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Optimal Procurement Policy With Trade Credit Financing, Capacity Constraints and Stock-dependent Demand

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Abstract — In this paper, optimum procurement policy is worked out when trade credit is offered to the retailer by the supplier and demand is stock-dependent. The own warehouse (*OW*) capacity is limited and excess inventory is stored in rented warehouse (*RW*). The rented warehouse has modern infra-structure facility. Hence, the unit holding cost in rented warehouse is assumed to be higher than that in the owned warehouse. A decision rule is constructed to hire a rented warehouse or not. The uniqueness of the optimal solution is discussed. The numerical examples are presented to validate the proposed problem. The managerial insights are deduced using sensitivity analysis. It is observed that the retailer should stock more and increase time for a higher ordering cost. On the other hand, lower ordering cost facilitates the retailer to go for owned warehouse only.

Keywords— Inventory, EOQ, stock-dependent demand, two-warehouse, trade credit.

1. INTRODUCTION

The trade credit analysis has attracted researchers. This promotional tool stimulates the demand, attracts customers. Goyal (1985) discussed an EOQ model under the condition of a permissible delay in payments. Shah *et al.* (2010) gave an up-to-date review article on trade credit. It has been proved by number of researchers that this promotional tool entice retailer to larger orders, so the question is where to stock this order. The review article by Shah *et al.* (2010) has citations which assumes that the retailer owns a single warehouse with unlimited floor space. However, due to high value of the space, any warehouse will have limited space.

Hartely (1976) discussed concept of two warehouses viz. owned warehouse (OW) and rented warehouse (RW).It was assumed that the units are first stocked in owned warehouse to its maximum capacity and rest in the rented warehouse. The holding cost of an item in the RW was assumed to be higher than OW. Sarma (1993) derived optimal strategies for two warehouses when replenishment rate is infinite. Goswami and Chaudhari (1992) allowed complete backlogged when demand increases linearly with time. Some other relevant articles are by Pakkala and Acharya (1992a, 1992b), Ishii and Nose (1996), Benkhrouf (1997), Yang (2004), Lee (2006), Yang (2006), Shah and Shah (2012) discussed inventory models in fuzzy environment.

Levin *et al.* (1972) quoted that "large piles of goods attract more customers". This scenario is termed as stock-dependent demand. Urban (2005) discussed optimal ordering policy when demand is stock-dependent. Zhou and Yang (2005) formulated a two-warehouse model with a stock-dependent demand. They incorporated transported cost incurred in transferring units from owned warehouse to rented warehouse. Zhou *et al.* (2012) discussed an uncooperative order inventory model for stock-dependent demand when trade credit is offered and limited one display space.

In this paper, the optimal ordering policy is derived for two-warehouse facility inventory system under stock-dependent demand and credit financing. The unit holding cost in rented warehouse (RW) is higher than that in the owned warehouse (OW). The objective function is profit maximization. The analytic results are derived to establish the existence and uniqueness of the cycle time. These results will aid to the decision maker to decide "whether or not to rent RW?" to stock larger order to maximize annual profits. This model can well set in the super malls, festival seasons etc. In next section, the notations and assumptions are given.

2. NOTATIONS AND ASSUMPTIONS

The following notations and assumptions are adopted to develop proposed mathematical model.

2.1 Notations

OW	Owned warehouse
RW	Rented warehouse
A	The ordering cost per order

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$R(I_k(t))$	The stock dependent demand rate $R(I_k(t)) = \alpha + \beta I_k(t)$,where $\alpha > 0$ denotes scale demand; $0 < \beta < 1$ denotes stock-dependent parameter; and $I_k(t)$ denotes inventory level at any instant of time t . where $k = 0$ or γ
W	The limited storage capacity of *OW*
P	The selling price per unit
C	The purchase cost per unit, with $C \le P$
h_0	The holding cost per unit time in *OW*
h_r	The holding cost per unit time in *RW*, $h_r \ge h_o$
T	The cycle time (a decision variable)
Q	The order quantity (a decision variable)
T_r	The time at which inventory reaches to zero in *RW*
T_o	$\frac{1}{\beta}\ln\left(1+\frac{\beta W}{\alpha}\right)$
M	The credit period offered by the supplier $M < T_o$
I_c	Interest charged per unit per annum
I_e	Interest earned per $ per annum
$Z(T)$	The annual net profit per unit time.

2.2 Assumptions

(1) The two-warehouse inventory system stocks single item.
(2) The planning horizon is infinite.
(3) Lead-time is zero or negligible. Shortages are not allowed.
(4) The *OW* has a finite capacity of W-units.
(5) The *RW* has infinite capacity.
(6) The items in *RW* are sold first.
(7) The holding cost of an item in *RW* is more than that in the *OW*.
(8) The supplier offers a credit period to the retailer. The retailer earns interest at the rate I_e on the generated revenue during permissible credit period. At the end of the credit period, the purchase dues are settled. On the unsold stock, the retailer has to pay interest charges at the rate I_c . (Shah (1993)).
(9) $I_o(t)$ denotes the inventory level during $(0, T_r)$ in the *OW*, in which the inventory is depleting due to stock-dependent demand. $I_r(t)$ denotes the inventory level during $(0, T_r)$ in the *RW*, in which the inventory level is changing due to stock-dependent demand. $I(t)$ is the inventory level during (T_r, T) and $I(T) = 0$.

