 0 \tag{14}$$

On solving Equation (13) with boundary condition we get

$$I_2(t) = -\frac{p}{\theta}\left(e^{\theta T_d} - 1\right)e^{-\theta t} + \frac{\mu}{c+\theta}\left(e^{cT_d + \theta T_d} - 1\right)e^{-\theta t} + \frac{p+\Delta p}{\theta}\left(1 - e^{\theta T_d - \theta t}\right) - \frac{\mu}{c+\theta}\left(e^{-ct} - e^{(c+\theta)T_d - \theta t}\right) \tag{15}$$

$$I_3(t) = \frac{\mu}{c+\theta}\left(e^{cH + \theta H - \theta t} - e^{ct}\right) \tag{16}$$

Using condition $I_2(T_p^d) = I_3(T_p^d)$,Production time after disruption T_p^d is

$$e^{\theta T_p^d} = \frac{pc + p\theta - \mu\theta + \Delta p\left(c+\theta\right)e^{\theta T_d} + \mu\theta e^{c\theta + H\theta}}{\left(p+\Delta p\right)\left(c+\theta\right)} \tag{17}$$

This proved the lemma*

Corollary 2.

If $\Delta p \leq \left(p\left(c+\theta\right)e^{-\theta H} - p\left(c+\theta\right) + \mu\theta\left(e^{cH} - e^{-\theta H}\right)\right) / \left(c+\theta\right)\left(1 - e^{\theta T_d - H\theta}\right)$, then T_p^d is in increasing trend in T_p

Differentiating to Equation (11).

$$\frac{dT_p^d}{dT_d} = \frac{\Delta p e^{\theta T_d}\left\{\Delta p\left(c+\theta\right)e^{\theta T_d} + \mu\theta e^{c\theta + H\theta} + pc + \left(p-\mu\right)\theta\right\}}{\left(p+\Delta p\right)^2} \geq 0 \tag{18}$$

Therefore, increases in T_d leads the production time with disruption T_p^d increases that is reduced incurred cost.

Corollary 3.

If $\Delta p \geq \left(p\left(c+\theta\right)e^{-\theta H} - p\left(c+\theta\right) + \mu\theta\left(e^{cH} - e^{-\theta H}\right)\right) / \left(c+\theta\right)\left(1 - e^{\theta T_d - H\theta}\right)$, and θ <<1, then T_p^d is in increasing trend in parameter *c*.

If θ <<1, then $$T_p^d = \frac{\Delta p T_d - \mu + \mu e^{cH}}{c\left(p+\Delta p\right)} \tag{19}$$

i.e. $$\frac{dT_p^d}{dc} = \frac{\mu + \mu e^{cH}\left(Hc^2 - 1\right)}{\left(p+\Delta p\right)c^2} \geq 0 \tag{20}$$

If parameter *c* increases then production system has to manufacture more items.
This proved the corollary*

Lemma 3.

If $I_2\left(H\right) < 0$, then replenishment time T_r and order quantity Q_r are

$$e^{-\theta T_r}\left(pc + p\theta - \mu\theta + \Delta p\left(c+\theta\right)e^{\theta T_d}\right) + \mu\theta e^{cT_r} + \left(p+\Delta p\right)\left(c+\theta\right) = 0 \tag{21}$$

$$Q_r = I_3\left(T_r\right) = \frac{p+\Delta p}{\theta}\left(1 - e^{\theta H - \theta T_r}\right) - \frac{\mu}{c+\theta}\left(e^{cT_r} - e^{cH + \theta H - \theta T_r}\right) \tag{22}$$

Proof:

If $I_2\left(H\right) < 0$, then production system does not fulfill the exponential demand

or $-p \leq \Delta p < \dfrac{p\left(c+\theta\right)e^{-\theta H} - p\left(c+\theta\right) + \mu\theta\left(e^{cH} - e^{-\theta H}\right)}{\left(c+\theta\right)\left(1 - e^{\theta T_d - H\theta}\right)}$, that is there will be shortages in the system.

Suppose T_r and Q_r (see Fig. 4) are time of placing an order and order quantity respectively.

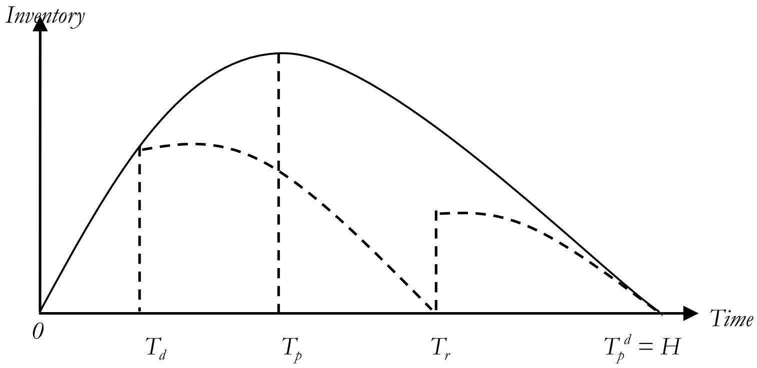

Figure 4. Production system after disruption, $T_p^d = H$

Then $I_2(T_r) = 0$ (by Equation (15)

$$\left(\frac{\Delta p - p}{\theta} e^{\theta T_r} + \frac{\mu}{c+\theta} e^{cT_r + \theta T_r}\right) = \frac{\mu}{c+\theta} - \frac{p}{\theta} - \frac{\Delta p}{\theta} e^{\theta T_d}$$

or $$e^{-\theta T_r}\left(pc + p\theta - \mu\theta + \Delta p\left(c+\theta\right)e^{\theta T_d}\right) + \mu\theta e^{cT_r} + \left(p + \Delta p\right)\left(c+\theta\right) = 0 \tag{23}$$

Then presentation of differential equation in this situation is

$$\frac{dI_3\left(t\right)}{dt} + \theta I_3\left(t\right) = p + \Delta p - \mu e^{ct}, T_r \le t \le H \quad \text{boundary condition} \quad I_3\left(H\right) = 0 \tag{24}$$

Above equation gives

$$I_3\left(t\right) = \frac{p+\Delta p}{\theta}\left(1 - e^{\theta H - \theta t}\right) - \frac{\mu}{c+\theta}\left(e^{cT_r} - e^{cH + \theta H - \theta t}\right)$$

Hence, the order quantity $Q_r = I_3(T_r)$, will be

i. e. $$Q_r = I_3\left(T_r\right) = \frac{p+\Delta p}{\theta}\left(1 - e^{\theta H - \theta T_r}\right) - \frac{\mu}{c+\theta}\left(e^{cT_r} - e^{cH + \theta H - \theta T_r}\right) \tag{25}$$

This proved the lemma*

Corollary 4.

If $-p \le \Delta p < \left(p\left(c+\theta\right)e^{-\theta H} - p\left(c+\theta\right) + \mu\theta\left(e^{cH} - e^{-\theta H}\right)\right) / \left(c+\theta\right)\left(1 - e^{\theta T_d - H\theta}\right)$,then T_r is decreasing in T_d

Differentiating to Equation (21)

$$\frac{dT_r}{dT_d} = \frac{\Delta p\left(c\theta T_r - c - \theta\right)}{-c\theta\left(-\theta e^{cT_r} + p + \Delta p + \Delta p T_d\right) + \mu} \le 0 \tag{26}$$

Therefore T_r is decreasing in T_d

This proved the corollary*

Corollary 5.

If $-p \le \Delta p < \left(p\left(c+\theta\right)e^{-\theta H} - p\left(c+\theta\right) + \mu\theta\left(e^{cH} - e^{-\theta H}\right)\right) / \left(c+\theta\right)\left(1 - e^{\theta T_d} e^{-H\theta}\right)$, Then Q_r is decreasing in T_d

Differentiating to Equation (16)

$$\frac{dQ_r}{dT_d} = \left\{\left(p + \Delta p\right)e^{\theta H - \theta T_r} - \frac{\mu\theta}{c+\theta}\left(c e^{cT_r} - \theta e^{cH + \theta H - \theta T_r}\right)\right\}\frac{dT_r}{dT_d} \le 0 \tag{27}$$

Since $\frac{dT_r}{dT_{d\,r}}$ is decreasing in T_d

This proved the corollary*

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5. APPLICATION AND SENSITIVE ANALYSIS

For application we assumed a particular case when constant production rate p = 350 units per day, initial demand of product μ = 300 units per day, disruption in production Δp = -150, rate of deterioration θ = 0.2, c = - 0.15, H = 30 days, and assumed that production system disrupted after T_d = 10 days.

On applying the proposed model we get $I_2(H)$ = 957.61 > 0 and thus by Equations (6) and (11), production time before disruption is T_p = 12.8 days, and production time after disruption is T_p^d = 38.78 days. The reproduction time after disruption is higher than the production time before disruption so, either the management needs to maintain more stocks to consume for 9 days.

Following figures shows the sensitiveness with respective to θ and T_d.