3. MATHEMATICAL MODEL

In this section, we want to discuss conditions in which hiring *RW* is favorable to stock more items to maximize annual net profit. If $Q \le W$, then there is no need of *RW*. Otherwise, W units are stocked in *OW* and remaining in *RW*. Two scenarios are to be analyzed, viz. (A) the single-warehouse inventory system, and (B) the two-warehouse inventory system. If we denote $T_o = \frac{1}{\beta}\ln\left(1+\frac{\beta W}{\alpha}\right)$, the inequality $Q \le W$ holds if and only if $T_o \ge T$.

3.1 Single-warehouse inventory model $(T_o \geq T)$ (Fig.1)

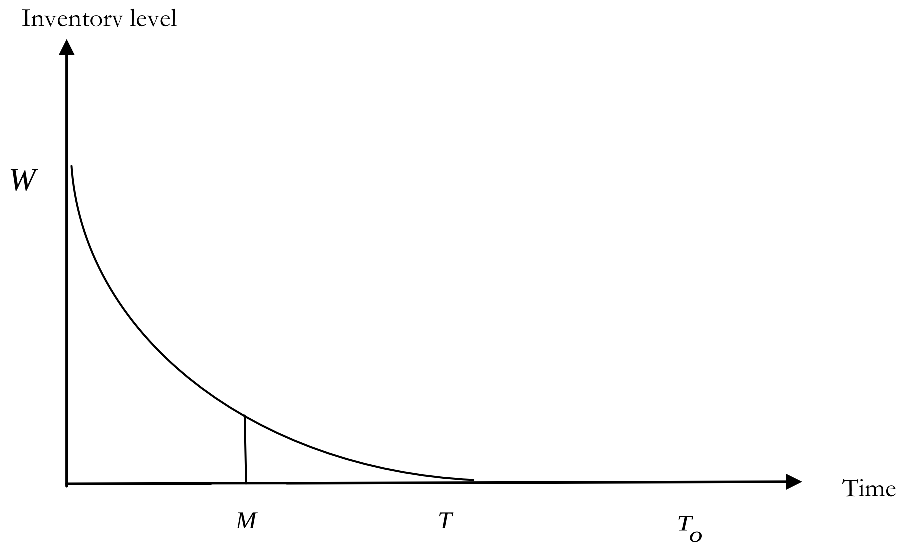

Figure 1. $T_o \geq T$

Initially, the inventory system has Q units. During the period $(0, T)$ the inventory level depletes in the *OW* due to stock-dependent demand. The rate of change of inventory level at any instant of time t is governed by the differential equation

$$\frac{dI_o(t)}{dt} + (\beta) I_o(t) = -\alpha, \ 0 \leq t \leq T. \tag{1}$$

Using boundary condition $I_o(T) = 0$, the solution of differential equation (1) is

$$I_o(t) = \frac{\alpha}{\beta}\left[e^{\beta(T-t)} - 1\right], \ 0 \leq t \leq T \tag{2}$$

and the purchase quantity is

$$Q = I_o(0) = \frac{\alpha}{\beta}\left[e^{\beta T_o} - 1\right] \tag{3}$$

The components of annual net profit per unit time of the inventory system are

(1) Sales revenue $= (P - C)\frac{Q}{T}$

(2) Ordering cost $= \frac{A}{T}$

(3) Holding cost in the *RW*=0 because only single-warehouse is used.

(4) Holding cost in the *OW* $= \frac{h_o}{T}\int_0^T I_o(t)dt = \frac{h_o \alpha}{\beta^2 T}\left[e^{\beta T} - \beta T - 1\right]$

For calculations of interest earned and interest charged two cases may arise.

Case 1: $T \leq M$

In this case, the retailer has sold all the units before the permissible delay period. So the interest charges are zero and the interest earned per unit time is $\frac{PI_e}{T}\left[\int_0^T R(I_o(t))tdt + Q(M-T)\right]$

Case 2: $M \leq T$

In this case, the retailer has to pay M. During $[0, M]$, the interest earned per unit time is $\frac{PI_e}{T}\left[\int_0^M R(I_o(t))tdt\right]$ and during $[M, T]$, the interest paid at the rate I_c on the unsold stock is $\frac{CI_c}{T}\int_M^T I_o(t)dt$.