Case I: When $-p \leq \Delta p < \left(p\left(c+\theta\right)e^{-\theta H} - p\left(c+\theta\right) + \mu\theta\left(e^{cH} - e^{-\theta H}\right)\right) / \left(c+\theta\right)\left(1 - e^{\theta T_d - H\theta}\right)$

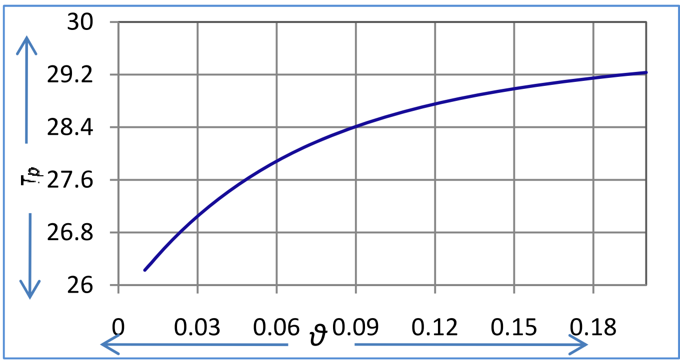

Figure 5. T_p with repective to θ

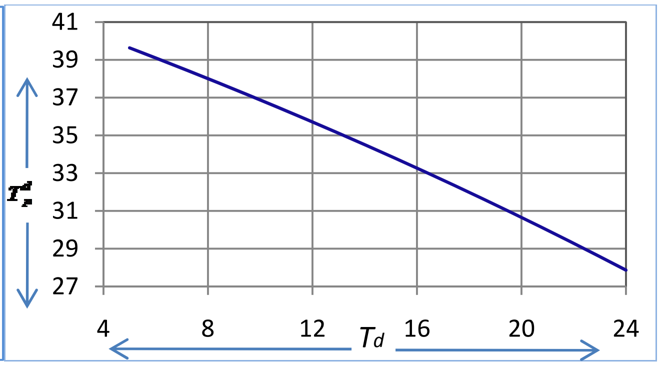

Figure 6. T_p^d with repective to T_d

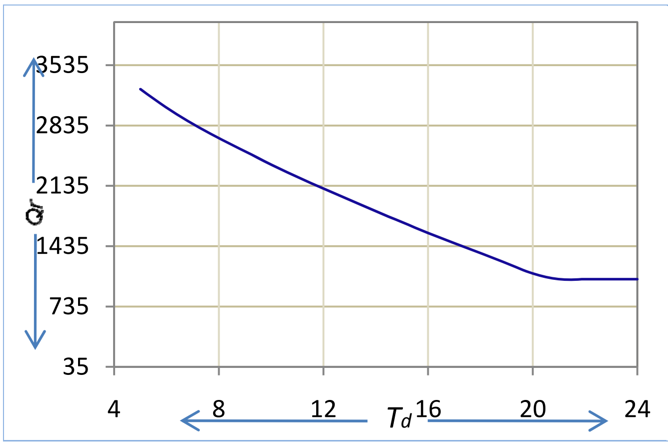

Figure 7. Q_r with repective to T_d

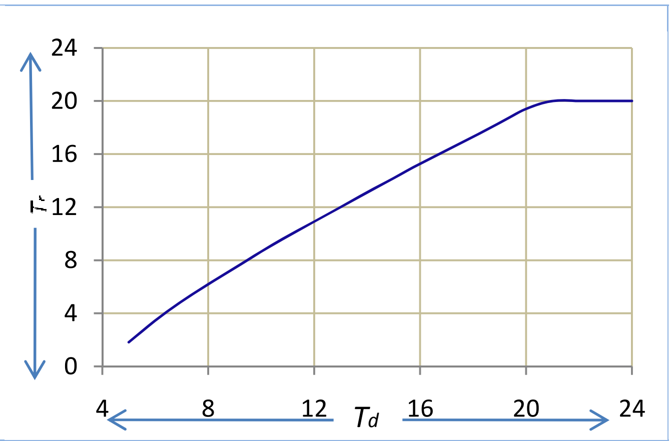

Figure 8. T_r with repective to T_d

From Fig. 5, T_p is increasing in θ, production time is direct proportional to deterioration means it needed to the manufacturer more items. So it is the effective way to reduce the cost as keeping lower deterioration and same followed for T_p^d (see Fig. 6). If $I_2(H) < 0$, then there are shortages occurs in the system and it needs to order quantity Q_r from the spot market at time T_r (see Fig. 7), also Q_r decereases in interval $4 \leq T_d \leq 20$, and there after remains constant. Time of placing the order (T_r) increases as T_d increases(see Fig. 8) in range $4 < T_d \leq 20$, and there after remains constant, and thus delay in disruption time produces fewer shortages which reduces the incurred cost.