The annual net profit per unit time, *Z(T)* is sales revenue minus sum of ordering cost, holding cost in *RW*, holding cost in *OW*, interest charged plus interest earned. Consequently, the net profit per unit time is

$$Z(T) = \begin{cases} Z_1(T), & \text{if } 0 < T < M \\ Z_2(T), & \text{if } M \leq \text{T} \end{cases} \tag{4}$$

where

$$Z_1(T) = (P-C)\frac{\alpha}{\beta T}\left[e^{\beta T}-1\right] - \frac{A}{T} - \frac{h_o\alpha}{\beta^2 T}\left[e^{\beta T}-\beta T-1\right] + \frac{PI_e\alpha}{T}\left[T^2+\left(e^{\beta T}-\beta T-1\right)+\frac{1}{\beta}\left(e^{\beta T}-1\right)\left(M-T\right)\right] \tag{5}$$

And

$$Z_2(T) = (P-C)\frac{\alpha}{\beta T}\left[e^{\beta T}-1\right] - \frac{A}{T} - \frac{h_o\alpha}{\beta^2 T}\left[e^{\beta T}-\beta T-1\right] + \frac{PI_e\alpha}{T}\left[M^2+\frac{1}{\beta^2}\left(e^{\beta T}-(1+\beta M)e^{\beta(T-M)}\right)-\frac{CI_c\alpha}{\beta^2 T}\left(e^{\beta(T-M)}-\beta(T-M)-1\right)\right] \tag{6}$$

One can check that $Z_1(M) = Z_2(M)$, Z(T) is well-defined and continuous. The first and second order derivative of $Z_1(T) = Z_2(T)$ are

$$Z_1'(T) = \frac{1}{T^2}\left\{\begin{array}{l} A+\dfrac{\alpha}{\beta^2}\left[(1-\beta T)e^{\beta T}-1\right]\left[h_o-(P-C)\beta-PI_e\beta\right] \\ +PI_e\alpha\left[T^2+Te^{\beta T}(M-T)-\dfrac{M}{\beta}\left(e^{\beta T}-1\right)\right]\end{array}\right\} \tag{7}$$

$$Z_1''(T) = \frac{1}{T^3}\left\{-2A-\alpha\left[\frac{2}{\beta}\left((1-\beta T)e^{\beta T}-1\right)+e^{\beta T}T^2\right]\left[h_o-(P-C)\beta-PI_e\beta\right]\right\} + \frac{PI_e\alpha}{T^3}\left[\beta T^2e^{\beta T}(M-T)-2MTe^{\beta T}+\frac{2M}{\beta}(e^{\beta T}-1)\right] \tag{8}$$

$$Z_2'(T) = \frac{1}{T^2}\left\{A+\frac{\alpha}{\beta^2}\left[(1-\beta T)e^{\beta T}-1\right]\left[h_o-(P-C)\beta-PI_e\beta\right]+\frac{CI_c\alpha}{\beta^2}\left[e^{\beta(T-M)}(1-\beta T)+\beta M-1\right]\right\}$$
$$+\frac{1}{T^2}\left\{PI_e\alpha\left[-M^2+\frac{1}{\beta^2}\left((1-\beta T)e^{\beta(T-M)}-(1-\beta T)e^{\beta T}+\beta Me^{\beta(T-M)}(1-\beta T)\right)\right]\right\} \tag{9}$$

$$Z_2''(T) = \frac{1}{T^3}\left\{-2A+\left(h_o-(P-C)\beta-PI_e\beta\right)\left[-2e^{\beta T}+3\beta Te^{\beta T}+2-\beta^2T^2e^{\beta T}\right]\right\}$$
$$+\frac{1}{T^3}\left\{\frac{CI_c\alpha}{\beta^2}\left[-2e^{\beta(T-M)}+2\beta Te^{\beta(T-M)}-2\beta M+2+\beta T^2e^{\beta(T-M)}\right]\right\}$$
$$+\frac{PIe\alpha}{T^3}\left[-M^2+\frac{1}{\beta^2}\left(-2e^{\beta(T-M)}+2e^{\beta T}-2\beta Me^{\beta(T-M)}-2\beta Te^{\beta T}+2\beta^2TMe^{\beta(T-M)}+2\beta TMe^{\beta(T-M)}\right)\right]$$
$$+\frac{PI_e\alpha}{T^3}\frac{1}{\beta^2}\left(\beta^2T^2e^{\beta T}-\beta^3T^2Me^{\beta(T-M)}-\beta^2T^2e^{\beta(T-M)}\right) \tag{10}$$

If $\beta = 0,$ equations (4) - (10) are same as those given in Liao and Huang (2010) without deterioration case. When $P = C$, then these equations are consistent with those given in Hwang and Shinn (1997). Using lemma 1 and 2 of Chung et al. (2001), we have $Z_1''(T) < 0$ for all T, and $Z_2''(T) < 0$ for all $T \geq M$ respectively. Hence, we have following theorem.

Theorem 1 Let $T \leq T_o$

1. $Z_1(T)$ is concave on $(0, M)$.
2. $Z_2(T)$ is concave on $[M, \infty)$.
3. $Z(T)$ is concave on $(0, \infty)$.

Also, let

$$\Delta = A + \frac{\alpha}{\beta^2}\left[(1-\beta M)e^{\beta M}-1\right]\left[h_o-(P-C)\beta-PI_e\beta\right] + PI_e\alpha\left[M^2-\frac{M}{\beta}\left(e^{\beta M}-1\right)\right]$$

Similar to theorem 2 of Chung et al. (2001), we have following theorem.

Theorem 2

1. If $\Delta > 0$, then $T^* = T_2^*$.
2. If $\Delta < 0$, then $T^* = T_1^*$.
3. If $\Delta = 0$, then $T^* = T_1^* = T_2^* = M$.

Thus, we have proved that for $T \leq T_o$, single warehouse is advantageous i.e. the retailer should use *OW* and order $Q^* \leq W$.

3.2 Two warehouse inventory model $(T_o < T)$ (Fig 2).

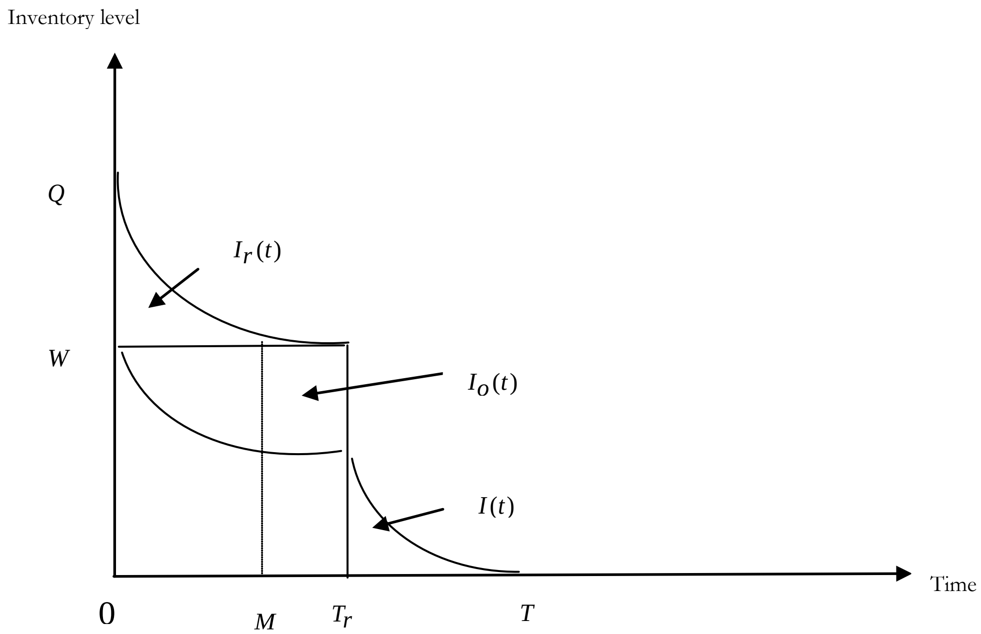

Figure 2. $T_o < T$

$Q > W$, two-warehouse facility should be used. Out of Q units received in the beginning of the cycle, W units are stocked in the *OW* and remaining $Q - W$ units are stored in the *RW*. During $(0, T_r)$ we first give away items from *RW* and then from *OW*. The rate of change of inventory level in *RW* during $(0, T_r)$ can be governed by the differential equation,

$$\frac{dI_r(t)}{dt} + \beta I_r(t) = -\alpha\,, \quad 0 \le t \le T_r \tag{11}$$

with boundary conditions $I_r(T_r) = 0$. The solution of equation (11) is

$$I_r(t) = \frac{\alpha}{\beta}\left[e^{\beta(T_r - t)} - 1\right]\,, \; 0 \le t \le T_r \tag{12}$$

During $(0, T_r)$ the rate of change of inventory level, $I_o(t)$ in *OW* is described by the differential equation,

$$\frac{dI_o(t)}{dt} = \beta I_o(t)\,, \quad 0 < t < T_r \tag{13}$$

with initial condition $I_o(0) = W$. The solution of equation (13) is

$$I_o(t) = We^{-\beta t}\,, \; 0 \le t \le T_r \tag{14}$$

During (T_r, T), the rate of change of inventory is governed by the differential equation,

$$\frac{dI(t)}{dt} + \beta I(t) = -\alpha\,, \; T_r \le t \le T \tag{15}$$

with boundary condition $I(T) = 0$. The solution of equation (16) is

$$I(t) = \frac{\alpha}{\beta}\left[e^{\beta(T - t)} - 1\right],\; T_r \le t \le T \tag{16}$$

The order quantity during the cycle time is

$$Q = I_r(0) + I_o(0) = \frac{\alpha}{\beta}\left[e^{\beta T_r} - 1\right] + W \tag{17}$$

Similar to section 3.1, the different cost components of the annual net profit per unit time are

1. Sales revenue $= \dfrac{(P-C)}{T}\left[\dfrac{\alpha}{\beta}\left[e^{\beta T_r} - 1\right] + W\right]$
2. Ordering cost $= \dfrac{A}{T}$
3. Holding cost in *RW*= $= \dfrac{h_r}{T}\int_0^{T_r} I_r(t)dt = \dfrac{h_r \alpha}{\beta^2 T}\left[e^{\beta T_r} - \beta T_r - 1\right]$