Case II: When $\Delta p > \left(p\left(c+\theta\right)e^{-\theta H} - p\left(c+\theta\right) + \mu\theta\left(e^{cH} - e^{-\theta H}\right)\right) / \left(c+\theta\right)\left(1 - e^{\theta T_d - H\theta}\right)$

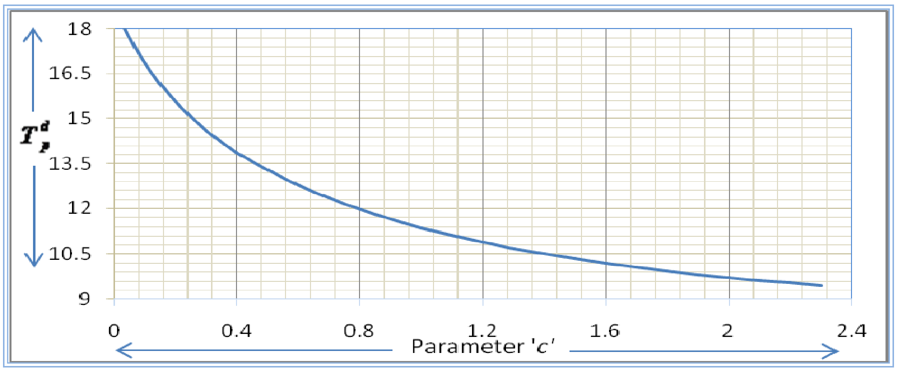

Figure 9. T_p^d with respective to parameter '*c*'

From Fig. 9, time T_p^d decrease as *c* increases, which lead that when demand, is in increasing trend management needs to start reproduction earlier than the time in which demand is in decreasing order. For *c* positive, the problem is easily solvable when production system gets disrupted earlier than the production system gets disrupted later, because of high demand and higher costumer expectations makes complexity for management. However, if *c* is negative, the outcome is just opposite because of less demand and lesser costumer's expectations.

If management could not effort to start the production earlier, then order quantity (Q_r) will be higher, which increases the total cost.

6. COMPARISON

To compare the performance of model we are taking different value of *c*. Consider positive and negative values for increasing and decreasing demand rates respectively.

Table 1. Comparison with increasing/decreasing trend of demand

Parameter 'c'	*Demand Trend*	$I_2(H)$	T_p	T_r	Q_r	T_p^d	$I_2(H)$
0.02	*Increasing*	$I_2(H)<0$	31.75	10.18	77524	-	-
-0.02	*Decreasing*	$I_2(H)>0$	26.75	-	-	32.49	99
0.03	*Increasing*	$I_2(H)<0$	33.03	10.73	34113.16	-	-
-0.03	*Decreasing*	$I_2(H)>0$	25.54	-	-	28.26	1769
0.05	*Increasing*	$I_2(H)<0$	35.61	08.79	303616	-	-
-0.05	*Decreasing*	$I_2(H)>0$	23.16	-	-	33.38	568
0.15	*Increasing*	$I_2(H)<0$	48.90	06.49	8388800	-	-
-0.15	*Decreasing*	$I_2(H)>0$	12.80	-	-	38.78	958
0.10	*Increasing*	$I_2(H)<0$	42.20	07.39	1755146	-	-
-0.10	*Decreasing*	$I_2(H)>0$	17.59	-	-	35.36	868

As per Table 1, one can observe those exponential increasing/decreasing demand rates are quite different. For increasing demand (c = 0.02) at rate μ = 300 unit per day gives $I_2(H)$ < 0 and order quantity Q_r = 77,524 units per day. Whereas for negative value of *c*, new production time is 32.49 days and there is surplus amount of inventory items. Thus for positive and negative value of *c*, $I_2(H)$ is negative and positive, respectively. Order quantity is highly sensitive to demand parameter *c*, but adverse to replenishment time, T_p^d and $I_2(H)$ both are highly sensitive to negative trend of demand. This means, if demand rate is increases management need to order more from the spot market beside this if demand rate decreases it need to stop the production earlier.

7. CONCLUSION, RECOMMENDATIONS AND FUTURE RESEARCH

In this paper, we proposed a production inventory model and analyzed the impact of exponential demand on disruption over a production system, and found that increasing demand pattern with disruption makes a significant role in comparisons to decreasing demand with disruption. Further study revealed that demand rate highly affects to the managerial policy for a disrupted production system.

If demand parameter c is negative then it is easy to solve the problem for management whenever system gets disrupted. If c is positive then demand rate increases and problem is complicated. For increasing demand rate the management needs to order more quantities of inventory item from the spot market. In contrary, for decreasing demand rate, it needs to stop the production. Thus, the performance of any disruption based production policy depends both on demand variations and production uncertainties.

The proposed model may be further extended by incorporating the more realistic assumptions like time dependent production along with probabilistic demand rate. Furthermore, one can consider a production system that generates defective items and having variable deteriorations with disruption.

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