4. Holding cost in $OW = \frac{h_o}{T}\left[\int_0^{T_r} I_o(t)dt + \int_{T_r}^{T} I(t)dt\right] = \frac{h_o}{T}\left[\frac{W}{\beta}\left(1-e^{-\beta T_r}\right)+\frac{\alpha}{\beta^2}\left(e^{\beta(T-T_r)}-\beta(T-T_r)-1\right)\right]$

5. Interest earned is same as given in single ware house case for $M < T$.

6. Interest charged $= \frac{CI_c}{T}\left[\int_M^{T_r} I_r(t)dt + \int_M^{T_r} I_o(t)dt + \int_{T_r}^{T} I(t)dt\right]$

$$= \frac{CI_c}{T}\left[\frac{\alpha}{\beta^2}\left(e^{\beta(T_r-M)}-\beta(T_r-M)-1\right)+\frac{W}{\beta}\left(e^{-\beta M}-e^{-\beta T_r}\right)+\frac{\alpha}{\beta^2}\left(e^{\beta(T-T_r)}-\beta(T-T_r)-1\right)\right]$$

Hence, the annual net profit per unit time is,

$$Z_3(T) = (P-C)\frac{Q}{T} - \frac{A}{T} - \frac{h_r\alpha}{\beta^2 T}\left[e^{\beta T_r}-\beta T_r-1\right] - \frac{h_o}{T}\left[\frac{W}{\beta}\left(1-e^{-\beta T_r}\right)+\frac{\alpha}{\beta^2}\left(e^{\beta(T-T_r)}-\beta(T-T_r)-1\right)\right]$$
$$-\frac{CI_c}{T}\left[\frac{\alpha}{\beta^2}\left(e^{\beta(T_r-M)}-\beta(T_r-M)-1\right)+\frac{W}{\beta}\left(e^{-\beta M}-e^{-\beta T_r}\right)+\frac{\alpha}{\beta^2}\left(e^{\beta(T-T_r)}-\beta(T-T_r)-1\right)\right]$$
$$+\frac{PI_e\alpha}{T}\left[M^2+\frac{1}{\beta^2}\left(e^{\beta T}-(1+\beta M)e^{\beta(T-M)}\right)\right] \tag{18}$$

Using continuity, we have $I_o(T_r) = I(T_r)$, which gives T_r to be a function of T as $T_r = \frac{1}{\beta}\ln\left(\frac{\alpha e^{\beta T}-\beta W}{\alpha}\right)$ (19)

Also,

$$\frac{dT_r}{dT} = \frac{\alpha e^{\beta T}}{\alpha e^{\beta T}-\beta W} > 1 \tag{20}$$

Substituting value of T_r from equation (19) into equation (18), we obtain annual net profit per unit time to be a function of T.

The necessary condition for $Z_3(T)$ to be maximum is to set $Z_3^{'}(T)$ to be zero.

Thus, we obtain $\frac{dZ_3(T)}{dT} = \frac{1}{T^2} f_3(T)$ (21)

where,

$$f_3(T) = A + (P-C)\left[W+\alpha T e^{\beta T_r}\frac{dT_r}{dT}-\frac{\alpha}{\beta}(e^{\beta T_r}-1)\right] + \frac{h_r\alpha}{\beta^2}\left[e^{\beta T_r}-\beta T_r-1+\beta T(e^{\beta T_r}+1)\frac{dT_r}{dT}\right]$$
$$+h_o\left[\frac{W}{\beta}\left(1-e^{-\beta T_r}\right)+\frac{\alpha}{\beta^2}\left(e^{\beta(T-T_r)}-\beta(T-T_r)-1\right)-WTe^{-\beta T_r}\frac{dT_r}{dT}+\frac{\alpha T}{\beta}\left(e^{\beta(T-T_r)}-1\right)(1-\frac{dT_r}{dT})\right]$$
$$+CI_c h(T) - PI_e\alpha\left[M^2+\frac{1}{\beta^2}\left((1-\beta T)e^{\beta T}-(1-\beta T)e^{\beta(T-M)}-\beta M(1-\beta T)e^{\beta(T-M)}\right)\right] \tag{22}$$

where

$$h(T) = \frac{\alpha}{\beta^2}\left(e^{\beta(T-M)}-\beta(T_r-M)-1\right) - \beta T\left(e^{\beta(T-M)}-1\right)\frac{dT_r}{dT} + \frac{W}{\beta}\left(e^{-\beta M}-e^{-\beta T_r}\left(1+\beta T\frac{dT_r}{dT}\right)\right)$$
$$+\frac{\alpha}{\beta^2}\left(e^{\beta(T-T_r)}-\beta(T-T_r)-1-\beta T\left(e^{\beta(T-T_r)}-1\right)\left(1-\frac{dT_r}{dT}\right)\right)$$

Clearly, both $f_3(T)$ and $Z_3(T)$ have the same sign and domain. Let T_3^* if it exists be the solution of $f_3(T) = 0$. we claim following theorem.

Theorem 3

1. If $f_3(T_o) > 0, T_3^*$ is the unique cycle time which maximizes $Z_3(T)$ on $[T_o, \infty)$.
2. If $f_3(T_o) \le 0$, then $Z_3(T)$ is decreasing on $[T_o, \infty)$. So, $T_3^* = T_o$.

Proof :
1. Proof follows from Shah (1993) and Thomas and Finney (1996).
2. Proof is similar to that of Liao and Huang (2010).

4. OPTIMIZATION OF TWO-WAREHOUSE MODEL

The annual net profit per unit time is

$$Z(T)=\begin{cases}Z_1(T), & if\ 0<T\le M\\ Z_2(T), & if\ M<T\le T_o\\ Z_3(T), & if\ T_o<T\end{cases} \tag{23}$$

Also, at $T=T_o$ and $T_w=0$ the capacity of the warehouse is $W=\frac{\alpha}{\beta}\left(e^{\beta T_o}-1\right)$. $Z(T)$ is continuous function except at $T=T_o$.

We have, $$Z_1^{'}(M)=Z_2^{'}(M)=\frac{1}{M^2}\left[A+\frac{\alpha}{\beta^2}\left((1-\beta M)e^{\beta M}-1\right)\left(h_o-(P-C)\beta-PI_e\beta\right)+PI_e\alpha\left[M^2-\frac{M}{\beta}\left(e^{\beta M}-1\right)\right]\right] \tag{24}$$

$$Z_2^{'}(T_o)=\frac{1}{T_o^2}\left[A+\frac{\alpha}{\beta^2}\left(\left(1-\beta T_o\right)e^{\beta T_o}-1\right)\left(h_o-(P-C)\beta-PI_e\beta\right)\right]+\frac{1}{T_o^2}\left[PI_e\alpha\left(T_o^2+T_oe^{\beta T_o}(M-T_o)-\frac{M}{\beta}\left(e^{\beta T_o}-1\right)\right)\right]$$
$$-\frac{1}{T_o^2}\left[\frac{CI_c\alpha}{\beta^2}\left(\left(\beta T_o-1\right)e^{\beta(T_o-M)}-\beta M+1\right)\right] \tag{25}$$

and

$$Z_3^{'}(T_o)=\frac{1}{T_o^2}f_3(T_o). \tag{26}$$

For simplicity, let

$$f_2(M)=\left[A+\frac{\alpha}{\beta^2}\left((1-\beta M)e^{\beta M}-1\right)\left(h_o-(P-C)\beta-PI_e\beta\right)+PI_e\alpha\left[M^2-\frac{M}{\beta}\left(e^{\beta M}-1\right)\right]\right]$$

and

$$f_2(T_o)=A+\frac{\alpha}{\beta^2}\left(\left(1-\beta T_o\right)e^{\beta T_o}-1\right)\left(h_o-(P-C)\beta-PI_e\beta\right)$$
$$+PI_e\alpha\left(T_o^2+T_oe^{\beta T_o}(M-T_o)-\frac{M}{\beta}\left(e^{\beta T_o}-1\right)\right)-\frac{CI_c\alpha}{\beta^2}\left(\left(\beta T_o-1\right)e^{\beta(T_o-M)}-\beta M+1\right)$$

Since $Z_2(T)$ is concave on $[M,\infty)$, $Z_2^{'}(T)$ is decreasing on $[M,\infty)$ and $f_2(M)>f_2(T_o)$.

Also, $f_2(T_o)<f_3(T_o)$ and

$$f_2(M)>0 \text{ if and only if } T_1^*>M \tag{27}$$

$$f_2(M)>0 \text{ if and only if } T_2^*>M \tag{28}$$

$$f_2(T_o)>0 \text{ if and only if } T_2^*>T_o \tag{29}$$

$$f_3(T_o)>0 \text{ if and only if } T_3^*>T_o \tag{30}$$

So we have following decision making Table 1 for optimum cycle time.

Table 1. Optimum cycle time

	$f_2(M)$	$f_2(T_o)$	Optimal Cycle time (Whichever gives maximum profit)
$f_3(T_o)>0$.	≤ 0	≤ 0	T_1^* or T_3^*
	>0	≤ 0	T_2^* or T_3^*
	>0	>0	T_3^*

	$f_2(M)$	$f_2(T_o)$	Optimal Cycle time (Whichever gives maximum profit)
$f_3(T_o)\le 0$	≤ 0	≤ 0	T_1^* or T_o
	>0	≤ 0	T_2^* or T_o

In next section, we discuss two examples to illustrate the mathematical formulation to decide the choice of rented warehouse.

5. NUMERICAL RESULTS

Example1

Let $\alpha = 400$ units/year, $\beta = 2\%, h_0 = \$0.2$ unit/year, $h_r = \$0.5$ unit/year, $M = 0.1$ year, $P = \$20$ /unit/year, $C = \$5$ /unit/year, $I_c = 15\%, I_e = 12\%, W = 300$ units/year. Then $f_3(T_o) = 71.30 > 0$. The optimal solution is given in Table 2 for different values of ordering cost, A.

Table 2. Optimal solution for Example 1

A	T_o	$f_2(M)$	$f_2(T_o)$	T^*	Q^*	$Z(T^*)$
1	0.744	< 0	< 0	$T_1^* = 0.053$	20.53	6057
15	0.744	> 0	< 0	$T_2^* = 0.063$	25.22	6122
225	0.744	> 0	> 0	$T_3^* = 0.319$	128.24	5394

The observations are

(a) Optimal cycle time and purchase quantity are very sensitive to ordering cost. The annual profit per unit time decreases with increase in ordering cost. It suggests that the retailer to stock more items in a larger cycle time.

(b) For $A = 1$ and $A = 15$, the order quantity is lower than the storage capacity W. so the retailer should not go for rented warehouse. In other words, the lower ordering cost is advantageous for the retailer to go with owned warehouse.

Example 2

Let $\alpha = 60$ units/year, $\beta = 2\%, h_o = \$4$ /unit/year, $h_r = \$5$ /unit/year, $M = 0.1$ year, $P = \$5$ /unit/year, $C = \$0.5$ /unit/year, $I_c = 14\%, I_e = 12\%, W = 15$ unit/year. Then $f_3(T_o) = -112.05 \le 0$, Using table 1, we have optimal solution in Table 3 for different values of A.

Table 3. Optimal solution for Example 2

A	T_o	$f_2(M)$	$f_2(T_o)$	T^*	Q^*	$Z(T^*)$
1	0.2494	< 0	< 0	0.092	5.54	251.94
4	0.2494	> 0	> 0	0.171	10.25	229.6

Observations are similar to those stated for Example 1.

Using data of example 1, we carry out sensitivity analysis to find out critical inventory parameters which forces the retailer to opt for rented warehouse also. Fig.3, Fig.4 and Fig.5 shows variation in optimum purchase quantity, total annual profit per unit time and optimal cycle time respectively. It is observed that optimal cycle time is inversely sensitive to scale demand, marginally sensitive to purchase cost and directly related to warehouse's display capacity and deterioration rates of units in warehouses. The total profit is hindered by unit purchase cost while escalates by the scale demand. The equilibrium is to be maintained by the retailer in these critical parameters.

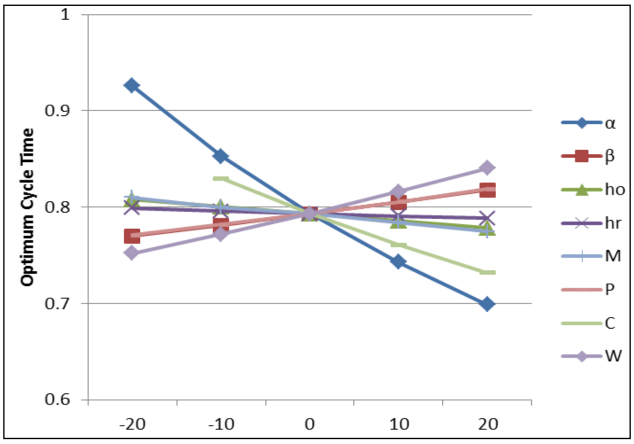

Figure 3. Percentage change in inventory parameters

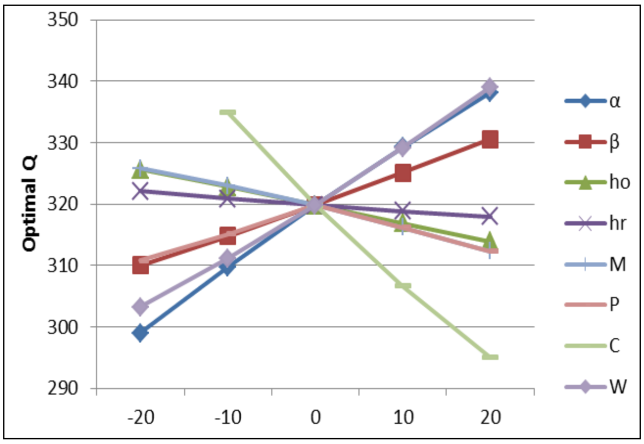

Figure 4. Percentage change in inventory parameters

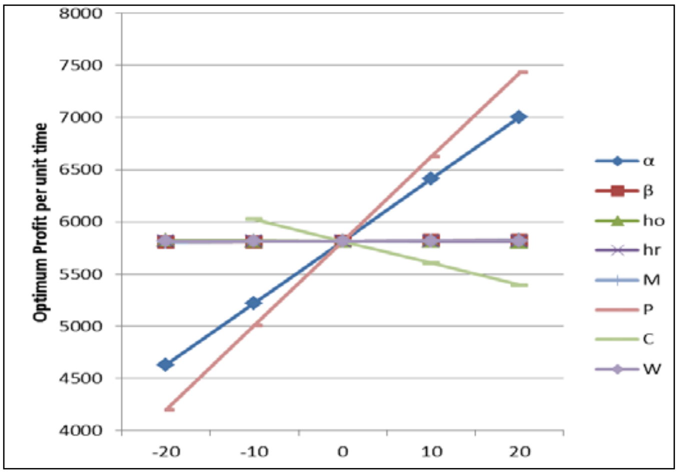

Figure 5. Percentage change in inventory parameters

6. CONCLUSION

This study aims to analyze the need of rented warehouse when own warehouse has limited storage capacity and demand is stock dependent. The retailers are attracted by the policy of delay payment and goes for larger order. The decision policy is suggested to the retailer to use the rented warehouse to maximize profit. The numerical examples and sensitivity analysis will help the decision maker to take the favorable decision. The future study should consider time-dependent shortages, time-dependent deterioration of units, credit linked to order quantity, etc.

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REFERENCES

1. Benkherouf, L. (1997). A deterioration order inventory model for deteriorating items with two storage facilities. *International Journal of Production Economics*, 48:167-175.
2. Chung,K.J., Chang, S.L and Yang, W.D. (2001). The optimal cycle time for exponentially deteriorating products under trade credit financing. *The Engineering Economist*, 46: 232-242.
3. Goswami,A. and Caudhari, K.S. (1982). An economic order quantity model for items with two levels of storage for a linear trend in demand. *Journal of the Operational Research Society*, 43: 157-167.
4. Goyal,S. K. (1985). Economic order quantity under conditions of permissible delay in payments. *Journal of the Operational Research Society*, 36: 335-338.

5. Hartely,V.R. (1976). *Operations Research – A Managerial Emphasis*. Santa Monica, CA.
6. Hwang, H. and Shinn, S.W. (1997). Retailer's pricing and lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payments. *Computer and Operations Research*, 24: 539-547.
7. Ishii, H. and Nose, T. (1996). Perishable inventory control with two types of customers and different selling prices under the warehouse capacity constraints. *International Journal of Production Economics*, 44: 167-176.
8. Lee, C.C. (2006). Two warehouse inventory model with deterioration under FIFO dispatching policy. *European Journal of Operational Research*, 174: 861-873.
9. Levin, R.I., Mclaughlin, C.P., Lamone, R.P. and Kottas, J.F. (1972). Productions/Operations Management Contemporary Policy for Managing Operating Systems, Macgraw-Hill, New York.
10. Liao, J.J. and Huang, K.N. (2010). Deterministic inventory model for deteriorating items with trade credit financing and capacity constraints. *Computers and Industrial Engineering*, 59: 611-618.
11. Pakkala, T.P.M and Acharya, K.K. (1992a). A deterministic inventory model for deteriorating items with two warehouses and finite replenishment rate. *European Journal of Operational Research*, 57: 71-76.
12. Pakkala, T.P.M and Acharya, K. K. (1992b). Discrete time inventory model for deteriorating items with two warehouses. *OPSEARCH*, 29: 90-103.
13. Sarma, K.V.S. (1987). A deterministic inventory model with two level of storage and an optimum release rate. *OPSEARCH*, 20: 175-180.
14. Sarma K.V.S (1987). A deterministic order level inventory model for deteriorating items with two storage facilities. *European Journal of Operational Research*, 29: 70-73.
15. Shah, N.H. (1993). A lot size model for exponentially decaying inventory when delay in payments is permissible. *Cahiers du CERO*, 35: 115-123.
16. Shah, N.H., Soni, H. and Jaggi, C.K. (2010). Inventory models and trade credit: Review. *Control and Cybernetics*, 39(3): 867-884.
17. Shah, N.H. and Shah, A.D. (2012). Optimal ordering transfer policy for deteriorating inventory items with fuzzy stock-dependent demand. *Mexican Journal of Operation Research*, 1: 29-44.
18. Thomas, C.B. and Finney, R.L (1996). Calculus with analytic geometry (9thed) Addison Wesley Publishing Company, Inc.
19. Urban, T.L. (2005). Inventory models with inventory level dependent demand. A comprehensive review and unifying theory. *European Journal of Operational Research*, 164(3): 792-304.
20. Yang, H.L. (2004). Two-warehouse inventory models for deteriorating items with shortages under inflation, *European Journal of Operational Research*, 157: 344-356.
21. Yang, H.L. (2006). Two-warehouse partial backlogging inventory models for deteriorating items under inflation. *International Journal of Production Economics*, 103: 362-370.
22. Zhou, Y.W and Yang, S.L. (2005). A two-warehouse inventory model for items with stock level dependent demand rate. *International Journal of Production Economics*, 95: 215-228.
23. Zhou,Y.W and Zhong Y. and Li, J. (2012). An uncooperative order model for items with trade credit, inventory-dependent demand and limited displayed-shelf space. *European Journal of Operational Research*, 223: 76-85.